JUGULAR TRIG
BY
HEART

(Timeless Trigonometry for Calculus)

BY

DR. J. AUSTIN FRENCH
GEORGETOWN COLLEGE

R.E.A.L. PUBLICATIONS
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R.E.A.L. =
Righteousness Enhanced Accelerated Learning

"For He will finish the work and cut it short in righteousness; because a short work will the Lord make upon the earth." Romans 9:28

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Published by R.E.A.L. Publications
February 2009

DEDICATED TO:

THE GLORY OF GOD'S GRACE

MY WIFE, BELINDA

MIGHTY MICROPEDIAS BY R.E.A.L. EDUCATION

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TRUTH GEMS FOR TEACHER AND STUDENT by Dr. J. Austin French. This micropedia consist of 53 Truth Gems from the Word of God directed at teaching and learning. Each Truth Gem and its explanation take one page. Since God is the Most High, this means His teachings are the most high teachings. No one knows better than the Creator how man was made, what he needs, what is the best way to teach man, and what is the best way for man to learn. Many of these truth gems start out each teaching session in the Math by Heart trilogy described below.

ALGEBRA II BY HEART by Dr. J. Austin French. This collection of two mighty micropedias consists of a micropedia text, a micropedia of supplementary materials, and DVDs of 53 teaching sessions. Both micropedias are free on the web or can be ordered on the web in paper copy or on one CD. The DVDs can be purchased from the web. This is a College Algebra course, which means it is a strong Algebra II course for high school. This is not what is called Intermediate Algebra (=Algebra I in high school) in some colleges.

ALGEBRA HEART-TEST QUESTIONS by Dr. J. Austin French. This is a collection of 874 multiple choice questions with answers. This tests the entire book Algebra II by Heart. The questions come exactly from the text. Each question has a reference to the location on the specific page of the text that the question covers.

CALCULUS I BY HEART by Dr. J. Austin French. This collection of two mighty micropedias consists of a micropedia text, a micropedia of supplementary materials, and DVDs of 38 teaching sessions. This is a rigorous first course in calculus. It is a first college calculus course. It can be used for high school students who have finished Algebra I, Algebra II, and have had some trigonometry (trigonometry is taught in pre-calculus or advanced math courses in high school). The topic is differential calculus. Both micropedias are free on the web or can be ordered on the web in paper copy or on one CD. The DVDs can be purchased from the web.

LOGIC FOR UNDERSTANDING MATHEMATICS by Dr. J. Austin French and Dr. Earl Dennis. This collection of two mighty micropedias consists of a micropedia text, a micropedia of supplementary materials, and DVDs of 31 detailed teaching sessions. The mystery of how to do proofs is revealed. Logic is taught and then that connection to math proof is made plain. Proofs are illustrated in the area of elementary set theory. It is for the advanced high school student through college. Math maturity to have done excellently in Algebra II is the only recommended prerequisite background. Both micropedias are free on the web or can be ordered on the web in paper copy or on one CD. The DVDs can be purchased from the web.

JUGULAR TRIG BY HEART (Timeless Trigonometry for Calculus) by. Dr. J. Austin French. This is a micro micropedia that contains just the trigonometry needed so that one is not hindered from making an A in Calculus because of lack of trig knowledge. Trig functions, trig identities, trig graphs, and inverse trig functions are covered.

WHY PENCIL AND BIG PRINT?

To "make it plain" (Hab 2:2). It is my desire to make the things taught to be easily mentally digestible. There are some wonderful meals fixed with love for me by my wife that are so blessed and digestible that I joke that the stomach can be by-passed and the food just be put into me intravenously! This book is intended to be like that for the mind...immediately absorbed by the mind.

This all began when I was teaching a class with computer generated notes. I then switched to pencil and big print. The response was unanimous; they liked the pencil and big print notes much better. It was said that when they did their homework, they had to recopy the computer generated notes to understand better, but with the pencil and big print notes they did not have to recopy them to understand.

A secondary reason for pencil and big print is that many texts are **encyclopedic**...containing far more information than can and needs to be consumed to know the mathematics excellently. So I go for the jugular and put in no more and no less than is needed to thrive mathematically, hence, a **micropedia**, not an encyclopedia!

You are seeing the note-taking style that served me well in getting a math Ph.D. and beyond.

Another reason I use pencil and big print is that I believe there is an anointing of clarity that comes with these notes and it is known that the "anointing teaches you" (1 Jn 2:27).

Rather than this being a second rate, antiquated learning system, I am giving you absolutely the best I know for you to learn with wisdom and joy. Drink it in.

Austin French

TABLE OF CONTENTS

TRUTH GEM

BE IN THE WILL OF GOD FOR WHAT YOU DO

A. ROM 15:32 ... that I may come to you with joy by the WILL OF GOD, and may be refreshed together with you.

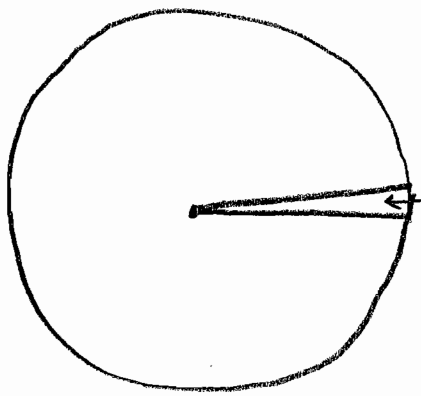
B. I come to you in the WILL OF GOD with joy and we will have refreshing math.

C. Being in the WILL OF GOD taking this course and faithfully, wisely studying you will flourish.

D. HEB. 10:36 For you have need of endurance, so that after you have done the WILL OF GOD you may receive the promise.

DEGREE MEASURE FOR ANGLES

A.

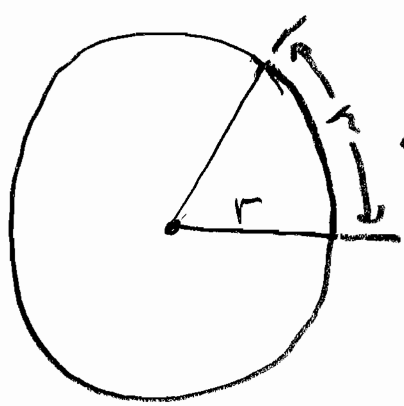


1 DEGREE = 1°

B. 360 OF THE PIE-SHAPED SLICES FILL THE CIRCLE

RADIAN MEASURE FOR ANGLES

A.



1 RADIAN = 1 rad

B. SINCE THE CIRCUMFERENCE IS $2\pi r$ ($\pi \approx 3.14$) THEN 2π OF THE PIE-SHAPED SLICES FILL THE CIRCLE (APPROX 6.28)

CONVERSION BETWEEN RADIANS AND DEGREES

A. $2\pi \text{ rad} = 360^\circ$

B. $\pi \text{ rad} = 180^\circ$ ← IMPORTANT

C. CONVERT 37° TO RADIANS

$1^\circ = \frac{\pi}{180} \text{ rad}$ DIVIDE BY 180

$37^\circ = \frac{37\pi}{180} \text{ rad}$ MULTIPLY BY 37

D. CONVERT RADIANS TO DEGREES.

CONVERT $\frac{\pi}{3}$ rad TO DEGREES

$\pi \text{ rad} = 180^\circ$

DIVIDE BY 3

$\frac{\pi}{3} \text{ rad} = \frac{180^\circ}{3} = 60^\circ$

CONVERT 2 rad TO DEGREES

$\pi \text{ rad} = 180^\circ$

$1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$

$2 \text{ rad} = \frac{2(180)^\circ}{\pi} = \frac{360}{\pi}^\circ$

E. FREQUENTLY USED CONVERSIONS

$\frac{\pi}{3}$ rad $= 60°$ $\frac{\pi}{4}$ rad $= 45°$ } KNOW

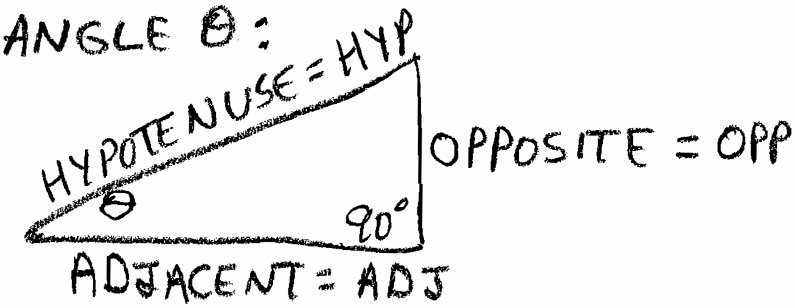
$\frac{\pi}{6}$ rad $= 30°$ $\frac{\pi}{2}$ rad $= 90°$ } THESE

$\frac{2\pi}{3}$ rad $= 120°$ $\frac{3\pi}{2}$ rad $= 270°$

WHEN DEGREE MEASURE IS MEANT THE "°" SYMBOL WILL APPEAR. WHEN RADIAN MEASURE IS MEANT IT IS LEFT OFF.

RIGHT TRIANGLE TRIG

A. FOR ANGLE θ:



B. TRIG FUNCTION DEFINITIONS

$\text{sine}(\theta) = \sin\theta = \frac{\text{OPP}}{\text{HYP}}$

$\text{cosine}(\theta) = \cos\theta = \frac{\text{ADJ}}{\text{HYP}}$

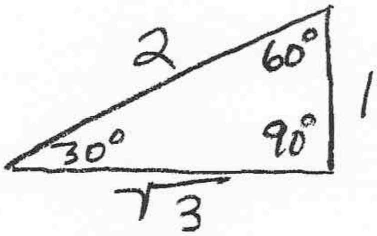
$\text{tangent}(\theta) = \tan\theta = \frac{\text{OPP}}{\text{ADJ}} = \frac{\sin\theta}{\cos\theta}$

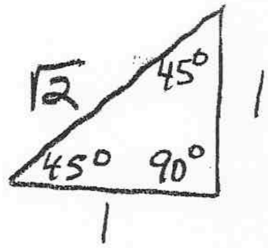
$\text{cosecant}(\theta) = \csc\theta = \frac{\text{HYP}}{\text{OPP}} = \frac{1}{\sin\theta}$

$\text{secant}(\theta) = \frac{\text{HYP}}{\text{ADJ}} = \frac{1}{\cos\theta} = \sec\theta$

$\text{cotangent}(\theta) = \cot\theta = \frac{\text{ADJ}}{\text{OPP}} = \frac{\cos\theta}{\sin\theta}$

C. FAMOUS TRIANGLES





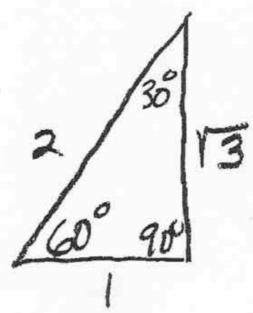
D. EVALUATING 30°, 60°, 45°:

$\sin 30° = \sin \frac{\pi}{6} = \frac{1}{2}$ $\quad \frac{OPP}{HYP}$

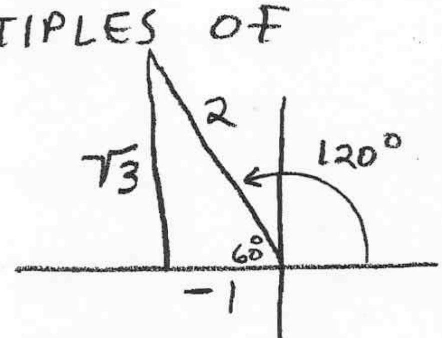
$\csc 45° = \csc \frac{\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$ $\quad \frac{HYP}{OPP}$

$\cot 60° = \cot \frac{\pi}{3}$

$= \frac{1}{\sqrt{3}}$ $\quad \frac{ADJ}{OPP}$



E. EVALUATING SOME MULTIPLES OF 30°, 60°, 45°:



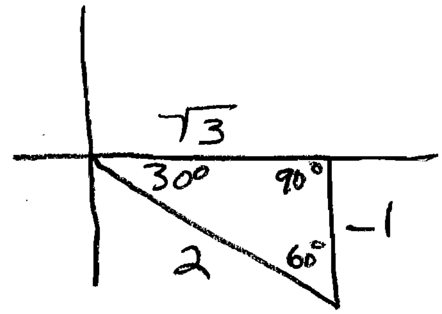
$\cos 120° = \cos \frac{2\pi}{3}$

$= -\frac{1}{2}$ $\quad \frac{ADJ}{HYP}$

ALWAYS CONSIDER THE HYPOTENUSE POSITIVE

$\tan -30^\circ = \tan -\frac{\pi}{6}$

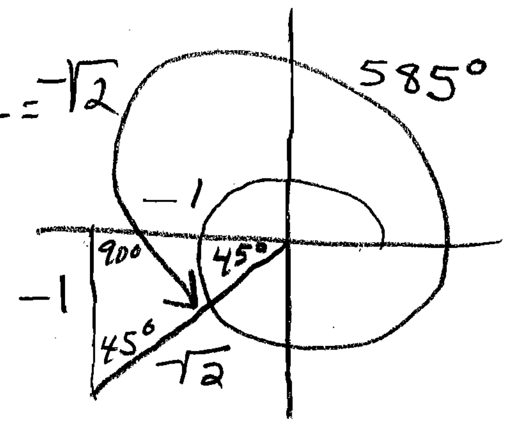
$= \frac{-1}{\sqrt{3}}$ $\quad \frac{\text{OPP}}{\text{ADJ}}$



$\sec 585^\circ = \sec \frac{13\pi}{4} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

HYP/ADJ

REMEMBER: THE HYPOTENUSE IS ALWAYS POSITIVE.



HOMEWORK

A. CONVERT TO RADIANS

$135^\circ \quad -270^\circ \quad 570^\circ \quad -23^\circ$

B. CONVERT TO DEGREES

$\frac{3\pi}{4}$ rad $\quad \frac{-7\pi}{6}$ rad $\quad \frac{8\pi}{3}$ rad $\quad -5$ rad

C. EVALUATE

$\sin 60^\circ \quad \cot 45^\circ \quad \tan \frac{5\pi}{6}$ rad

$\cos -120^\circ \quad \sec -\frac{7\pi}{6}$ rad $\quad \csc \frac{8\pi}{3}$ rad

ALERT:

SOME HOMEWORK PROBLEMS JUST ASSIGNED ARE WORKED ON THE NEXT PAGES. IT IS RECOMMENDED THAT YOU DO YOUR HOMEWORK BEFORE READING ON.

WORKED HOMEWORK

A. CONVERT TO RADIANS

P. 5

$570°$ ←

$\pi \text{ rad} = 180°$

$\frac{\pi}{180} \text{ rad} = 1°$

$\frac{570\pi}{180} \text{ rad} = 570°$

$\frac{19}{6}\pi \text{ rad} = 570°$

$-23°$

$\pi \text{ rad} = 180°$

$\frac{\pi}{180} \text{ rad} = 1°$

$-\frac{23\pi}{180} \text{ rad} = -23°$

B. CONVERT TO DEGREES

P. 5

$-\frac{7\pi}{6} \text{ rad}$

$\pi \text{ rad} = 180°$

$1 \text{ rad} = \frac{180}{\pi}°$

$-\frac{7\pi}{6} \text{ rad} = -\frac{7\pi}{6}\left(\frac{180}{\pi}\right)° = -7(30°) = -210°$

WORKED HOMEWORK (CONT.)

B. (cont.) CONVERT TO DEGREES

p.5

-5 rad

$\pi \text{ rad} = 180^\circ$

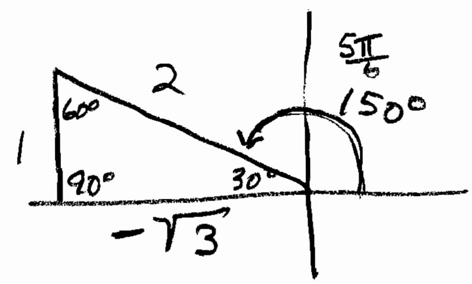
$1 \text{ rad} = \dfrac{180}{\pi}^\circ$

$-5 \text{ rad} = -\dfrac{5(180)}{\pi}^\circ = \dfrac{-900}{\pi}^\circ$

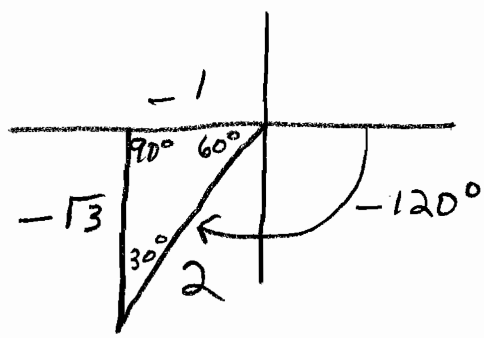
C. EVALUATE

p.5

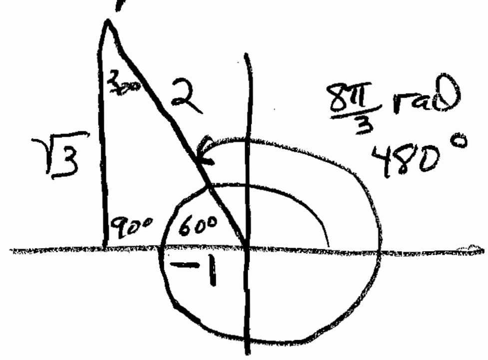
$\tan \frac{5\pi}{6} = \dfrac{1}{-\sqrt{3}} \quad \dfrac{\text{OPP}}{\text{ADJ}}$



$\cos -120^\circ = \dfrac{-1}{2} \quad \dfrac{\text{ADJ}}{\text{HYP}}$



$\csc \frac{8\pi}{3} = \dfrac{2}{\sqrt{3}} \quad \dfrac{\text{HYP}}{\text{OPP}}$



TRUTH GEM

WISDOM

A. PR 1:7 WISDOM IS THE PRINCIPAL THING; THEREFORE GET WISDOM. AND IN ALL YOUR GETTING, GET UNDERSTANDING.

B. <u>DEFINITIONS</u>:

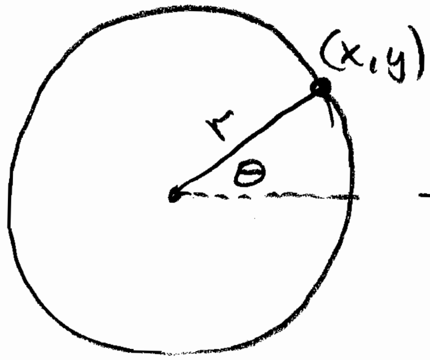
1. <u>KNOWLEDGE</u>: FACTS, GAINED INFORMATION
2. <u>UNDERSTANDING</u>: WHY A FACT IS A FACT.
3. <u>WISDOM</u>: BEING LED BY THE SPIRIT. KNOWING WHAT TO DO AT ANY MOMENT.

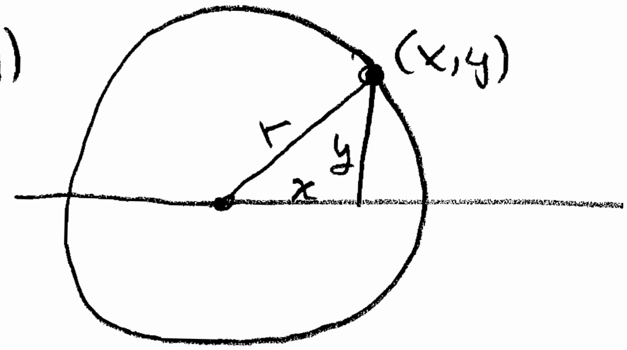
C. PRAY FOR WISDOM <u>IN</u> <u>FAITH</u>; JAMES 1:5

IF ANY OF YOU LACKS WISDOM, LET HIM ASK OF GOD, WHO GIVES TO ALL LIBERALLY AND WITHOUT REPROACH, AND IT WILL BE GIVEN HIM.

TRIG FUNCTIONS DEFINED BY USING CIRCLES

A.





$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x}$

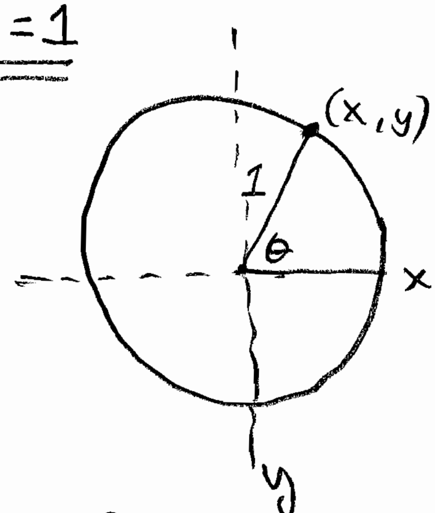
$\tan\theta = \frac{y}{x}$ $\qquad$ $\cot\theta = \frac{x}{y}$

B. NOTE FOR UNIT CIRCLE $\underline{\underline{r=1}}$

$\sin\theta = y$ $\qquad$ $\csc\theta = \frac{1}{y}$

$\cos\theta = x$ $\qquad$ $\sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x}$ $\qquad$ $\cot\theta = \frac{x}{y}$



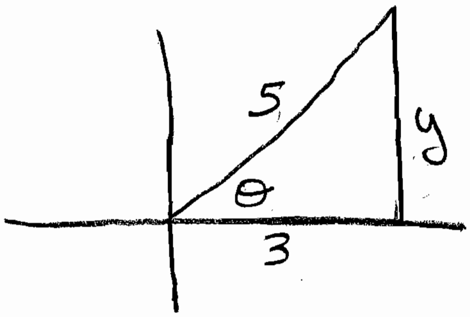
SINCE $x^2+y^2=1$,

$\cos^2\theta + \sin^2\theta = 1$ $\qquad$ $x^2+y^2=1$

$\cos^2\theta = [\cos\theta]^2$ $\qquad$ $\sin^2\theta = [\sin\theta]^2$

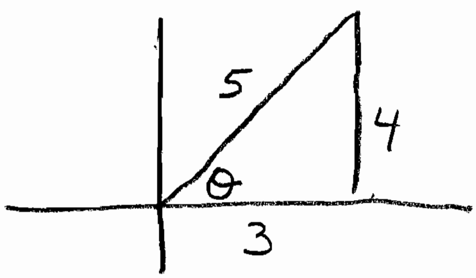
PROBLEMS FINDING TRIG FUNCTION VALUES

A. FOR $\cos\theta = \frac{3}{5}$ AND $0 < \theta < \frac{\pi}{2}$, FIND THE OTHER 5 TRIG FUNCTION VALUES.



BY PYTHAGOREAN THEOREM

$$5^2 = 3^2 + y^2$$
$$25 = 9 + y^2$$
$$16 = y^2$$
$$\sqrt{16} = \sqrt{y^2} = |y|$$
$$4 = y \text{ SINCE } y > 0^{*}$$



$\cos\theta = \frac{3}{5} \quad \frac{\text{ADJ}}{\text{HYP}}$

$\sec\theta = \frac{5}{3} \quad \frac{\text{HYP}}{\text{ADJ}}$

$\tan\theta = \frac{4}{3} \quad \frac{\text{OPP}}{\text{ADJ}}$

$\cot\theta = \frac{3}{4} \quad \frac{\text{ADJ}}{\text{OPP}}$

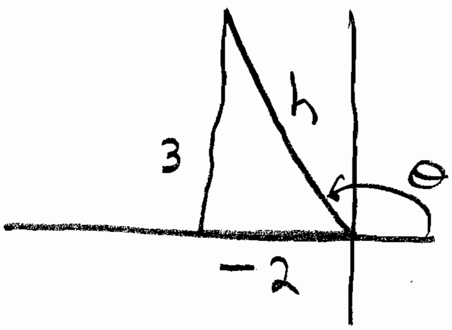
$\csc\theta = \frac{5}{4} \quad \frac{\text{HYP}}{\text{OPP}}$

$\sin\theta = \frac{4}{5} \quad \frac{\text{OPP}}{\text{HYP}}$

* SOME OTHER PROBLEMS $y < 0$ SO

$$\sqrt{y^2} = |y| = -y$$

B. FOR $\cot\theta = \frac{-2}{3}$ AND $\frac{\pi}{2} < \theta < \pi$, FIND THE OTHER 5 TRIG FUNCTION VALUES



BY PYTHAGOREAN THEOREM

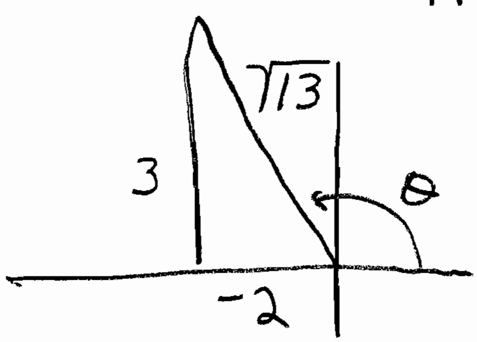
$$h^2 = 2^2 + 3^2$$

$$h^2 = 4 + 9$$

$$h^2 = 13$$

$$|h| = \sqrt{h^2} = \sqrt{13}$$

$h = \sqrt{13}$ SINCE $h > 0$



$\sin\theta = \frac{3}{\sqrt{13}}$ $\frac{OPP}{HYP}$

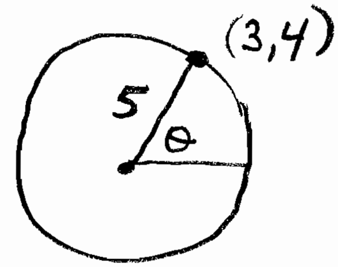
$\csc\theta = \frac{\sqrt{13}}{3}$ $\frac{HYP}{OPP}$

$\cos\theta = \frac{-2}{\sqrt{13}}$ $\frac{ADJ}{HYP}$

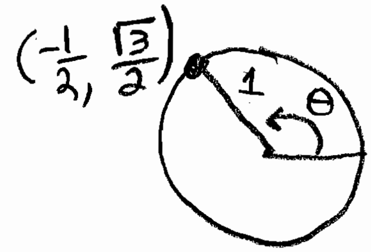
$\sec\theta = \frac{\sqrt{13}}{-2}$ $\frac{HYP}{ADJ}$

$\tan\theta = \frac{3}{-2}$ $\frac{OPP}{ADJ}$

HOMEWORK



A. THE POINT $(3,4)$ IS ON THE CIRCLE WITH RADIUS 5. FIND $\cos\theta$ AND $\csc\theta$.



B. THE POINT $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ IS ON THE UNIT CIRCLE.

FIND $\sin\theta$ AND $\cot\theta$.

C. FOR $0 < \theta < \frac{\pi}{2}$ AND $\tan\theta = \frac{5}{6}$, FIND THE OTHER 5 TRIG FUNCTION VALUES.

D. FOR $\pi < 0 < \frac{3\pi}{2}$ AND $\csc\theta = \frac{4}{-3}$, FIND THE OTHER 5 TRIG FUNCTION VALUES.

ALERT:

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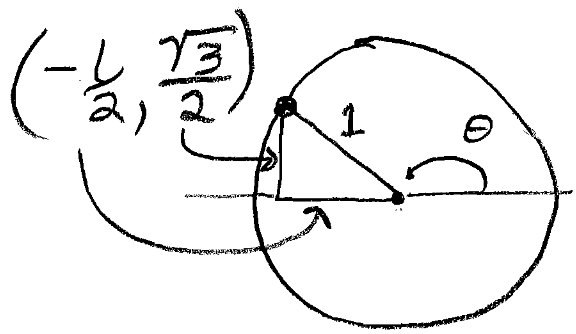
WORKED HOMEWORK

$\frac{B}{p.12}$ THE POINT $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ IS ON THE UNIT CIRCLE. FIND $\sin\theta$ AND $\cot\theta$

$$\sin\theta = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$\frac{\text{OPP}}{\text{HYP}}$



$$\cot\theta = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$$

$\frac{\text{ADJ}}{\text{OPP}}$

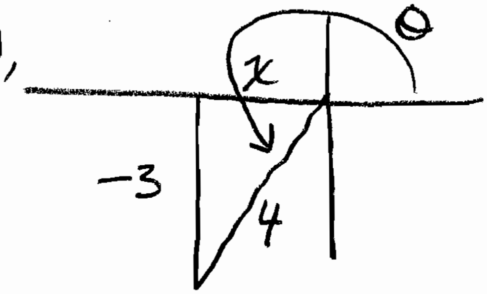
OR RECALL FOR THE <u>UNIT</u> CIRCLE

$$\sin\theta = y = \frac{\sqrt{3}}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$$

WORKED HOMEWORK (CONT.)

D p. 12 FOR $\pi < \theta < \frac{3\pi}{2}$ AND $\csc\theta = \frac{4}{-3}$, FIND THE OTHER 5 TRIG FUNCTION VALUES BY PYTHAGOREAN THEOREM,



$$x^2 + 3^2 = 4^2$$

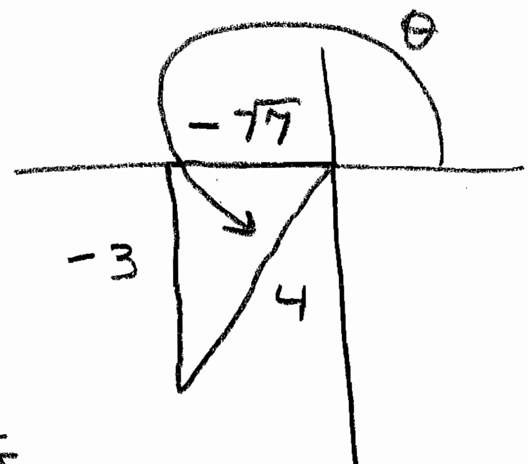
$$x^2 + 9 = 16$$

$$x^2 = 7$$

$$|x| = \sqrt{x^2} = \sqrt{7}$$

$-x = \sqrt{7}$ SINCE $x < 0$, $|x| = -x$

$$x = -\sqrt{7}$$



$\sin\theta = \frac{-3}{4}$ $\frac{OPP}{HYP}$

$\cos\theta = \frac{-\sqrt{7}}{4}$ $\frac{ADJ}{HYP}$

$\tan\theta = \frac{-3}{-\sqrt{7}} = \frac{3}{\sqrt{7}}$ $\frac{OPP}{ADJ}$

$\cot\theta = \frac{-\sqrt{7}}{-3} = \frac{\sqrt{7}}{3}$ $\frac{ADJ}{OPP}$

$\sec\theta = \frac{4}{-\sqrt{7}}$ $\frac{HYP}{ADJ}$.

TRUTH GEM

MEDITATE

A. PS 1:2 BUT HIS DELIGHT IS IN THE LAW OF THE LORD AND IN HIS LAW HE MEDITATES* DAY AND NIGHT.

* PONDERS BY TALKING TO HIMSELF.

B. WHAT YOU ARE IN THE WILL OF GOD TO LEARN CAN BE MEDITATED.

C. PICK A DEFINITION, THEOREM, OR STEPS OF A PROBLEM DERIVATION.

1. SLOWLY STUDY IT WORD FOR WORD SEEKING UNDERSTANDING.
2. SPEAK IT OUT SO YOU CAN HEAR IT.
3. WRITE IT DOWN OVER AND OVER.
4. REVIEW IT.

TRIG IDENTITIES

A. $\boxed{\sin^2\theta + \cos^2\theta = 1}$

THIS WAS SHOWN IN B, PAGE 9

<u>IDENTITY</u>: TRUE FOR ALL VALUES OF θ FOR WHICH IT IS DEFINED

$$\left.\begin{aligned} \sin^2 37^\circ + \cos^2 37^\circ &= 1 \\ \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{6} &= 1 \end{aligned}\right\}$$ WORKS FOR ANY θ

B. $\boxed{1 + \tan^2\theta = \sec^2\theta}$

<u>DERIVATION</u>:

$$\sin^2\theta + \cos^2\theta = 1$$ DIVIDE BY $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$$

$$(\tan\theta)^2 + 1 = (\sec\theta)^2$$

$$1 + \tan^2\theta = \sec^2\theta$$

C. $1 + \cot^2\theta = \csc^2\theta$

PROVE FOR HOMEWORK USING PREVIOUS IDENTITIES.

D. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

GIVEN WITHOUT PROOF

E. $\sin 2\theta = 2\sin\theta\cos\theta$

PROOF:

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

Let $\theta = \alpha = \beta$

$\sin(\theta+\theta) = \sin\theta\cos\theta + \sin\theta\cos\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

F. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

GIVEN WITHOUT PROOF.

G. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

PROVE FOR HOMEWORK USING PREVIOUS IDENTITIES.

H. $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

PROOF: $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos 2\theta \overset{G}{=} \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$$

$$\cos 2\theta = (\cos^2\theta) - 1 + \cos^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$1 + \cos 2\theta = 2\cos^2\theta$$

$$\frac{1}{2} + \frac{1}{2}\cos 2\theta = \cos^2\theta$$

I. $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

PROVE FOR HOMEWORK USING PREVIOUS IDENTITIES

ALERT:

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WORKED HOMEWORK

$\frac{C}{p.17}$ $1 + \cot^2\theta = \csc^2\theta$ ← PROVE

HINT: $\sin^2\theta + \cos^2\theta = 1$

DIVIDE BY $\sin^2\theta$.

$\frac{G}{p.18}$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ ← PROVE

HINT:

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

LET $\theta = \alpha = \beta$

$\frac{I}{p.18}$ $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ ← PROVE

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ ← G, p.18

$\cos 2\theta = (1 - \sin^2\theta) - \sin^2\theta$ ← SINCE $\sin^2\theta + \cos^2\theta = 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

TRUTH GEM

DO NOT BE COMFORTABLE WITH THE FAMILIAR THAT IS NOT GOOD.

A. FROM COL. 3:9-10 "... PUT OFF THE OLD MAN WITH HIS DEEDS AND... PUT ON THE NEW MAN."

B. THERE COMES A TIME WHEN YOU NO LONGER ACCEPT AS YOU BEHAVIOR THAT IS NOT GOOD THAT IS VERY FAMILIAR TO YOU

C. IT IS A NEW DAY AND IF THINGS IN THE PAST THAT MAY HAVE BEEN FAMILIAR TO YOU START CREEPING IN, THINGS LIKE

1. OVERCOMMITTING YOURSELF
2. PROCRASTINATION
3. LACK OF DISCIPLINE
4. IRRESPONSIBILITY IN STUDY HABITS
5. UNWISE STUDY
6. BELITTLING YOURSELF

⋮

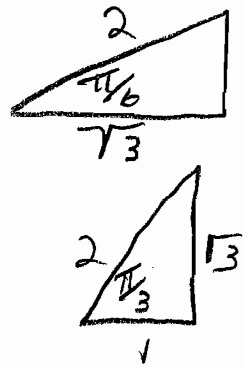
THEN WITH FIRE SAY "OLD THINGS HAVE PASSED AWAY" 2 COR 5:17. IT IS A NEW DAY. I HAVE PUT ON THE NEW MAN.

TRIG GRAPHS

A. THE GRAPH OF $y = \sin\theta$

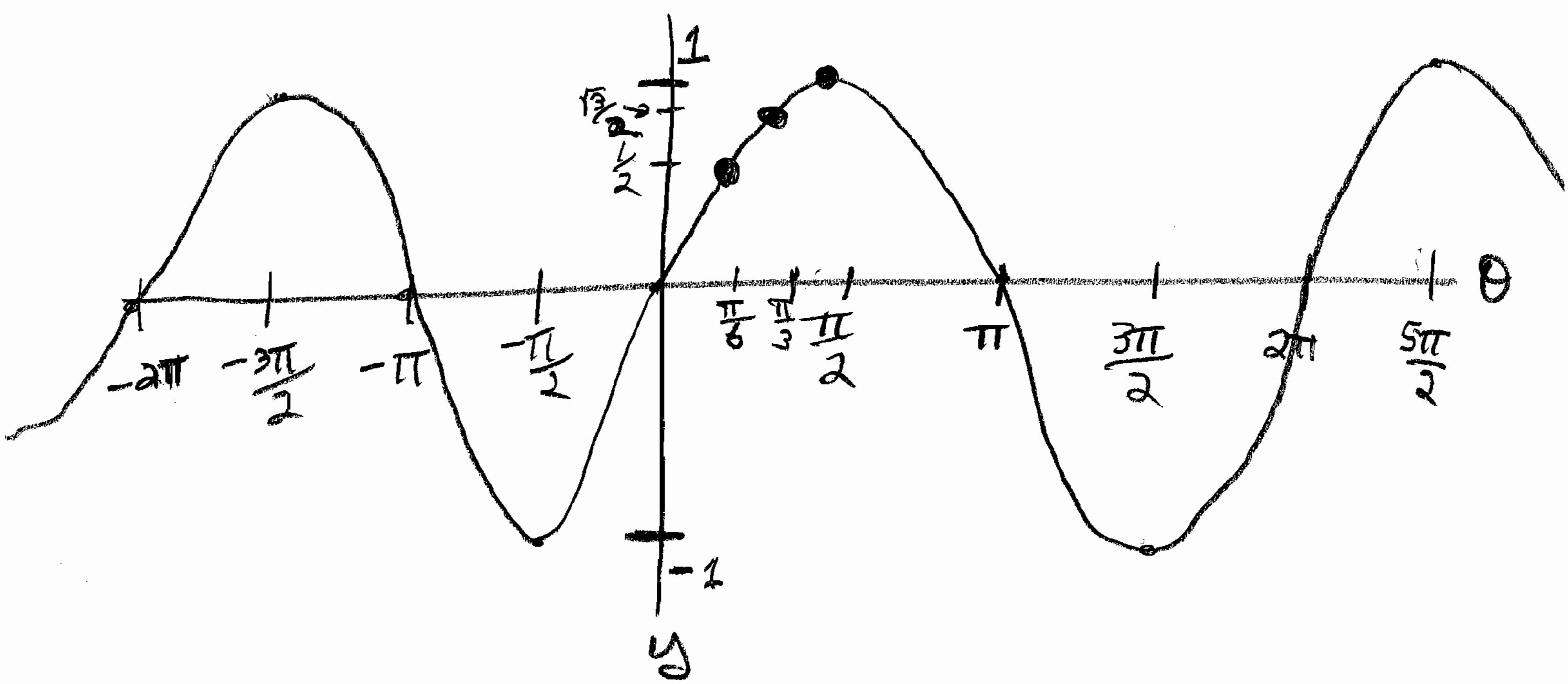
$\sin\frac{\pi}{6} = \frac{1}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$



θ	y
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$

YOU COULD PLOT THESE POINTS AND MANY OTHERS AND SEE THE GRAPH OF $y = \sin\theta$ IS



<u>NOTE</u>: 1. THE DOMAIN OF sin IS $(-\infty, \infty)$ <u>ALL</u> THE REALS (i.e. $\text{dom}(\sin) = (-\infty, \infty)$).

2. THE RANGE OF sin IS $[-1, 1]$ (i.e. $\text{ran}(\sin) = [-1, 1]$)

3. THE sin GRAPH IS <u>PERIODIC</u> WITH A <u>PERIOD</u> OF 2π. IT REPEATS AFTER 2π.

4. KNOWING THE GRAPH GIVES YOU THE INFORMATION TO EVALUATE sin FOR MULTIPLES OF $\frac{\pi}{2}$ (i.e. $\sin\left(\frac{k\pi}{2}\right)$)

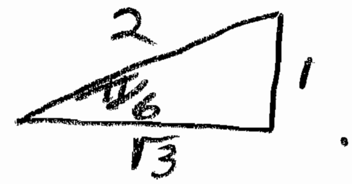
$\sin\left(\frac{\pi}{2}\right) = 1$ $\quad \sin(\pi) = 0$ $\quad \sin \frac{3\pi}{2} = -1$

$\sin(0) = 0$ $\quad \sin\left(-\frac{\pi}{2}\right) = -1$ $\quad \sin(-\pi) = 0$

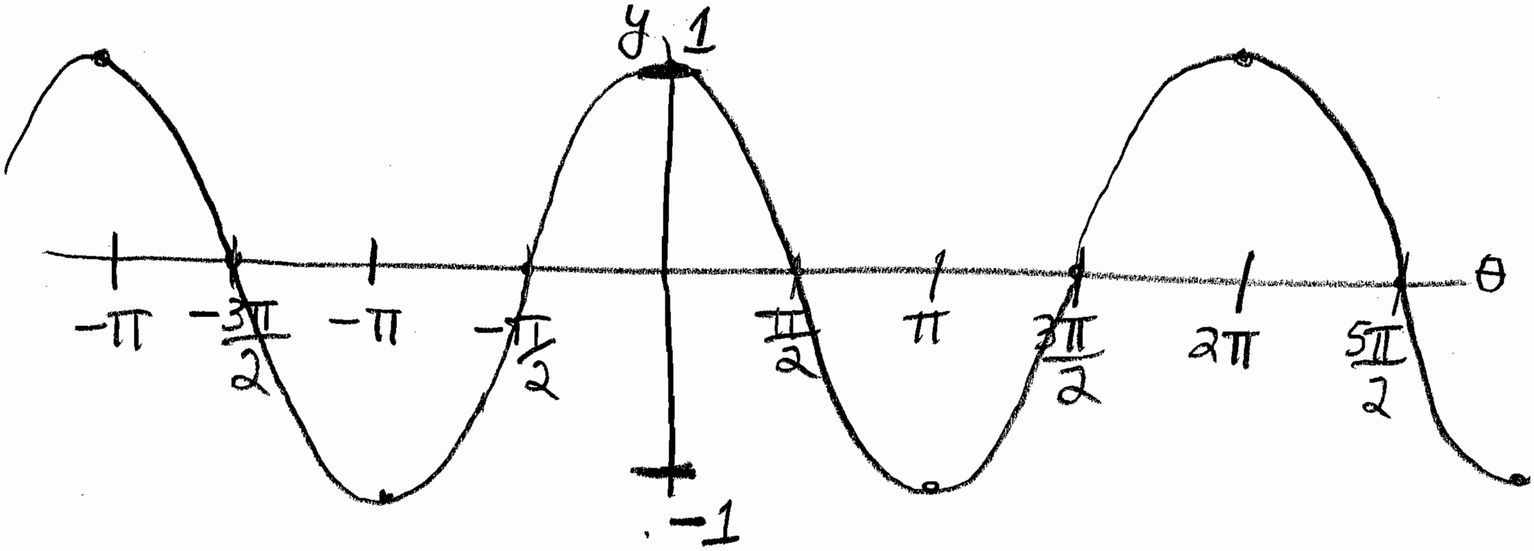
5. MEMORIZE THE GRAPH OF $y = \sin\theta$ TO EVALUATE MULTIPLES OF $\frac{\pi}{2}$

6. USE RIGHT TRIANGLES TO EVALUATE MULTIPLES OF $\frac{\pi}{6}$, $\frac{\pi}{3}$, AND $\frac{\pi}{4}$ THAT ARE <u>NOT</u> MULTIPLES OF $\frac{\pi}{2}$.

7. NOTE ON THE GRAPH THE POINT $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ WAS PLOTTED TO ILLUSTRATE $\sin\frac{\pi}{6} = \frac{1}{2}$ WHICH WAS FOUND USING



B. THE GRAPH OF $y = \cos\theta$



NOTE: 1) $\mathrm{dom}(\cos) = (-\infty, \infty)$

2) $\mathrm{ran}(\cos) = [-1, 1]$

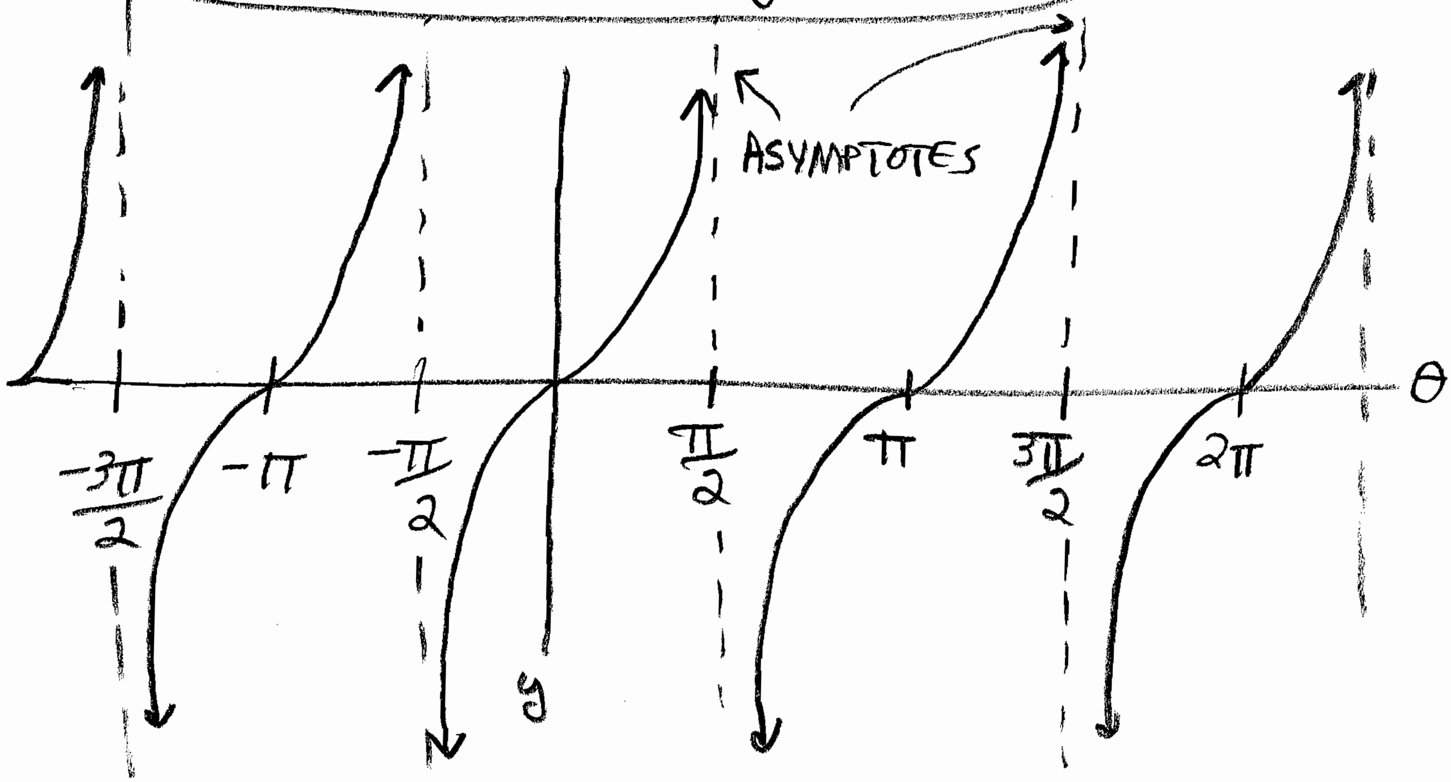
3) COS HAS A PERIOD OF 2π

4) $\cos(0) = 1 \quad \cos\left(\frac{\pi}{2}\right) = 0 \quad \cos(-\pi) = -1$

5) USE THE GRAPH OF $y = \cos\theta$ TO EVALUATE COS AT MULTIPLES OF $\frac{\pi}{2}$ MEMORIZE THE GRAPH.

IN GENERAL, MEMORIZE THE GRAPHS OF THE 6 TRIG FUNCTIONS THAT YOU ARE BEING GIVEN. USE THE GRAPHS, AMONG OTHER THINGS, TO EVALUATE THE TRIG FUNCTIONS FOR MULTIPLES OF $\frac{\pi}{2}$

c. THE GRAPH OF $y = \tan\theta$



NOTE: 1) $\text{dom}(\tan) = \{\theta \,/\, \theta \neq \frac{\pi}{2} + k\pi,\ k \text{ INTEGER}\}$

2) $\text{ran}(\tan) = (-\infty, \infty)$

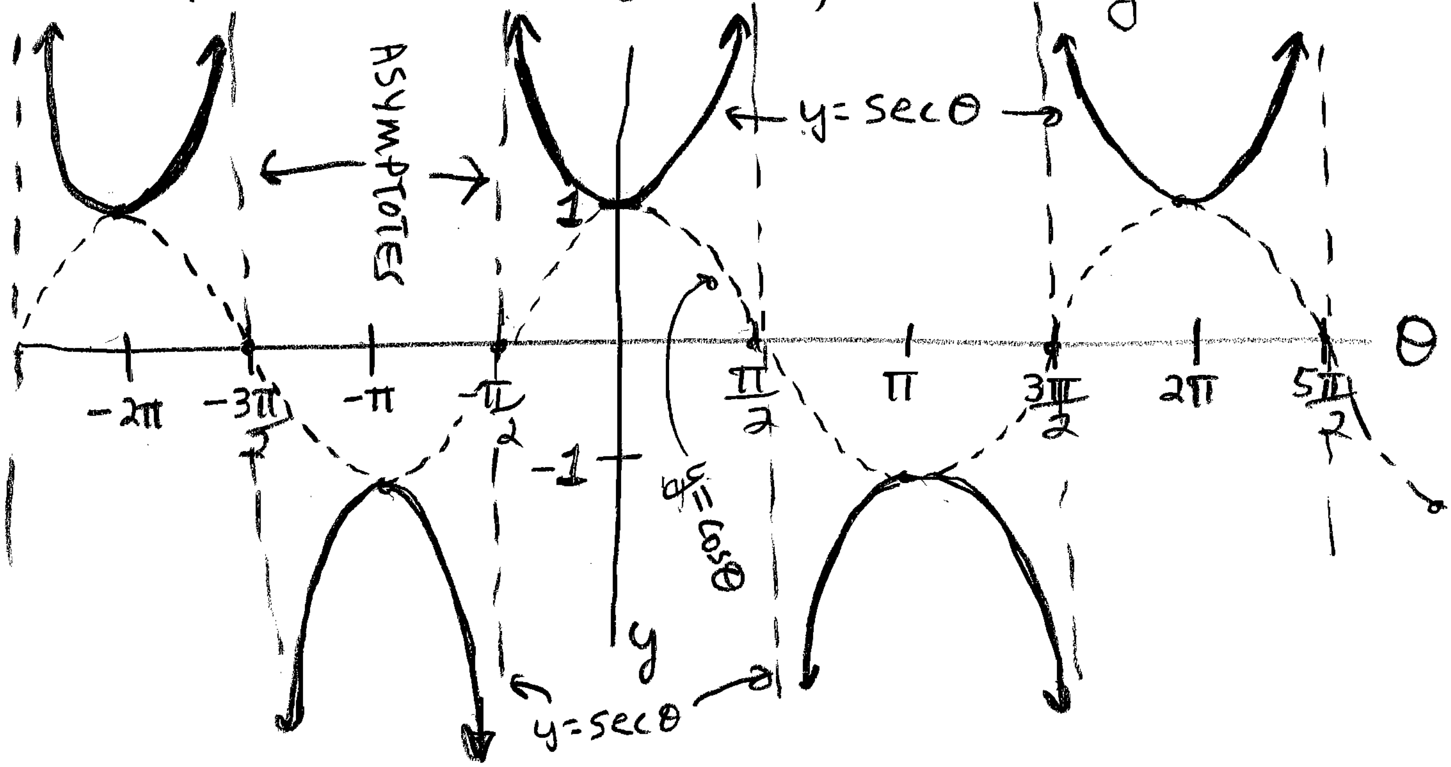
3) tan is periodic with a period of π

4) $\tan(0) = 0$, $\tan\left(\frac{\pi}{2}\right)$ = UNDEFINED

$\tan(-\pi) = 0$, $\tan\left(\frac{3\pi}{2}\right)$ = UNDEFINED

D. THE GRAPH OF $y = \sec\theta = \dfrac{1}{\cos\theta}$

HINT: GRAPH IS OVER, UNDER $y = \cos\theta$



NOTE: 1) $\mathrm{dom}(\sec) = \{\theta \,/\, \theta \neq \frac{\pi}{2} + k\pi,\ k \text{ INTEGER}\}$

2) $\mathrm{ran}(\sec) = (-\infty, -1] \cup [1, \infty)$

3) sec is periodic with period 2π

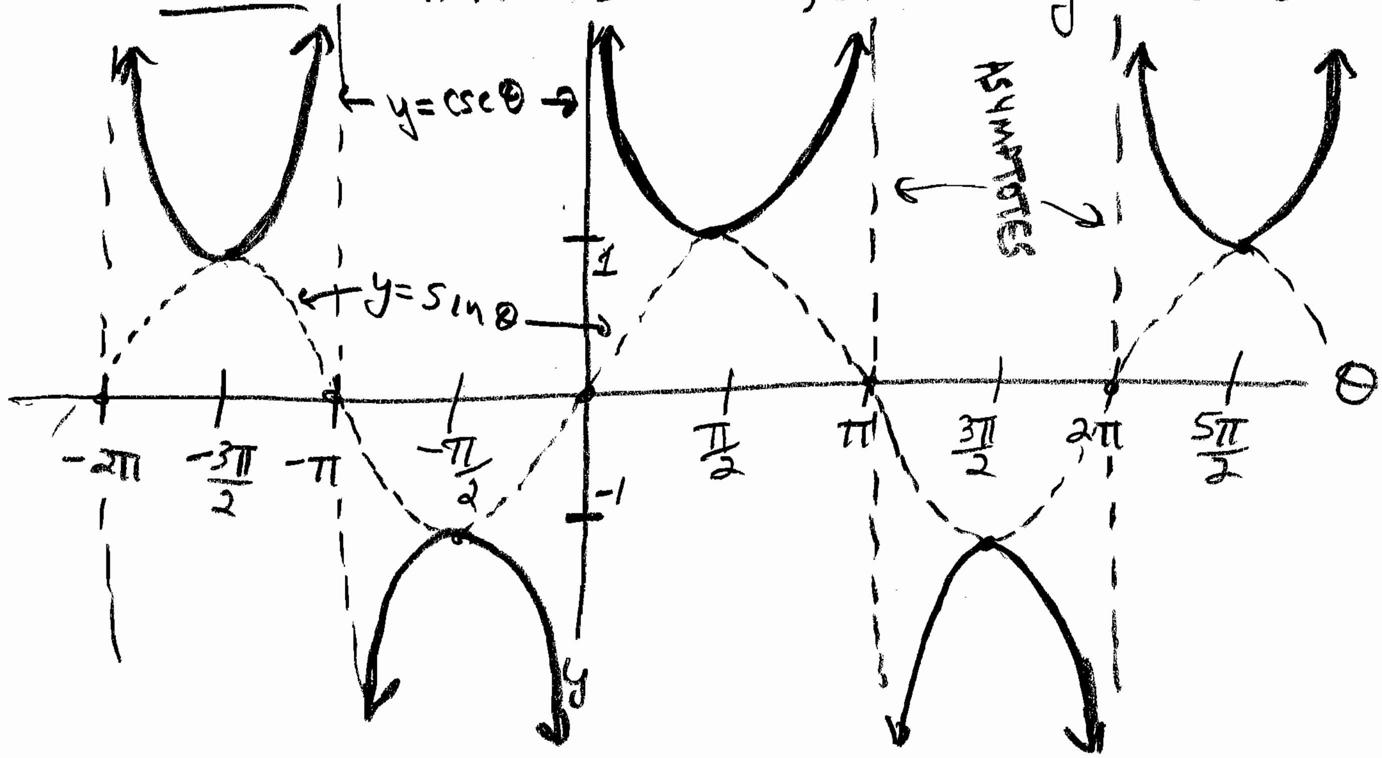
4) $\sec(0) = 1 \qquad \sec\left(\frac{\pi}{2}\right) =$ UNDEFINED

$\sec(\pi) = -1 \qquad \sec(-2\pi) = 1$

$\sec\left(-\frac{3\pi}{2}\right) =$ UNDEFINED.

E. THE GRAPH OF $y = \csc\theta = \dfrac{1}{\sin\theta}$

HINT: GRAPH IS OVER, UNDER $y = \sin\theta$



NOTE: 1) $\mathrm{dom}(\csc) = \{\theta \mid \theta \neq k\pi,\ k \text{ INTEGER}\}$

2) $\mathrm{ran}(\csc) = (-\infty, -1] \cup [1, \infty)$

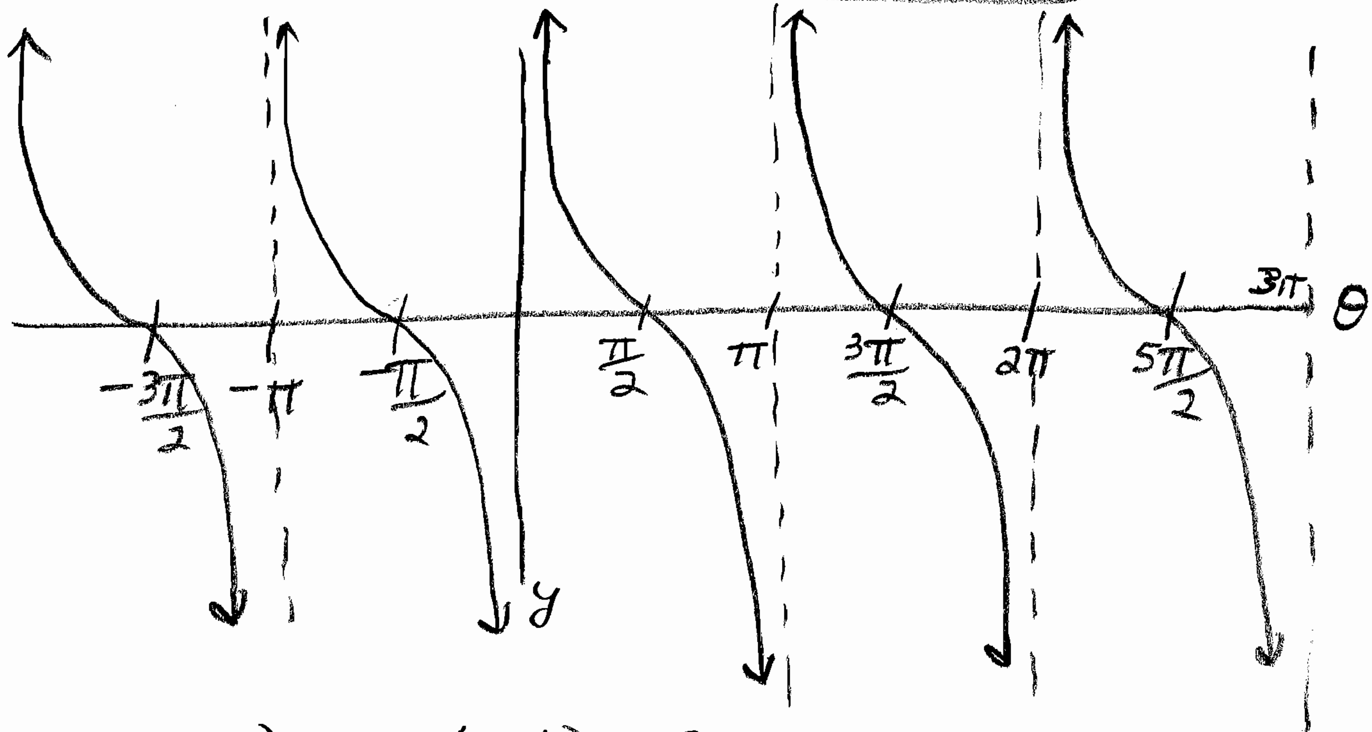
3) sec is periodic with period 2π

4) $\csc\left(\frac{\pi}{2}\right) = 1$ $\csc(\pi) =$ UNDEFINED

$\csc(0) =$ UNDEFINED $\csc\left(\frac{3\pi}{2}\right) = -1$

$\csc(-\pi) =$ UNDEFINED $\csc\left(-\frac{3\pi}{2}\right) = 1$

F. THE GRAPH OF $y = \cot\theta = \dfrac{1}{\tan\theta}$



NOTE: 1) $\mathrm{dom}(\cot) = \{\theta \mid \theta \neq k\pi,\ k \text{ INTEGER}\}$

2) $\mathrm{ran}(\cot) = (-\infty, \infty)$

3) cot is periodic with period π

4) $\cot(\frac{\pi}{2}) = 0$ $\quad \cot(\pi) = $ UNDEFINED

$\cot(\frac{3\pi}{2}) = 0$ $\quad \cot(-\frac{\pi}{2}) = 0$

$\cot(-\pi) = $ UNDEFINED

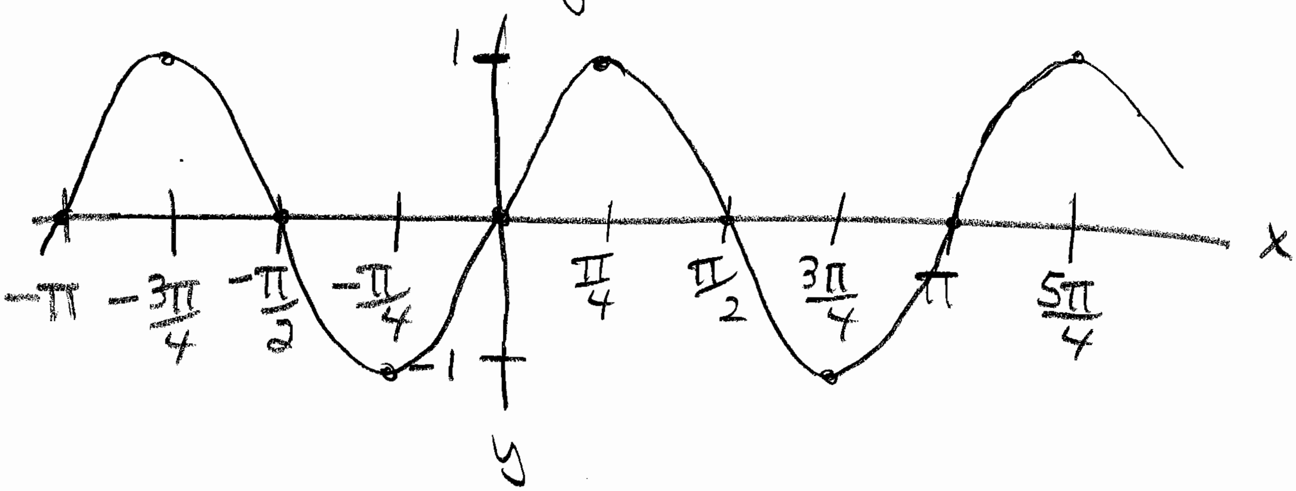
G. PERIOD ADJUSTMENT: THE EFFECT OF $\underline{k}$ IN $y = \sin(\underline{k}x)$.

1. $y = \sin(2x) = \sin 2x$

   LET $x = \frac{\pi}{4}$. $y = \sin 2(\frac{\pi}{4}) = \sin \frac{\pi}{2} = 1$

   SO THE $y = \sin 2x$ GRAPH REACHES ITS PEAK AT $\frac{\pi}{4}$ NOT $\frac{\pi}{2}$ AS IN THE $y = \sin x$ GRAPH

2. GRAPH OF $y = \sin 2x$



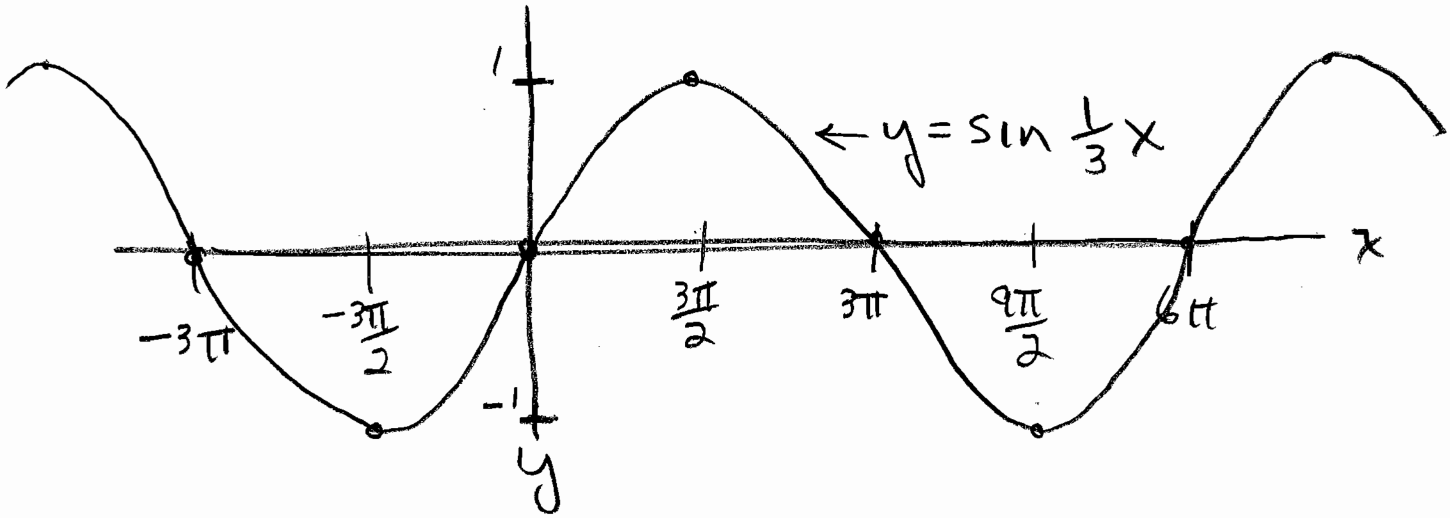
3. THE PERIOD OF $y = \sin 2x$ IS $\pi = \frac{2\pi}{2}$

4. THE PERIOD OF $y = \sin kx$ IS $\frac{2\pi}{k}$

5. $y = \sin \frac{1}{3}x$ GRAPH

PERIOD $= \frac{2\pi}{\frac{1}{3}} = 3(2\pi) = 6\pi$

FIRST PEAK AT $\frac{6\pi}{4} = \frac{3\pi}{2}$



6. THE PERIODS OF $y = \cos kx$, $y = \tan kx$, $y = \cot kx$, $y = \sec kx$, AND $y = \csc kx$ ARE ADJUSTED SIMILARLY.

HOMEWORK

A. FOR EACH, DRAW THE APPROPRIATE RIGHT TRIANGLE AND GIVE THE EXACT ANSWER. (REVIEW)

$\sin\frac{\pi}{3}$ $\quad\cos\frac{2\pi}{3}$ $\quad\tan\frac{5\pi}{6}$ $\quad\cot\frac{7\pi}{6}$

$\sec\frac{4\pi}{3}$ $\quad\csc\frac{11\pi}{6}$ $\quad\cos\left(-\frac{4\pi}{3}\right)$ $\quad\sin\left(-\frac{7\pi}{6}\right)$

B. FOR EACH, SKETCH THE APPROPRIATE GRAPH FROM MEMORY AND GIVE THE EXACT ANSWER.

$\sin\frac{3\pi}{2}$ $\quad\cos\frac{5\pi}{2}$ $\quad\tan\left(\frac{3\pi}{2}\right)$

$\cot\pi$ $\quad\sec(-\pi)$ $\quad\csc(-2\pi)$

$\cos\left(-\frac{\pi}{2}\right)$ $\quad\tan(-\pi)$ $\quad\csc\left(-\frac{5\pi}{2}\right)$

C SKETCH EACH GRAPH. STATE ITS PERIOD.

$y=\cos 2x$ $\quad y=\cos 3x$ $\quad y=\cos\frac{1}{5}x$

$y=\tan 2x$ $\quad y=\cot\frac{1}{2}x$ $\quad y=\sec 2x$

ALERT:

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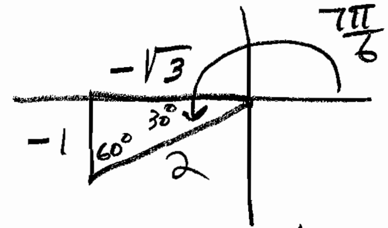
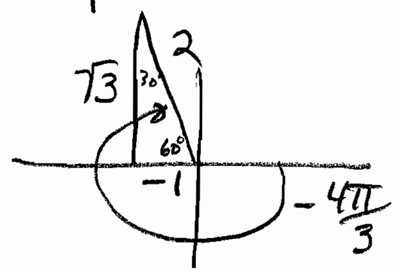
SOME WORKED HOMEWORK PROBLEMS

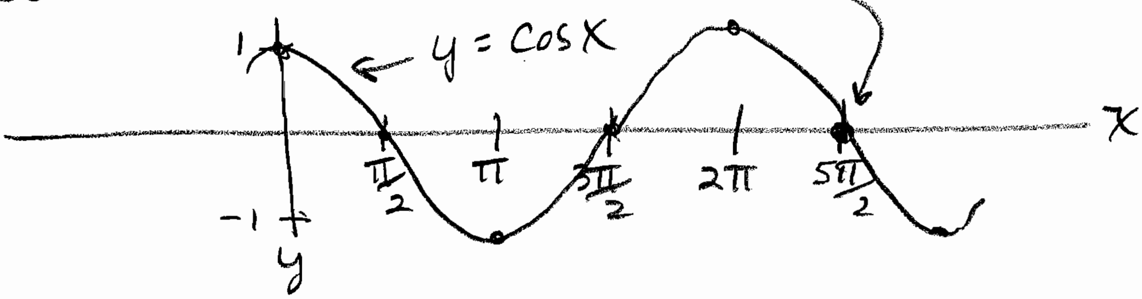
$\dfrac{A}{p.30}$ $\quad \cot \dfrac{7\pi}{6} = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$ $\quad \dfrac{ADJ}{OPP}$



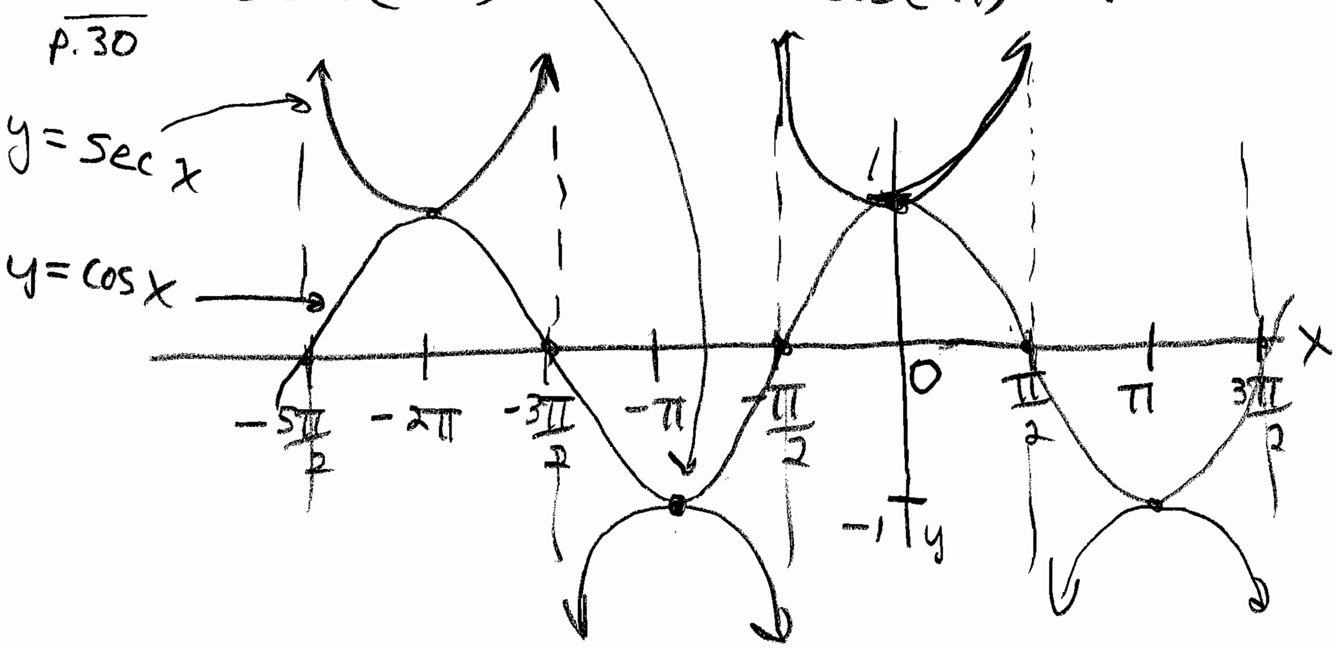
$\dfrac{A}{p.30}$ $\quad \cos\left(\dfrac{-4\pi}{3}\right) = \dfrac{-1}{2}$ $\quad \dfrac{ADJ}{HYP}$



$\dfrac{B}{p.30}$ $\quad \cos \dfrac{5\pi}{2} = 0$

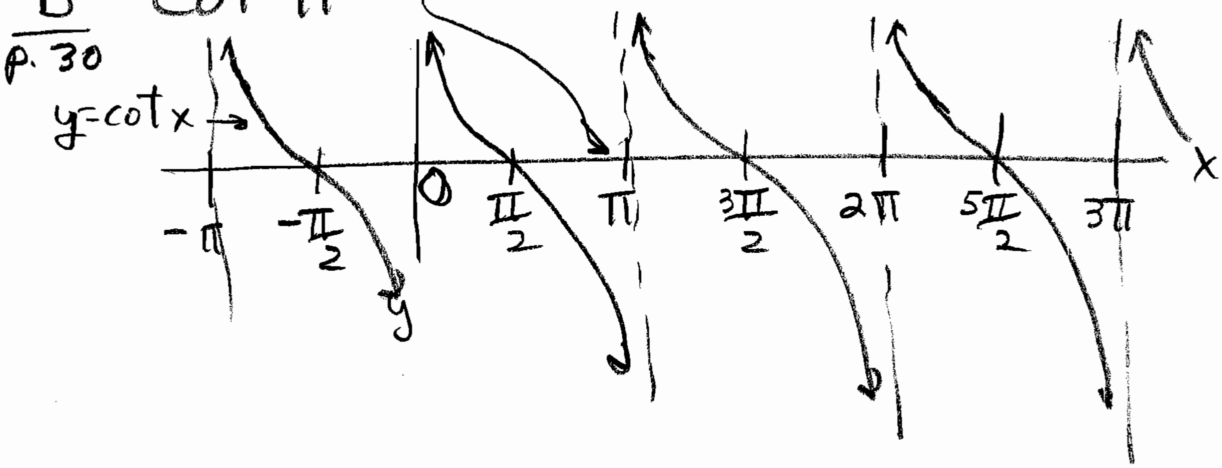


$\dfrac{B}{p.30}$ $\quad \sec(-\pi) = -1 = \dfrac{1}{\cos(-\pi)} = \dfrac{1}{-1} = -1$



HOMEWORK WORKED (CONT.)

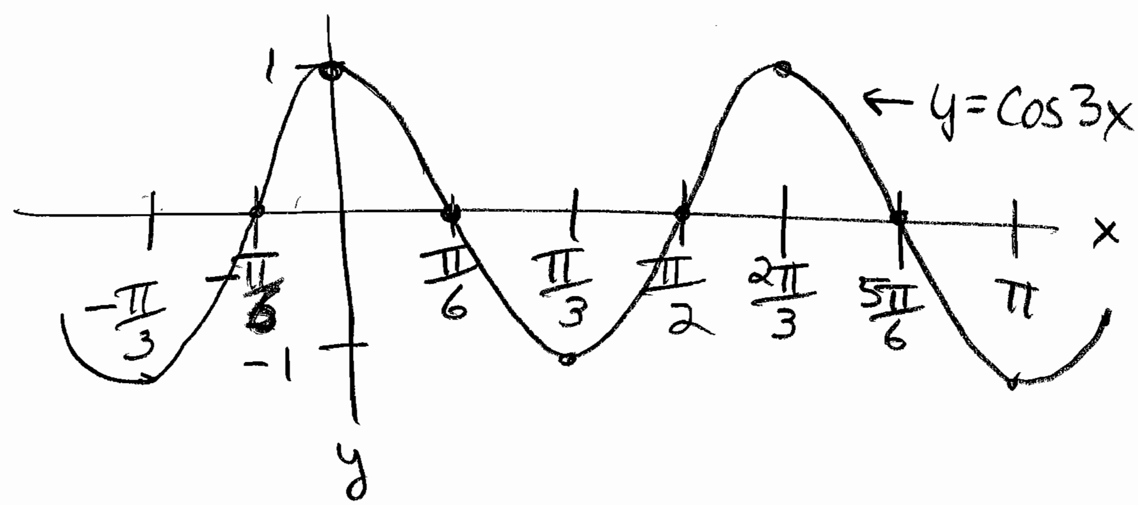
$\frac{B}{p.\,30}$ $\cot \pi$ = UNDEFINED



$\frac{C}{p.\,30}$ $y = \cos 3x$ graph

1^{st} X-AXIS CROSSING $\dfrac{\frac{\pi}{2}}{3} = \dfrac{\frac{\pi}{2}}{\frac{3}{1}} = \frac{1}{3}\cdot\frac{\pi}{2} = \frac{\pi}{6}$

PERIOD $= \frac{2\pi}{3}$

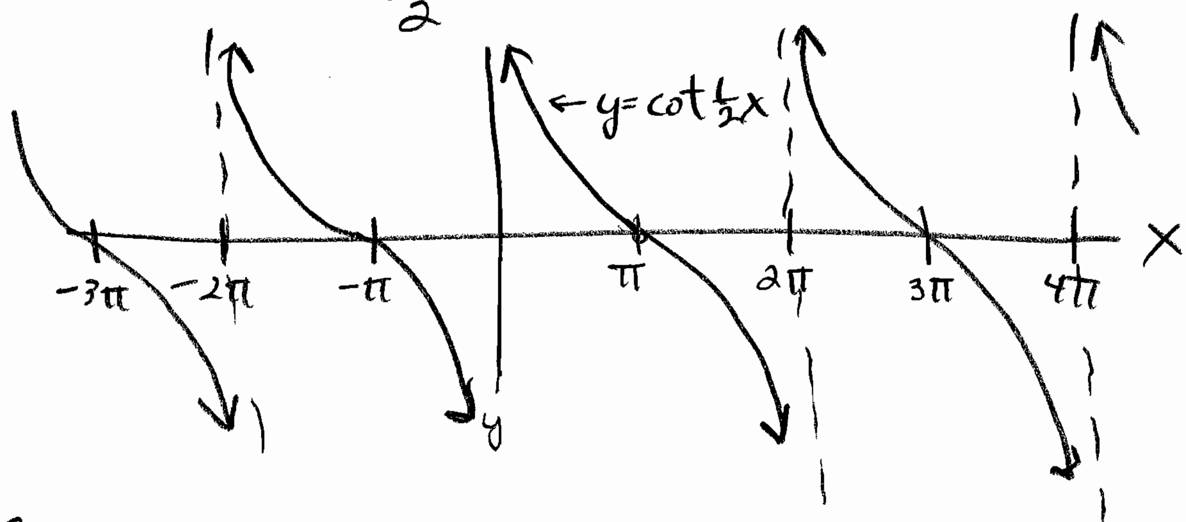


HOMEWORK WORKED (CONT.)

$\frac{C}{p.30}$ $y = \cot \frac{1}{2}x$

1^{ST} x-AXIS CROSSING $\dfrac{\frac{\pi}{2}}{\frac{1}{2}} = \frac{\pi}{2} \cdot \frac{2}{1} = \pi$

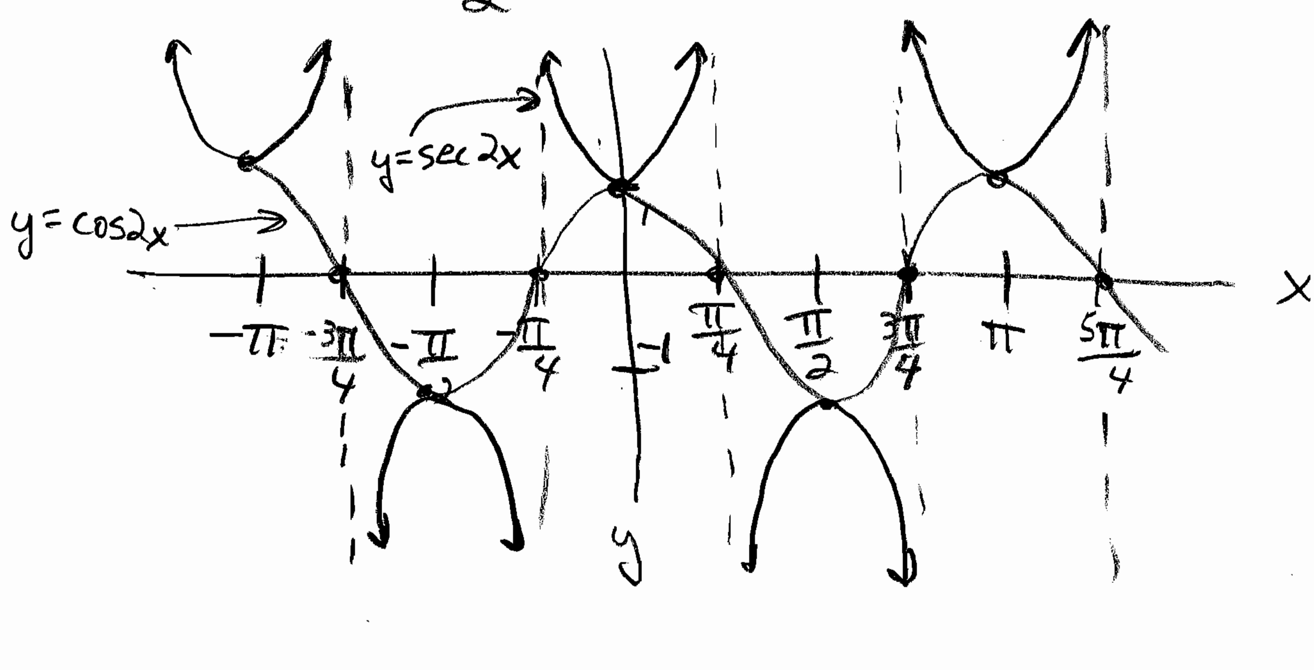
PERIOD $= \dfrac{\pi}{\frac{1}{2}} = 2\pi$



$\frac{C}{p.30}$ $y = \sec 2x$

1^{ST} ASYMPTOTE $\dfrac{\frac{\pi}{2}}{2} = \dfrac{\frac{\pi}{2}}{\frac{2}{1}} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

PERIOD $= \frac{2\pi}{2} = \pi$



TRUTH GEM

HUNGER AND THIRST FOR THE RIGHT THINGS TO BE PROPERLY FILLED.

A. (MT. 5:6) BLESSED ARE THOSE WHO HUNGER AND THIRST FOR RIGHTEOUSNESS, FOR THEY SHALL BE FILLED.

B. SOME TRY TO GET STUDENTS TO LEARN BY TRYING TO "MAKE-EM, MAKE-EM, MAKE-EM"; LEARN BY FORCE FEEDING ← IMPROPER FUTILE WAY TO TRY TO GET STUDENTS FILLED.

C. IF YOU TRULY HUNGER AND THIRST TO LEARN THIS GOOD MATERIAL, YOU WILL BE FILLED.

D. THE TRULY HUNGRY WILL HUNGER TO KNOW BETTER WAYS TO LEARN (SOME ARE FOUND AT www.arealeducation.com : FREE BOOK TRUTH GEMS FOR TEACHER AND STUDENT)

E. THE REALLY TRULY HUNGRY WILL GO TO THE SOURCE OF THE TRUTH GEM BOOK: PRAYING TO GOD THAT THE EYES OF YOUR UNDERSTANDING BE ENLIGHTENED, THEN SEEKING IN THE BIBLE.

F. (COL 2:3) "IN WHOM (JESUS) ARE HIDDEN ALL OF THE TREASURES OF WISDOM AND KNOWLEDGE." LOOK NO FURTHER.

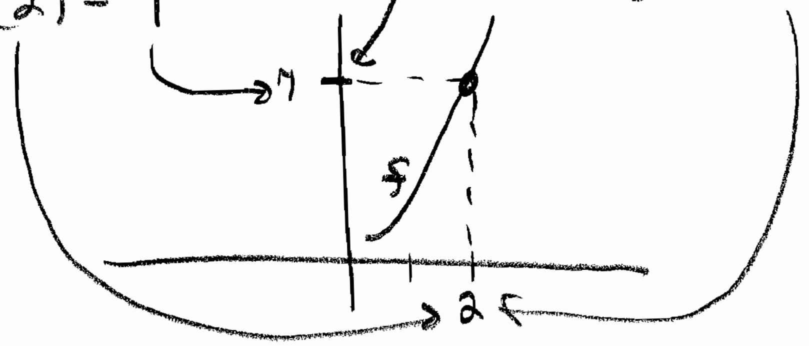
INVERSE TRIG FUNCTIONS

A. f^{-1} IS THE NOTATION FOR THE INVERSE OF THE FUNCTION f

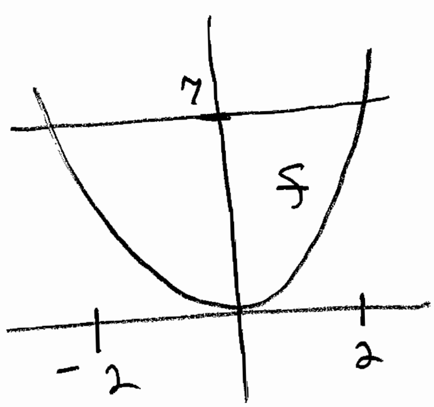
B. THE INVERSE REVERSES WHAT f DOES.

f DOES   REVERSE

$f(2)=7$ $f^{-1}(7)=2$



C. NO HORIZONTAL LINE CAN HIT THE GRAPH OF f TWICE AND f^{-1} BE A FUNCTION



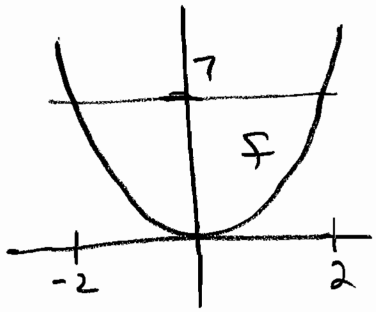
$f(2)=7$

BUT $f^{-1}(7)= ?$

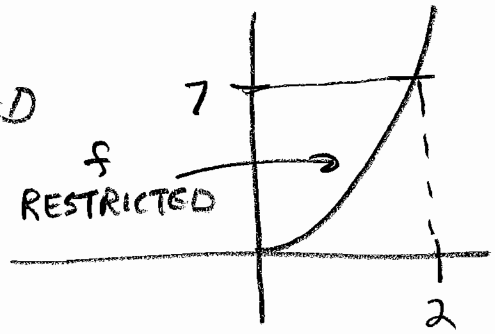
2 OR −2

f^{-1} NOT A FUNCTION

D. A GRAPH CAN BE RESTRICTED SO f^{-1} IS A FUNCTION

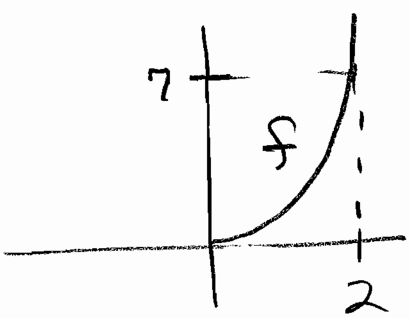


RESTRICTED TO



E. ALL THE TRIG FUNCTIONS WILL BE RESTRICTED SO THAT THE INVERSE IS A FUNCTION. IT IS IMPORTANT TO LEARN THE UPCOMING RESTRICTIONS.

F. YOU CAN EVALATE f^{-1} BY LOOKING AT THE GRAPH OF f.

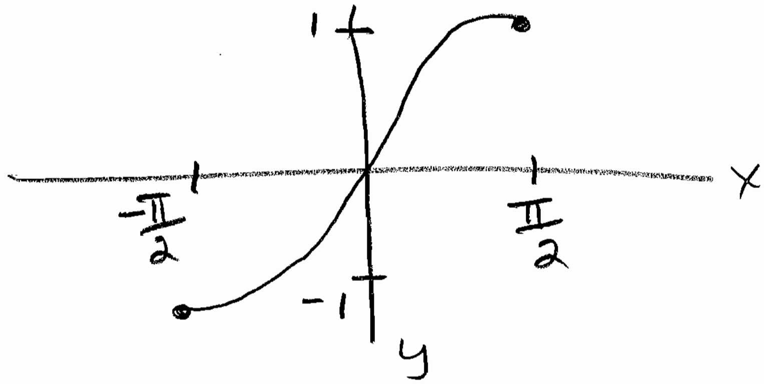


LOOKING AT THE GRAPH OF f, $f(2)=7$

SO $f^{-1}(7)=2$

G. THE INVERSE OF SIN

1. NAME: arcsin OR $\sin^{-1}$
2. RESTRICTION: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
3. GRAPH OF RESTRICTED SIN



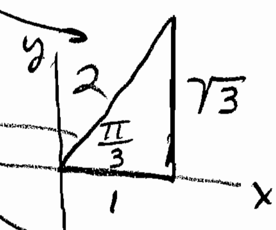
4. EVALUATING $\sin^{-1}$ LOOKING AT SIN GRAPH

$\sin^{-1}(1) = \frac{\pi}{2}$ $\quad \arcsin(-1) = -\frac{\pi}{2}$ $\quad \sin^{-1}(0) = 0$

5. EVALUATING $\sin^{-1}$ LOOKING AT TRIANGLES

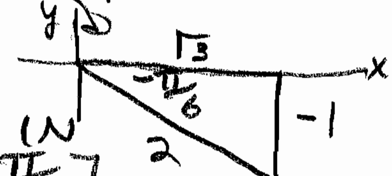
LOOKS FAMILIAR

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$



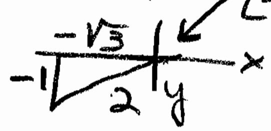
LOOKS FAMILIAR

$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$



DO NOT ANSWER $\frac{11\pi}{2}$ ← NOT IN $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

DO NOT DRAW

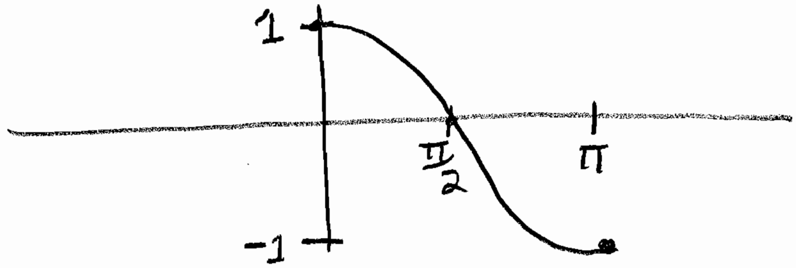


H. THE INVERSE OF COS

1. NAME: arccos OR $\cos^{-1}$

2. RESTRICTION: $[0, \pi]$

3. GRAPH OF RESTRICTED COS.



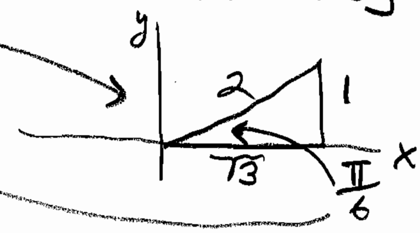
4. EVALUATING $\cos^{-1}$ LOOKING AT COS GRAPH

$\cos^{-1}(1) = 0 \quad \arccos(-1) = \pi \quad \cos^{-1}(0) = \frac{\pi}{2}$

5. EVALUATING $\cos^{-1}$ LOOKING AT TRIANGLES

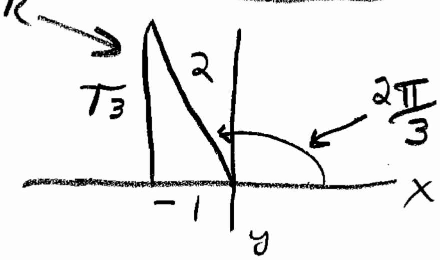
LOOKS FAMILIAR

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$



LOOKS FAMILIAR

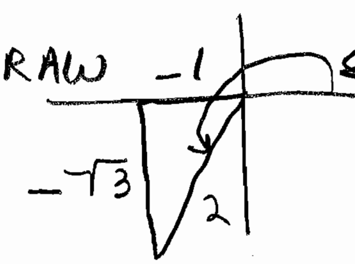
$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$



DO NOT ANSWER $+\frac{4\pi}{3}$

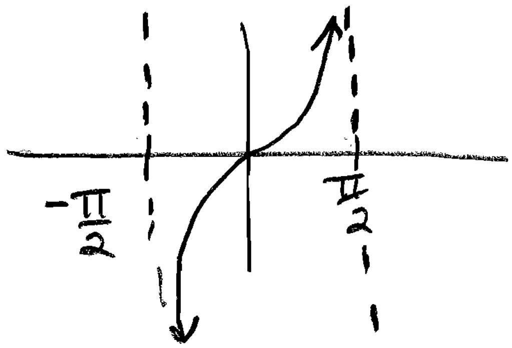
NOT IN $[0, \pi]$

DO NOT DRAW



I. THE INVERSE OF tan

1. NAME: arctan OR $\tan^{-1}$
2. RESTRICTION: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
3. GRAPH OF RESTRICTED tan.



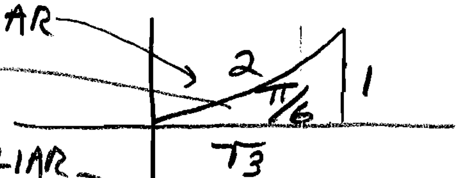
4. EVALUATING $\tan^{-1}$ LOOKING AT tan GRAPH

$\arctan(0) = 0$

5. EVALUATING $\tan^{-1}$ LOOKING AT TRIANGLES

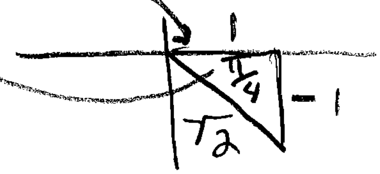
LOOKS FAMILIAR

$\arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$



LOOKS FAMILIAR

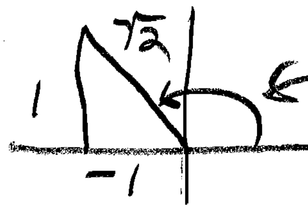
$\tan^{-1}(-1) = \tan^{-1}\left(\frac{-1}{1}\right) = \frac{\pi}{4}$



DO NOT ANSWER $\frac{3\pi}{4}$

DO NOT DRAW

NOT IN $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

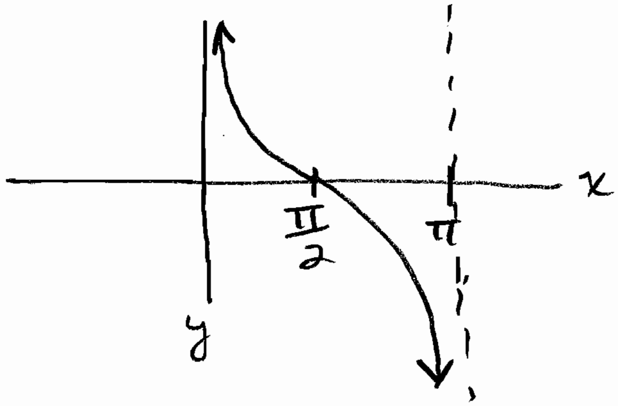


J. THE INVERSE OF cot.

1. NAME: arccot OR $\cot^{-1}$

2. RESTRICTION: $(0, \pi)$

3. GRAPH OF RESTRICTED cot



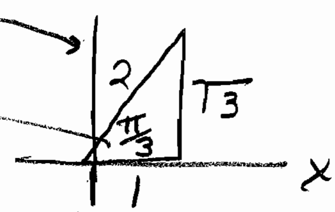
4. EVALUATING $\cot^{-1}$ LOOKING AT cot GRAPH

$$\operatorname{arccot}(0) = \frac{\pi}{2}$$

5. EVALUATING $\cot^{-1}$ LOOKING AT TRIANGLES

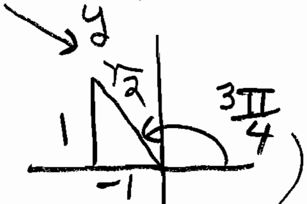
LOOKS FAMILIAR

$$\operatorname{arccot}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}$$



LOOKS FAMILIAR

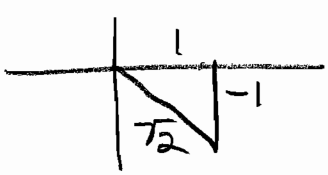
$$\cot^{-1}(-1) = \cot^{-1}\left(\frac{-1}{1}\right) = \frac{3\pi}{4}$$



DO NOT ANSWER $-\frac{\pi}{4}$

DO NOT DRAW

NOT IN $(0, \pi)$

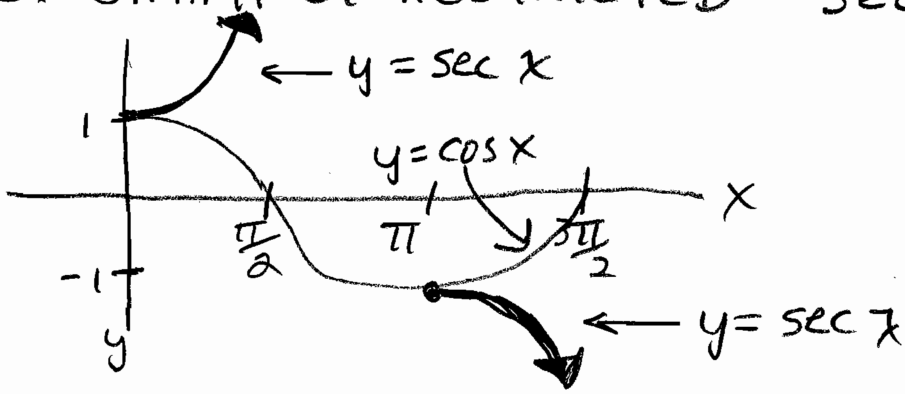


K. THE INVERSE OF sec

1. NAME: arcsec OR $\sec^{-1}$

2. RESTRICTION: $[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$

3. GRAPH OF RESTRICTED sec



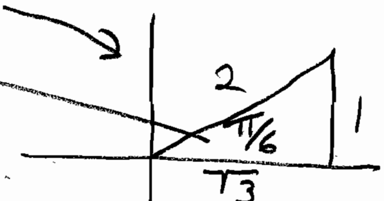
4. EVALUATING $\sec^{-1}$ LOOKING AT sec GRAPH

$\text{arcsec}(1) = 0 \qquad \sec^{-1}(-1) = \pi$

5. EVALUATING $\sec^{-1}$ LOOKING AT TRIANGLES

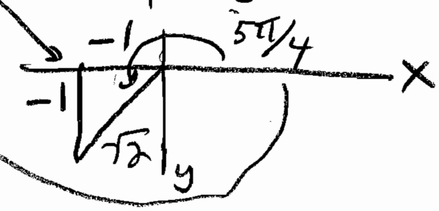
LOOKS FAMILIAR

$\text{arcsec}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$



LOOKS FAMILIAR

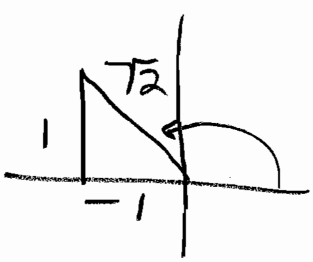
$\sec^{-1}(-\sqrt{2}) = \sec^{-1}\left(\frac{\sqrt{2}}{-1}\right) = \frac{5\pi}{4}$



DO NOT ANSWER $\frac{3\pi}{4}$

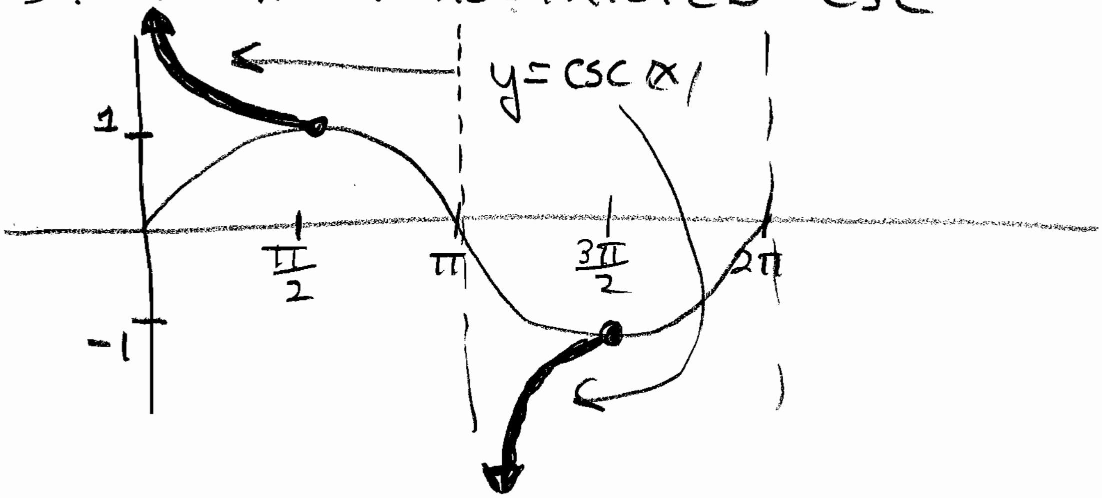
DO NOT DRAW

NOT IN $[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$



L. THE INVERSE OF CSC

1. NAME: arccsc OR $\csc^{-1}$
2. RESTRICTION: $(0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2})$
3. GRAPH OF RESTRICTED CSC



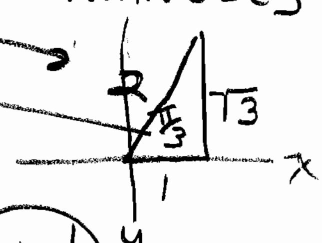
4. EVALUATING $\csc^{-1}$ LOOKING AT CSC GRAPH

$\operatorname{arccsc}(1) = \frac{\pi}{2}$ $\qquad$ $\csc^{-1}(-1) = \frac{3\pi}{2}$

5. EVALUATING $\csc^{-1}$ LOOKING AT TRIANGLES

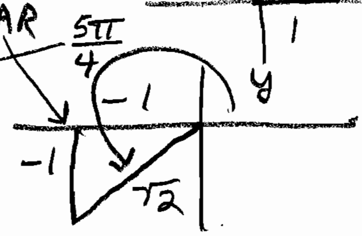
LOOKS FAMILIAR

$\operatorname{arccsc}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$



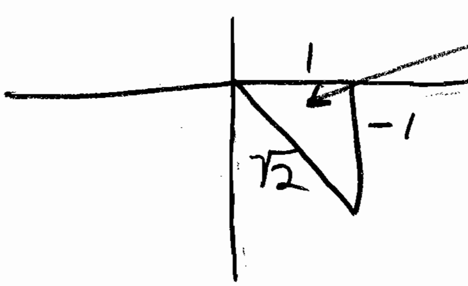
LOOKS FAMILIAR

$\csc^{-1}(-\sqrt{2}) = \csc\left(\frac{\sqrt{2}}{-1}\right) = \frac{5\pi}{4}$



DO NOT ANSWER $-\frac{\pi}{4}$

DO NOT DRAW

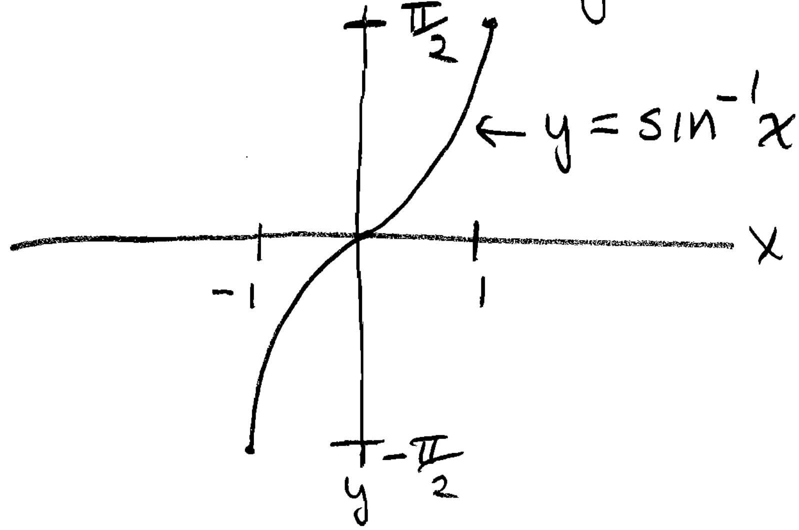


NOT IN $(0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2})$

M. GRAPHS OF INVERSE TRIG FUNCTIONS

1. THE GRAPH OF f^{-1} IS THE GRAPH OF f REFLECTED ABOUT $y = x$

2. THE GRAPH OF $y = \arcsin x$



3. IT IS RARE THAT YOU NEED TO KNOW WHAT THE GRAPHS OF THE INVERSE TRIG FUNCTIONS LOOK LIKE TO WORK ANY PROBLEM IN ALL OF CALCULUS. THE INFORMATION IN THE PREVIOUS PAGES OF THIS MICRO MICROPEDIA IS PLENTY SO THAT AN "A" IN CALCULUS WILL NOT BE HINDERED BY LACK OF TRIG KNOWLEDGE.

HOMEWORK

A. FOR EACH, SKETCH THE GRAPH OF THE RESTRICTED TRIG FUNCTION AND LOOK AT THE GRAPH TO EVALUATE

$\sin^{-1}(-1)$ $\arccos(0)$ $\tan^{-1}(0)$

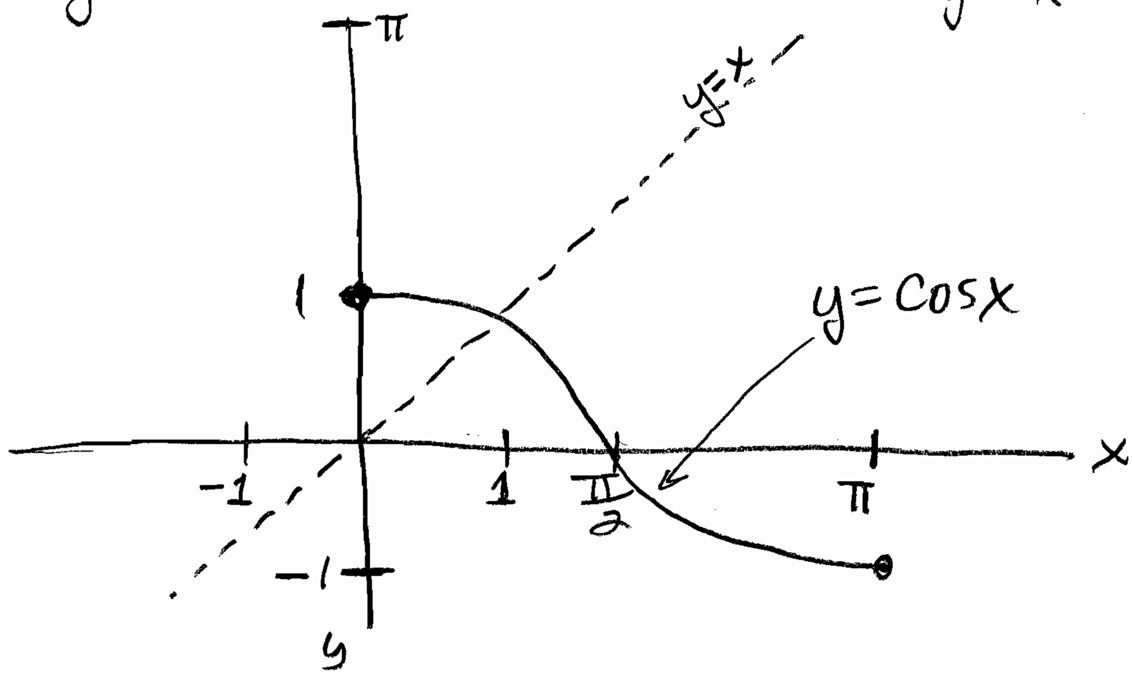
$\cot^{-1}(0)$ $\sec^{-1}(-1)$ $\csc^{-1}(1)$

B. FOR EACH, EVALUATE EXACTLY LOOKING AT A FAMILIAR TRIANGLE

$\arcsin\left(\frac{1}{\sqrt{2}}\right)$ $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ $\tan^{-1}(-\sqrt{3})$

$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ $\sec^{-1}(-2)$ $\operatorname{arccsc}\left(\frac{-2}{\sqrt{3}}\right)$

C. SKETCH THE GRAPH OF $y = \cos^{-1} x$ BY REFLECTING THE RESTRICTED GRAPH OF $y = \cos x$ ABOUT THE LINE $y = x$



ALERT:

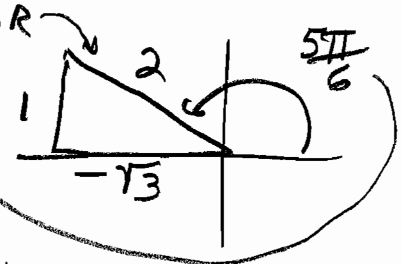
SOME HOMEWORK PROBLEMS JUST ASSIGNED ARE WORKED ON THE NEXT PAGES. IT IS RECOMMENDED THAT YOU DO YOUR HOMEWORK BEFORE READING ON.

SOME WORKED HOMEWORK PROBLEMS

ALL OF PROBLEM "A" FOR HOMEWORK ARE WORKED IN THE PREVIOUS NOTES

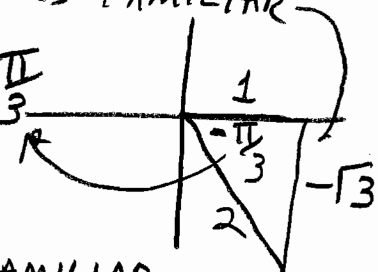
LOOKS FAMILIAR

B p. 44 $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$



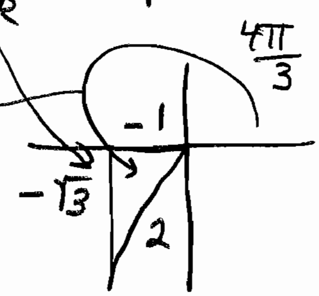
LOOKS FAMILIAR

B p. 44 $\tan^{-1}(-\sqrt{3}) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$



LOOKS FAMILIAR

B p. 44 $\sec^{-1}(-2) = \sec\left(\frac{2}{-1}\right) = \frac{4\pi}{3}$



C p. 44 GRAPH OF $y = \arccos x$