COLLEGE ALGEBRA

NO NOTE-TAKING*
VIA OVERHEADS

by

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* You can still take notes if you want to!

A

DEDICATION

To the One who makes things clear.

A

MIGHTY MICROPEDIA

FROM THE

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TRUTH GEMS FOR TEACHER AND STUDENT by Dr. J. Austin French. This micropedia consist of 53 Truth Gems from the Word of God directed at teaching and learning. Each Truth Gem and its explanation take one page. Since God is the Most High, this means His teachings are the most high teachings. No one knows better than the Creator how man was made, what he needs, what is the best way to teach man, and what is the best way for man to learn. Many of these truth gems start out each teaching session in the Math by Heart trilogy described below.

ALGEBRA II BY HEART by Dr. J. Austin French. This collection of two mighty micropedias consists of a micropedia text, a micropedia of supplementary materials, and DVDs of 53 teaching sessions. Both micropedias are free on the web or can be ordered on the web in paper copy or on one CD. The DVDs can be purchased from the web. This is a College Algebra course, which means it is a strong Algebra II course for high school. This is not what is called Intermediate Algebra (=Algebra I in high school) in some colleges.

CALCULUS I BY HEART by Dr. J. Austin French. This collection of two mighty micropedias consists of a micropedia text, a micropedia of supplementary materials, and DVDs of 38 teaching sessions. This is a rigorous first course in calculus. It is a first college calculus course. It can be used for high school students who have finished Algebra I, Algebra II, and have had some trigonometry (trigonometry is taught in pre-calculus or advanced math courses in high school). The topic is differential calculus. Both micropedias are free on the web or can be ordered on the web in paper copy or on one CD. The DVDs can be purchased from the web.

LOGIC FOR UNDERSTANDING MATHEMATICS by Dr. J. Austin French and Dr. Earl Dennis. This collection of two mighty micropedias consists of a micropedia text, a micropedia of supplementary materials, and DVDs of 31 detailed teaching sessions. The mystery of how to do proofs is revealed. Logic is taught and then that connection to math proof is made plain. Proofs are illustrated in the area of elementary set theory. It is for the advanced high school student through college. Math maturity to have done excellently in Algebra II is the only recommended prerequisite background. Both micropedias are free on the web or can be ordered on the web in paper copy or on one CD. The DVDs can be purchased from the web.

TABLE OF CONTENTS

[CHAPTER 1]

INTRODUCTORY THOUGHTS

A. YOU ARE FREED FROM NOTE-TAKING TO FOLLOW THE LINE OF THOUGHT.

B. ATTEMPT: REMOVE EXCUSES FOR NOT LEARNING.

1. TEACHER: WITHOUT EXCUSE FOR NOT MAKING THINGS CLEAR.
2. STUDENT: WITHOUT EXCUSE FOR
   a. NOT STUDYING
   b. NOT HAVING A GOOD SET OF NOTES
   c. NOT BEING CLEAR ON WHAT IS TO BE LEARNED.

C <u>GRADING</u>: NOT BASED ON THE <u>APPEARANCE</u> THAT YOU KNOW AND CAN DO MATH, BUT THAT YOU, YOURSELF, KNOW AND CAN DO MATH. RIGHTEOUS JUDGMENT (JN 7:24)

D MATH STUDYING WISDOM

1. STUDY A PROBLEM WORKED IN THE TEXT SO THAT YOU CAN WORK IT WITHOUT LOOKING, WITHOUT HESITATING. (TICKET FOR HELP, HAVE YOU DONE THAT?)
   a) FIND OUT WHERE YOU DO NOT HAVE UNDERSTANDING ON THE PROBLEM YOU HAVE JUST LEARNED TO WORK WITHOUT LOOKING... WHAT SKILL IS IT THAT YOU LACK?
   b. STOP AND MASTER THOSE SKILLS BY WORKING MANY PROBLEMS THAT TARGET THAT SKILL EXCLUSIVELY.
   c. NOW GO BACK AND MASTER THE MAIN PROBLEM.
2 REVIEW AND DEVELOP A BUFFET LINE OF PROBLEM TYPES YOU KNOW WELL. (SEED FOR MORE UNDERSTANDING)
3. CREATIVELY USE THE BUFFET LINE OF KNOWN PROBLEMS TO SOLVE UNKNOWN PROBLEM TYPES.
4. NOTE: IT REQUIRES DISCIPLINED <u>WORK</u> TO GET GOOD AT MATH.
   a. SOME NOT BAD AT MATH, JUST BAD AT RESPONSIBILITY
   b. EXPERIENCE THE <u>JOY</u> OF KNOWING SOMETHING <u>WELL</u>
   c. BE RESPONSIBLE A DAY AT A TIME FOR A SEMESTER.

JUGULAR PROBLEMS AND MINI-JUGULAR PROBLEMS

A PROBLEM OF THIS TYPE REQUIRES ONE TO KNOW MANY CONCEPTS IN ORDER TO WORK IT. THIS BOOK CONTAINS 9 JUGULAR PROBLEMS AND 2 MINI-JUGULAR PROBLEMS. IF ONE IS ABLE TO WORK ALL OF THESE PROBLEMS, MUCH ALGEBRA HAS BEEN LEARNED. THE PROBLEMS ARE NOW LISTED ALONG WITH THE SECTION IN WHICH THEY ARE FOUND. (J = JUGULAR ; MJ = MINI-JUGULAR)

(J) 1. (4,K) $\left(\dfrac{2x^3y^{-5}}{16x^{-7}y^{-10}}\right)^2 = 2^{\square}x^{\square}y^{\square}$ FILL IN THE BOXES

(J) 2. (4,T,3) FOR $x>0$, $y>0$ PUT IN SIMPLIFIED FORM

$$\frac{5x^2\sqrt[4]{y^2}}{\sqrt[4]{9xy^5}}$$

(MJ) 3. (5,M) FACTOR BY THE "REDUCE TO GROUP-THEN-FACTOR" METHOD

$$6x^2-13x-5$$

(J) 4. (5, Y) PUT IN SIMPLEST FORM WITH NO NEGATIVE EXPONENTS

$$\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} \div \frac{y^{-1} - x^{-1}}{3x} =$$

(J) 5. (6, J, 2, C) WRITE $\dfrac{\sqrt{3} - i\sqrt{2}}{\sqrt{5} + i\sqrt{3}}$ IN $a+bi$ FORM.

(J) 6. (7, N, 2) SOLVE

$$\frac{7}{3}(x+3)^{\frac{5}{3}}(2x-1)^{\frac{1}{6}} + \frac{5}{3}(2x-1)^{\frac{7}{6}}(x+3)^{2/3} = 0$$

(J) 7. (7, θ, 3) FIND ALL REAL SOLUTIONS TO

$$\sqrt{8+x} + \sqrt{1+x} - \sqrt{41+x} = 0$$

(MJ) 8. (9, H, 4) SOLVE $\left|\frac{3}{4} - \frac{2}{3}x\right| \geq \frac{1}{5}$

(J) 9. (9, J, 3) FIND ALL VALUES FOR x SO THAT $\sqrt{\dfrac{3x^2 - 4x - 4}{x+1}}$ IS REAL

(J) 10. (11,P) COMPLETING THE SQUARE, PUT THE EQUATION $y = 3x^2 - 30x + 79$ IN STANDARD FORM FOR A PARABOLA. FIND THE VERTEX, AXIS OF SYMMETRY, A PAIR OF SYMMETRIC PARTNERS, AND SKETCH.

(J) 11. (14,S) UNKNOWN IN THE EXPONENT: SOLVE $2^{3x-1} = 7^x$

WISDOM-KNOWLEDGE-UNDERSTANDING PROBLEM
WKU PROBLEM

YOU USE <u>WISDOM</u> TO PIECE TOGETHER MUCH OF THE <u>KNOWLEDGE</u> AND <u>UNDERSTANDING</u> RECEIVED IN THE COURSE TO SOLVE A COMPLEX PROBLEM: A FRESH CHALLENGE.

EXAMPLE: Find an equation for line L where L has slope equal to the y-coordinate of the vertex of the parabola

$$3y^2 + x = 30y - 71$$

and L passes through the point (p,q) where $5^{2p+1} = 8$ and q is the larger solution to $-2x^2 + 3x + 7 = 0$.

UNDERSTANDING DEFINITIONS

A. DEFINITIONS: CLEARY, EXACTLY DESCRIBE THE CONCEPT, NO MORE NO LESS

B. ALL DEFINITIONS "IF AND ONLY IF"

12 IF AND ONLY IF DOZEN

C. A DEFINITION WRITES\SPEAKS THE CONCEPT INTO EXISTENCE IN THE COURSE

D. DEFINITION: p IS A QUALOM IF AND ONLY IF p IS A POSITIVE INTEGER GREATER THAN 7

E. THEOREM: (FOLLOWS FROM A DEFINITION) EVERY QUALOM IS GREATER THAN 5

F. THERE IS ONLY ONE DEFINITION FOR A CONCEPT IN A COURSE ... THE EXACT WORDING WHEN ORIGINALLY DEFINED

G. THINGS EQUIVALENT TO A DEFINITION FALL UNDER THE THEOREM NAME

H. SO ON TESTS WHEN ASKED FOR A DEFINITION, GIVE THE EXACT WORDING, NOT SOMETHING EQUIVALENT (LIKE A POSITIVE INTEGER GREATER THAN $7\frac{1}{2}$).

[CHAPTER 2]

ALGEBRA BEGINNINGS

A. LISTING METHOD FOR DENOTING SETS

1. $\{2,3\}$ IS READ "THE SET WHOSE ELEMENTS ARE 2 AND 3".
2. $\{5\}$ IS READ "THE SET WHOSE ONLY ELEMENT IS 5".
3. $\emptyset$ IS THE EMPTY SET (NULL SET).
   $\emptyset = \{\ \}$. $\quad \{\emptyset\} \neq \emptyset$
4. $\{2,3\} = \{3,2\} = \{3,2,3,2,2\}$

B. SET-BUILDER NOTATION FOR SETS

1. $\in$ MEANS "IS AN ELEMENT OF" (ALSO, "IS IN" OR "BELONGS TO")
2. N DENOTES THE SET OF NATURAL NUMBERS. $N = \{1,2,3,\ldots\}$. N IS ALSO CALLED THE SET OF POSITIVE INTEGERS.
3. $\{x \mid x \in N \text{ and } x < 3\}$ IS READ "THE SET OF ALL x SUCH THAT x IS AN ELEMENT OF N AND x IS LESS THAN 3". (I.E. THE SET $\{1,2\}$)

C. THE SET OF <u>WHOLE</u> <u>NUMBERS</u>, W

1. $W = \{0, 1, 2, 3, \ldots\}$

2. 0 IS A WHOLE NUMBER BUT NOT A POSITIVE INTEGER.

D. THE SET OF INTEGERS, I

$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

E. THE SET OF <u>RATIONAL</u> <u>NUMBERS</u>, Q.

<u>DEFINITION</u>:

1. $Q = \left\{ \frac{a}{b} \,\middle|\, a \text{ IS AN INTEGER AND } b \text{ IS AN INTEGER AND } b \neq 0 \right\}$

2. Q FIRST LETTER OF QUOTIENT.

3. EXAMPLES OF RATIONALS:

$\frac{2}{3}, \frac{-4}{5}, 7 = \frac{7}{1}, 0 = \frac{0}{5} = \frac{0}{8}$

$\frac{25}{100} = .25 \qquad \frac{1}{3} = .33333\ldots$

4. NOTE: $23/99 = .23232323\ldots = .\overline{23}$

$$
\begin{array}{r|l}
 & .2323\ldots \\
99 & 23000 \\
 & 198 \\
 & \overline{320} \\
 & 297 \\
 & \overline{230} \\
 & 198 \\
 & \overline{320}
\end{array}
$$

A BAR OVER DIGITS MEANS THOSE ARE TO BE REPEATED

$7.2\overline{456} = 7.2456456456\ldots$

5. RATIONAL NUMBERS ARE THE REPEATING OR TERMINATING DECIMALS.

a. 52.135 TERMINATING

b. $4.2323\ldots = 4.\overline{23}$ REPEATING

c. NOTE $.\overline{23} \neq .23$ $\quad .23 = \frac{23}{100}$ $\quad .\overline{23} = \frac{23}{99}$

6. CHANGING A REPEATING DECIMAL TO A QUOTIENT OF INTEGERS (I.E. TO A FRACTION OF INTEGERS).

CHANGE $4.1345345\ldots = 4.1\overline{345}$ TO A FRACTION OF INTEGERS.

a. SET x TO THE NUMBER.

$$x = 4.1345345345\ldots$$

b. MULTIPLY x TO MOVE THE DECIMAL POINT <u>PAST</u> THE FIRST REPEAT. MULTIPLY x TO MOVE THE DECIMAL POINT IN FRONT OF THE FIRST REPEAT.

$$\begin{aligned} 10000x &= 41345.345345\ldots \\ -\quad 10x &= -\ 41.345345\ldots \\ \hline 9990x &= 41304 \end{aligned}$$

NOW SUBTRACT

$$x = \frac{41304}{9990} = \frac{20652}{4995} = 4.1\overline{345}$$

F. IRRATIONAL NUMBERS, Ir

1. AN IRRATIONAL NUMBER IS A NON-REPEATING AND NON-TERMINATING DECIMAL.
2. EXAMPLES

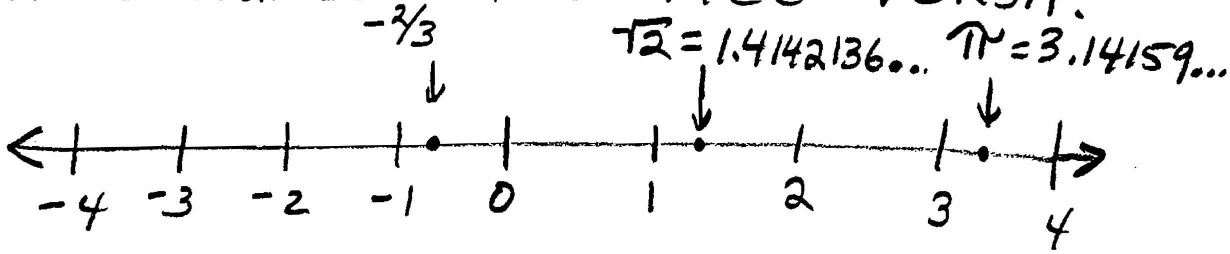
$$\pi = 3.14159\ldots$$
$$\sqrt{2} = 1.4142136\ldots$$
$$\sqrt{3} = 1.7320508\ldots$$
$$.020020002000020000002\ldots$$

3. NOTE $\pi \neq 3.14$ $\quad \pi \neq 22/7$

G. A <u>REAL NUMBER</u> IS A NUMBER THAT IS EITHER RATIONAL OR IRRATIONAL

R DENOTES THE SET OF REALS.

H. <u>NUMBER LINE</u>: FOR EVERY POINT ON THE NUMBER LINE THERE IS ASSOCIATED A REAL NUMBER AND VICE-VERSA.



I. SUBSET: $\subseteq$ "IS A SUBSET OF"

DEFINITION:

1. $H \subseteq K$ MEANS "EVERY ELEMENT OF H IS AN ELEMENT OF K"
2. $\{1,2\} \subseteq \{1,2,3,4\}$
3. $\{1,2\} \subseteq \{1,2\}$
4. $N \subseteq W \subseteq I \subseteq Q \subseteq R \qquad Ir \subseteq R$
5. TRUE OR FALSE

a.	$0 = \emptyset$	FALSE
b.	$2 \in \{1,2\}$	TRUE
c.	$2 \subseteq \{1,2\}$	FALSE
d	$\{2\} \in \{1,2\}$	FALSE
e.	$\{2\} \subseteq \{1,2\}$	TRUE
f.	$\frac{3}{5} \in Q$	TRUE
g.	$\pi \in I$	FALSE
h.	$\pi \in Ir$	TRUE
i.	$\pi \in R$	TRUE
j.	$\pi \subseteq R$	FALSE
k	$\{\pi\} \subseteq Ir$	TRUE

J. <u>HOMEWORK</u>: DISCOVERY! HOMEWORK IS AN OPPORTUNITY FOR INNER SATISFACTION. <u>OIS</u> = OPPORTUNITY FOR INNER SATISFACTION

1. HOW ARE EACH OF THE FOLLOWING READ?

   a. $\{3,4,5\}$ b. $\{3,4,5,\ldots\}$

   c. $\{7\}$ d. $\{\phi\}$

   e. $\{x \mid x \in N \text{ and } x<5\}$

2. WRITE USING THE LISTING METHOD

   a. $\{x \mid x \in W \text{ AND } x<4\}$

   b. $\{x \mid x \in W \text{ AND } x<2\}$

3. WRITE USING SET-BUILDER NOTATION

   a. $\{0,1,2,3,4\}$ b. $\{\ldots,-3,-2,-1\}$

4. NAME A RATIONAL NUMBER THAT IS NOT AN INTEGER.

5. NAME A REAL NUMBER THAT IS NOT A RATIONAL NUMBER.

6. WRITE AS A REPEATING DECIMAL

   a. $\frac{5}{7}$ b. $\frac{17}{990}$

7. WRITE AS A FRACTION OF INTEGERS.

a) $32.3\overline{14}$ b) $.05\overline{673}$ c) $.27$

8. EACH OF THE FOLLOWING IS A MEMBER OF WHICH OF THESE SETS: N, W, I, Q, Ir, R

Example: $2/3$ Q, R

a. -3 b. π c. $\frac{0}{7}$ d. $.36$

e. $.\overline{36}$ f. $.525225222522225\ldots$

9. BY THE LISTING METHOD NAME A SET T SUCH THAT $T \subseteq \{1,2,3,4,5\}$

10. TRUE OR FALSE

a. $\{3\} \in \{1,2,3\}$ b. $\{2,3\} \in \{1,2,3\}$

c. $3 \subseteq \{1,2,3\}$ d. $\{3\} \subseteq \{1,2,3\}$

e. $\{2,3\} \subseteq \{1,2,3\}$ f. $\{1,2,3\} \subseteq \{1,2,3\}$

g. $\{1,2,3\} = \{1,2,3\}$ h. $\{3,2,1\} = \{1,2,3\}$

i. $\sqrt{2} \in I$ j. $\sqrt{2} \in Ir$

k. $\sqrt{2} \subseteq Ir$ l. $Q \subseteq Ir$

m. $\{\sqrt{2}, \sqrt{3}\} \subseteq Ir$ n. $\{2, \sqrt{2}\} \subseteq Ir$

K. DUMMY VARIABLES: x, y, z ARE DUMMY VARIABLES: $\{x \mid x \in N \text{ AND } x<3\} = \{y \mid y \in N \text{ AND } y<3\} = \{z \mid z \in N \text{ AND } z<3\}$

REPLACE A DUMMY VARIABLE WITH 2:

$2 \in N$ AND $2<3$ TRUE 2 IS IN THE SET.

REPLACE A DUMMY VARIABLE WITH 4:

$4 \in N$ AND $4<3$ FALSE 4 IS NOT IN THE SET.

L. IFF MEANS IF AND ONLY IF

EXAMPLE: YOU HAVE 12 IFF YOU HAVE A DOZEN. (A COMMON FATE ON EITHER SIDE OF THE IFF.)

M. UNION $\cup$, INTERSECTION $\cap$

DEFINITION:

$H \cup K = \{x \mid x \in H \text{ OR } x \in K\}$

$H \cap K = \{x \mid x \in H \text{ AND } x \in K\}$

LET $A = \{2,3,4,5\}$ $B = \{3,5,6,7\}$

$A \cap B = \{3,5\}$ $A \cup B = \{2,3,4,5,6,7\}$

$(A \cap B) \cup \{0\} = \{0,3,5\}$

N. CLOSURE PROPERTIES

1. CLOSED FOR $+$: ADD ANY 2 REALS AND YOU GET A REAL $5+2=7$
2. CLOSED FOR $\cdot$: MULTIPLY ANY 2 REALS AND YOU GET A REAL. $5\cdot 2=10$
3. CLOSED FOR $-$: SUBTRACT ANY 2 REALS AND YOU GET A REAL. $5-2=3$
4. NOT CLOSED FOR $\div$: $5\div 0$ NOT DEFINED.

O. COMMUTATIVE PROPERTY DEFINITION

1. OF ADDITION : $a+b=b+a$

$$2+3=3+2 \qquad 5+\boxed{}=\boxed{}+5$$

$$5+(4+6)=(4+6)+5$$

2. OF MULTIPLICATION : $a\cdot b=b\cdot a$

$$2\cdot 3=3\cdot 2 \qquad 2\cdot\boxed{}=\boxed{}\cdot 2$$

$$2(x+3y)=(x+3y)\cdot 2$$

$$5+4\cdot 3=5+3\cdot 4$$

P. NOTE : THE MULTIPLICATION SYMBOL $\cdot$ CAN BE UNDERSTOOD IF LEFT OFF

$$x\cdot y=xy \qquad 3\cdot 2=(3)(2) \text{ NOT } 32$$

$$4\cdot(x+y)=4(x+y)$$

Q **ASSOCIATIVE PROPERTY DEFINITION**

1. OF ADDITION $L+(M+R) = (L+M)+R$

$2+(3+5)=(2+3)+5$

$(\boxed{}+x)+y = \boxed{}+(x+y)$

$((2+z)+x)+y = (2+z)+(x+y)$

2. OF MULTIPLICATION $L\cdot(M\cdot R)=(L\cdot M)\cdot R$

$2(xy)=(2x)y$ $\quad \boxed{}(xy)=(\boxed{}x)y$

$(2+z)(xy)=((2+z)x)y$

3. WHICH PROPERTY IS ILLUSTRATED?

a. $5+(3+x) = 5+(x+3)$

COMMUTATIVE PROPERTY OF +

b. $5+(3(xy)+2) = 5+((3x)y+2)$

ASSOCIATIVE PROPERTY OF •

R. **IDENTITIES**

1. ADDITIVE IDENTITY 0

$x+0 = x$

2. MULTIPLICATIVE IDENTITY 1

$x\cdot 1 = x$

3. ILLUSTRATED $x+3z = x\cdot 1+3z$

$(3+y)+0 = (3+y)$

S. DISTRIBUTIVE PROPERTY DEFINITION

$a(b+c) = ab + ac$ OR

$(a+b)c = ac + bc$

$2x + 2y = 2(x+y)$ CALLED FACTORING

$\boxed{}(x+y) = \boxed{}x + \boxed{}y$

$(e+f)(x+y) = (e+f)x + (e+f)y$

$= ex + fx + ey + fy$

$= ex + ey + fx + fy$

FOIL — FIRST, OUTSIDE, INSIDE, LAST

T. INVERSES

1. ADDITIVE: $-x$ "MINUS x", "THE NEGATIVE OF x", "THE OPPOSITE OF x"

   $x + (-x) = 0$ EVERY REAL HAS AN ADDITIVE INVERSE.

2. MULTIPLICATIVE: $\frac{1}{x}$ "THE RECIPROCAL OF x" $x \neq 0$

   $x \cdot \frac{1}{x} = 1$ 0 HAS NO MULTIPLICATIVE INVERSE

   $\frac{1}{0} \leftarrow$ BAD $\frac{0}{1} = 0 \leftarrow$ GOOD

U. NOTATION YOU CAN USE

1. $\frac{a}{b} = a \cdot \frac{1}{b} = a \div b \underset{\text{CAREFUL}}{=\!=} a/b$

$$\frac{3}{x+y} = 3 \cdot \frac{1}{x+y} = 3 \div (x+y) = 3/(x+y)$$

$$3/(x+y) \neq 3/x+y$$

2. $a - b = a + (-b)$

V. GETTING READY FOR HOMEWORK

1. MULTIPLICATIVE INVERSE FOR $\frac{2}{3}$

$\frac{2}{3} \cdot \boxed{\frac{3}{2}} = 1 \qquad \frac{2}{3} \cdot \frac{1}{\frac{2}{3}} = 1$ SO

$\frac{3}{2} = \frac{1}{\frac{2}{3}}$ ANSWER $\frac{3}{2}$

2. HINT FOR MULTIPLICATIVE INVERSE OF $0.222\overline{2} = 0.22222\ldots$
CONVERT TO A FRACTION BY METHOD PREVIOUSLY LEARNED, THEN TAKE THE RECIPROCAL

W. HOMEWORK (OIS)

1. NAME AN ELEMENT OF $\{q \mid q \in N \text{ and } q < 10\}$.
2. $H = \{3, 4, 7\}$ $K = \{5, 6, 7, 8\}$

   $H \cap K =$ $H \cup K =$

   $H \cap \{5\} =$ $H \cup \{5\} =$
3. CAN YOU DIVIDE BY 0? CAN 0 BE DIVIDED BY A NON-ZERO NUMBER?
4. STATE COMPLETELY THE PROPERTY

   a. $6x + 7 = x6 + 7$

   b. $(2x + y) + z = 2x + (y + z)$

   c. $5(xy + z) = 5(yx + z)$

   d. $5(xy + z) = 5(z + xy)$

   e. $5(xy + z) = 5xy + 5z$

   f. $5(x(mn) + 3) = 5((xm)n + 3)$

   g. $5(x+3) + z(x+3) = (5 + z)(x + 3)$

   h. $5(x+3) + [-5(x+3)] = 0$

   i $mx + x = mx + x \cdot 1$
5. FIND THE MULTIPLICATIVE INVERSES OF $7/9$, $.12\overline{12}$, -1

[CHAPTER 3]

PROPERTIES OF REALS & ABSOLUTE VALUES

A. UNDERSTANDING MINUS —

1. $-a = -1 \cdot a$ (HENCE, $-(x+y) = -1 \cdot (x+y)$)
2. $-(-a) = a$
3. $(-a)(-b) = ab$
4. $-(a-b) = b-a$

PROOF: $-(a-b) = -1 \cdot (a-b) =$
$-1(a+(-b)) = -1 \cdot a + (-1)(-b) =$
$-a + 1 \cdot b = -a + b = b+(-a) = b-a$

5. $-(ab) = (-a)b = a(-b)$
6. $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

NOTE: $\frac{x-y}{y-x} = \frac{x-y}{-(x-y)} = \frac{\cancel{x-y}}{(-1)\cancel{(x-y)}} = \frac{1}{-1} = -1$

7. $-b$ CAN BE POSITIVE!

LET $b = -5$. $-b = -(-5) = 5$

DO NOT THINK THAT A MINUS SIGN IN FRONT OF A NUMBER MEANS THE WHOLE THING IS A NEGATIVE NUMBER.

B. RELATIONS

1. $<$ IS LESS THAN
2. $>$ IS GREATER THAN
3. $\leq$ IS LESS THAN OR EQUAL TO
4. $\geq$ IS GREATER THAN OR EQUAL TO
5. NOTE: $x < y$ IFF $y > x$

   $5 < 8$ SO $8 > 5$
6. TRICHOTOMY ONLY 1 IS TRUE

   $x < y \qquad x = y \qquad x > y$
7. NEGATE $x < y$ YOU GET $x \geq y$

C. PROPERTIES OF EQUALS

1. $x = x$
2. IF $x = y$, THEN $y = x$

   EXAMPLE IF $0 = x^2 - 9$, $x^2 - 9 = 0$
3. IF $x = y$ AND $y = z$, THEN $x = z$

   TRANSITIVITY
4. CAN SUBSTITUTE EQUALS FOR EQUALS

   EXAMPLE: GIVEN $a = b$, SO

   $3a + 7 = 3b + 7$

D. ABSOLUTE VALUE BUILD-UP

YOUR TASK: GIVEN A REAL NUMBER, EITHER DO NOTHING TO IT OR PUT A MINUS IN FRONT OF IT. AFTER YOUR TASK IS COMPLETED THE RESULT IS TO BE ≥ 0.

TASK 1 CANDIDATE p, WHERE $p \geq 0$

YOU DO NOTHING. RESULT $p \geq 0$

TASK 2 CANDIDATE q, WHERE $q < 0$

YOU PUT A MINUS IN FRONT

RESULT $-q$, WHICH IS ≥ 0

E. ABSOLUTE VALUE DEFINITION

$|a|$ IS READ "ABSOLUTE VALUE OF a"

$$|a| = \begin{cases} a & \text{IF } a \geq 0 \\ -a & \text{IF } a < 0 \end{cases}$$

$|5| = 5$ SINCE $5 \geq 0$

$|-5| = -(-5) = 5$ SINCE $-5 < 0$

F. ABSOLUTE VALUE SIGN REMOVAL

IF INSIDE ABSOLUTE VALUE SIGN IS ≥ 0 WRITE IT DOWN. IF INSIDE ABSOLUTE VALUE SIGN IS < 0, WRITE IT DOWN AND PUT A MINUS IN FRONT OF IT. (DO ALL THIS EVEN IF RELUCTANT!) NOTE: THE RESULT WILL BE ≥ 0.

SUPPOSE $a < -2$ AND $b > 7$

$$|\underset{\text{NEG}}{a}| = -a \qquad |\underset{\text{POS}}{b}| = b$$

$$|\underbrace{\underset{\text{NEG}\cdot\text{POS}}{a \cdot b}}_{\text{NEG}}| = -a \cdot b$$

$$|\underbrace{\underbrace{-3}_{\text{NEG}} \cdot \underset{\text{NEG}}{a} \cdot \underset{\text{POS}}{b}}_{\text{POS}}| = -3ab$$

$$|b-a| = |\underbrace{b+(-a)}_{\substack{\text{POS} + \text{POS} \\ \text{POS}}}| = b-a$$

$$|a-b| = |\underbrace{\underset{\text{NEG} + \text{NEG}}{a+(-b)}}_{\text{NEG}}| = -(a-b) = b-a$$

3-22 A

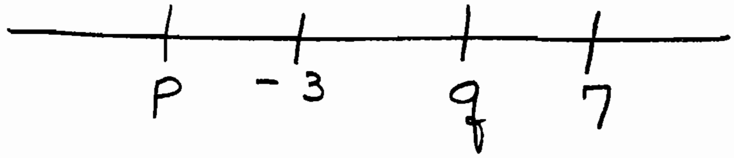
$\boxed{\text{L, R ANALYSIS}}$

$L < R$

$L - R < 0$ Left − Right is negative

$0 < R - L$ Right − Left is positive

EXAMPLES:



$\overset{L-R}{p - q}$ NEGATIVE $\qquad \overset{L-R}{p - q} < 0$

$\overset{R-L}{7 - q}$ POSITIVE $\qquad 7 - q > 0$

$q + 3 = \overset{R-L}{q - (-3)}$ positive $\qquad q + 3 > 0$

$\overset{R-L}{-3 - p}$ POSITIVE $\qquad -3 - p > 0$

NOW APPLY TO ABSOLUTE VALUES:

$$\Big|\overset{\substack{NEG\\ L-R}}{q - 7}\Big| = -(q-7) = 7 - q$$

$$|p+3| = \Big|\overset{\substack{NEG\\ L-R}}{p - (-3)}\Big| = -(p+3) = -p - 3$$

G. ABSOLUTE VALUE PROPERTIES

1. $|a| \geq 0$
2. $|ab| = |a| \cdot |b|$
3. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
4. $|a+b| \leq |a| + |b|$

H. ABSOLUTE VALUE AND DISTANCE

1. $|a|$ DISTANCE BETWEEN a AND ZERO.
2. $|a-b|$ DISTANCE BETWEEN a AND b

   a. DISTANCE BETWEEN 3 AND 5

   $$|3-5| = |-2| = -(-2) = 2$$

   b. DISTANCE BETWEEN -2 AND 1

   $$|-2-1| = |-3| = -(-3) = 3$$

I. PRECEDENCE OF OPERATIONS

HIGH 1. GROUPING SYMBOLS

2. $\times, \div$ (TIE BREAK LEFT TO RIGHT)

LOW 3. $+, -$ (TIE BREAK LEFT TO RIGHT)

EXAMPLES FOLLOW PUTTING IN THE UNDERSTOOD GROUPING SYMBOLS

1. $3 + 2 \cdot 4 = 3 + (2 \cdot 4) = 3 + 8 = 11$

2. $5 - 8 \div 2 = 5 - (8 \div 2) = 5 - 4 = 1$

3. $5 + 6 \div 2 \cdot 4 = 5 + (6 \div 2) \cdot 4 = 5 + 3 \cdot 4$
   $= 5 + (3 \cdot 4) = 5 + 12 = 17$
   SO $5 + 6 \div 2 \cdot 4 = 5 + ((6 \div 2) \cdot 4)$

4. $5 - 3 + 2 = (5 - 3) + 2 = 2 + 2 = 4$

5. $(5+7) \div 2 \cdot 4 = 12 \div 2 \cdot 4 = (12 \div 2) \cdot 4$
   $= 6 \cdot 4 = 24$
   SO $(5+7) \div 2 \cdot 4 = ((5+7) \div 2) \cdot 4$

6. $5 + 7 \div (2 \cdot 4) = 5 + 7 \div 8 = 5 + (7 \div 8)$
   $= 5 + \frac{7}{8} = \frac{47}{8}$
   SO $5 + 7 \div (2 \cdot 4) = 5 + (7 \div (2 \cdot 4))$

7. DO NOT THINK THE FOLLOWING SHOULD BE <u>EQUALITIES</u>

$$5 - 2x \neq 3x$$

$$4 + 3x \neq 7x$$

J. HOMEWORK (OIS)

1. SIMPLIFY EACH OF THE FOLLOWING

   a. $(-x)(-y)(z)(-w)$

   b. $\dfrac{p-q}{q-p}$

2. GIVEN $z>5$ AND $w<-3$. WRITE YOUR ANSWER WITHOUT ABSOLUTE VALUE SIGNS.

   $|z| =$ $\quad |w| =$ $\quad |zw| =$

   BE SURE TO GIVE BRIEF WRITTEN REASONS FOR YOUR ANSWER.

   $|3zw| =$ $\quad |-3zw| =$ $\quad |5-z| =$

   $|z-w| =$ $\quad |w-2z| =$ $\quad |w+3| =$

3. FIND A SPECIFIC VALUE FOR a AND b SO THAT $|a+b| < |a| + |b|$.

4. EVALUATE EACH OF THE FOLLOWING

   a. $7-2\cdot 3$ $\qquad$ b. $6\cdot 3+4$

   c. $2+3\cdot 4$ $\qquad$ d. $5-6\div 2\cdot 4$

   e. $5-4+2$ $\qquad$ f. $2\cdot 6\div 3-4\cdot 2$

   g. $2\cdot 6\div(3-4)\cdot 2$

   h. $2\cdot(6\div 3-4\cdot 2)$

4-26

[CHAPTER 4]

EXPONENTS AND RADICALS

A. USING PROPERTIES OF THE REALS TO SIMPLIFY EXPRESSIONS WITH VARIABLES.

1. $4x+3x = (4+3)x = 7x$

   DISTRIBUTIVE

2. $-2(x-4)-(x+4) =$

   $-2x+8-x-4 = -2x-x+8-4$

   $= (-2-1)x+4 = -3x+4$

3. $$\frac{3(x-2y)+5y}{2(y+2x)-y-7x} = \frac{3x-6y+5y}{2y+4x-y-7x}$$

   $$= \frac{3x-y}{y-3x} = \frac{-(y-3x)}{y-3x} = -1$$

B. POSITIVE INTEGER EXPONENTS DEF.

$a^1 = a \qquad a^2 = a \cdot a \qquad a^3 = a \cdot a \cdot a$

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

$a^{n+1} = a^n \cdot a$

1. $2^3 = 2 \cdot 2 \cdot 2 = 8$
2. $(-3)^2 = (-3)(-3) = 9$

C. UNARY MINUS AND PRECEDENCE OF EXPONENTIATION

1. UNARY MINUS: THE OPERATION OF TAKING THE ADDITIVE INVERSE OF A NUMBER.

OPERATOR	OPERAND	RESULT
$-$ UNARY MINUS	3	-3

2. BINARY MINUS: THE OPERATION OF SUBTRACTION (2 OPERANDS)

OPERATOR	OPERANDS	RESULT
$-$ BINARY MINUS	5, 2	$5-2 = 3$

3. PRECEDENCE: EXPONENTIATION BEFORE UNARY MINUS

$$-3^2 = -(3^2) = -(3 \cdot 3) = -9$$

$$(-3)^2 = (-3)(-3) = 9$$

D. PRECEDENCE LIST

HIGH 1. GROUPING SYMBOLS - INNERMOST FIRST

2. EXPONENTIATION

3. UNARY MINUS

4. $\times, \div$ (TIE BREAK LEFT TO RIGHT)

LOW 5 $+$, BINARY MINUS (TIE BREAK LEFT TO RIGHT)

EXAMPLE $-2^3-\left(3+(-5)\right)^2 = -2^3-(-2)^2$

$= -(2\cdot2\cdot2)-[(-2)\cdot(-2)] = -8-4 = -12$

E. $\text{NEGATIVE}^{\text{EVEN}} = \text{POSITIVE}$

$\text{NEGATIVE}^{\text{ODD}} = \text{NEGATIVE}$

1. $(-2)^4 = (-2)(-2)(-2)(-2) = 16$
2. $(-2)^3 = (-2)(-2)(-2) = -8$

F. $(\text{NON-ZERO})^0 = 1$

$5^0 = 1 \qquad (-7)^0 = 1$

$$\left(\frac{3x^2+1}{2x^4+5}\right)^0 = 1$$

G. NEGATIVE INTEGER EXPONENTS DEF.

$$a^{-n} = \frac{1}{a^n} \qquad n \text{ a positive integer}$$

1. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
2. $5^{-1} = \frac{1}{5^1} = \frac{1}{5}$
3. $\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}} = \frac{1}{\frac{1}{8}} = 1\cdot\frac{8}{1} = 8$

H. EXPONENT LAWS

1. $\boxed{a^m \cdot a^n = a^{m+n}}$

$$a^3 \cdot a^2 = (a\cdot a\cdot a)\cdot(a\cdot a) = a^5 = a^{3+2}$$

2. $\boxed{a^m = \frac{1}{a^{-m}}, \quad a^{-m} = \frac{1}{a^m}}$

$$a^3 = \frac{1}{a^{-3}} \qquad a^{-4} = \frac{1}{a^4}$$

3. $\boxed{\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}}$

$$\frac{5^3}{5^7} = 5^{3-7} = 5^{-4} \qquad \frac{5^3}{5^7} = \frac{1}{5^{7-3}} = \frac{1}{5^4}$$

4. $\boxed{(a^m)^n = a^{m \cdot n}}$

$(5^3)^2 = 5^{3 \cdot 2} = 5^6$

$(5^3)^2 = 5^3 \cdot 5^3 = 5^{3+3} = 5^6$

5. $\boxed{a^m b^m = (ab)^m}$

$(2x)^3 = 2^3 \cdot x^3$

$(2x)^3 = 2x \cdot 2x \cdot 2x = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x = 2^3 x^3$

6. $\boxed{\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n}$

$\left(\dfrac{x}{2}\right)^3 = \dfrac{x^3}{2^3} = \dfrac{x^3}{8}$

$\left(\dfrac{x}{2}\right)^3 = \dfrac{x}{2} \cdot \dfrac{x}{2} \cdot \dfrac{x}{2} = \dfrac{x^3}{2^3} = \dfrac{x^3}{8}$

7. $\boxed{a^0 = 1 \qquad a \neq 0}$ DEFINITION

I. PRACTICE USING THE PROPERTIES.

WRITE WITH NO NEGATIVE EXPONENTS.

1. $(2x^3)^5 = 2^5 \cdot (x^3)^5 = 32x^{15}$

2. $\left(\dfrac{5x^2}{3y^{-3}}\right)^4 = \dfrac{(5x^2)^4}{(3y^{-3})^4} = \dfrac{5^4x^8}{3^4y^{-12}}$

$$= \frac{625x^8y^{12}}{81}$$

3. $\left(\dfrac{2x^{-4}y^3}{8x^5y^{10}}\right)^2 = \left(\dfrac{1}{4x^{5+4}y^{10-3}}\right)^2$

$$= \left(\frac{1}{4x^9y^7}\right)^2 = \frac{1^2}{4^2x^{18}y^{14}} = \frac{1}{16x^{18}y^{14}}$$

J. JUGULAR PROBLEMS:

A GROUP OF PROBLEMS THAT CONTAIN MUCH OF THE ESSENCE OF THE COURSE. EACH ONE REQUIRES MANY PREVIOUS SKILLS TO WORK THE PROBLEM.

K. JUGULAR PROBLEM #1: FILL IN THE BOXES

$$\left(\frac{a^p x^q y^r}{b^s x^t y^u}\right)^m = a^{\square} b^{\square} x^{\square} y^{\square}$$

$$\left(\frac{2x^3y^{-5}}{16x^{-7}y^{-10}}\right)^2 = 2^{\square} x^{\square} y^{\square}$$ FILL IN THE BOXES

$$\left(\frac{2x^3y^{-5}}{16x^{-7}y^{-10}}\right)^2 = \left(\frac{x^{3+7}y^{-5+10}}{8}\right)^2 =$$

$$\left(\frac{x^{10}y^5}{2^3}\right)^2 = \left(2^{-3}x^{10}y^5\right)^2 = 2^{\boxed{-6}} x^{\boxed{20}} y^{\boxed{10}}$$

L. SOME ROOKIES THINK THE FOLLOWING SHOULD BE EQUALITIES. THEY AREN'T.

$$\frac{1}{3} \neq -3$$

$$\frac{1}{9} = \frac{1}{3^2} \neq -3^2$$

THE TRUTH IS $\frac{1}{3^2} = 3^{-2}$

$^{4-33}$

L. HOMEWORK (OIS)

1. SIMPLIFY $-3(x-4)-2(x+3)$
2. SIMPLIFY $-(x-5)+3(2-5x)$
3. SIMPLIFY $\dfrac{4(x-2y)+7y}{3(2x-6y)+19y-10x}$
4. EVALUATE $-3^2-(4-2)^3\div 2$
5. EVALUATE $-5^3+(4-5)^4\cdot 3$
6. $(-1)^{27}=$ $\qquad (-1)^{672}=$
7. $\left(\dfrac{5+(x^2+2)}{3+(4x^6+1)}+3\right)^0=$
8. WRITE WITH NO NEGATIVE EXPONENTS

   a. $(-2x^{-3})^2$ $\qquad$ b. $\left(\dfrac{5x^{-2}}{y^2}\right)^3$

   c. $(3x^2y^{-1})^5\cdot(-2x^{-3}y)^2$

   d. $\left(\dfrac{5x^{-2}y^3}{15x^5y^{-7}}\right)^4$

9. FILL IN THE BOXES (SHOW YOUR WORK)

a. $$\left(\frac{5x^{4}y^{-7}}{125x^{-3}y^{10}}\right)^{3} = 5^{\square}x^{\square}y^{\square}$$

b. $$\left(\frac{3x^{6}y^{5}}{81x^{-5}y^{10}}\right)^{4} = 3^{\square}x^{\square}y^{\square}$$

10. $p < -7 \qquad q > 5$

$|-3pq| =$ REMOVE ABSOLUTE VALUE SIGNS. EXPLAIN

11. $\{2,4,6,8\} \cap \{5,6,7,8\} =$

12. $\{x \mid x < 3 \text{ AND } x > 1\} \cap \{x \mid x > 2\} =$

M. SCIENTIFIC NOTATION

1. EXAMPLE 2.345×10^5

   HAVE ONLY ONE NONZERO DIGIT TO THE LEFT OF THE DECIMAL, MULTIPLIED BY A POWER OF 10.

2. PUT INTO SCIENTIFIC NOTATION

   $323.4 = 3.234 \times 10^2$

   THE DECIMAL WAS MOVED 2 TO THE LEFT SO MULTIPLYING BY 10^2 HAS THE EFFECT OF MOVING THE DECIMAL BACK TO ITS ORIGINAL POSITION

3. PUT .00045 IN SCIENTIFIC NOTATION

   $.00045 = 4.5 \times 10^{-4}$

   DECIMAL MOVED TO THE RIGHT 4 PLACES SO MULTIPLY BY 10^{-4}

4. PUT EACH PART IN SCIENTIFIC NOTATION AND SIMPLIFY

$$\frac{(.000063)(3000)}{(2100000)} = \frac{(6.3\times 10^{-5})(3.0\times 10^{3})}{2.1\times 10^{6}}$$

$$= (3)(3.0)10^{-5+3-6} = 9.0\times 10^{-8}$$

N. $n^{\underline{th}}$ ROOT OF x

1. NOTATION $x^{\frac{1}{n}} = \sqrt[n]{x}$

2. $\boxed{\begin{array}{l}\underline{\text{DEFINITION}} \text{ FOR } n \text{ AN } \underline{\text{ODD}} \\ \text{POSITIVE INTEGER} \quad x^{\frac{1}{n}} = \sqrt[n]{x} \\ x^{\frac{1}{n}} = b \text{ IFF } b^{n} = x\end{array}}$

a. $8^{\frac{1}{3}} = 2$ SINCE $2^3 = 8$

b. $(-27)^{\frac{1}{3}} = -3$ SINCE $(-3)^3 = -27$

c. $(-1)^{\frac{1}{5}} = \sqrt[5]{-1} = -1$ SINCE $(-1)^5 = -1$

d. $\sqrt[5]{32} = 2$ SINCE $2^5 = 32$

3) DEFINITION OF $\sqrt[n]{x}$ FOR n AN <u>EVEN</u> POSITIVE INTEGER AND $x \geq 0$.

$$x^{\frac{1}{n}} = \sqrt[n]{x} = b \text{ IFF } b \geq 0 \text{ AND } b^n = x$$

a. $(-2)^4 = 16$ AND $2^4 = 16$ SO IS $\sqrt[4]{16} = 2$ OR $\sqrt[4]{16} = -2$?

ANSWER : 2 SINCE $2 \geq 0$

THE DEFINITION SAYS THE ANSWER b MUST BE ≥ 0.

(SOME BOOKS CALL BOTH $2, -2$ n^{th} ROOTS AND 2 THE <u>PRINCIPAL</u> n^{th} ROOT. ACCORDING TO OUR DEFINITION $\sqrt[4]{16} = 2$)

b. $\sqrt{x}$ MEANS $\sqrt[2]{x}$

(READ "THE SQUARE ROOT OF x")

$\sqrt{9} = 3$ $\qquad$ $\sqrt{25} = 5$

c. $\sqrt[4]{\frac{1}{16}} = \frac{1}{2}$

d. <u>BAD</u> $\sqrt[\text{even}]{\text{NEGATIVE}} = (\text{NEGATIVE})^{\frac{1}{\text{even}}}$

$\sqrt[4]{-16}$ ← BAD. THERE IS NO $b \geq 0$ SUCH THAT $b^4 = -16$

e. $\sqrt{x^2} = |x|$ (NOT x !)

ILLUSTRATION: LET $x = -3$

$\sqrt{x^2} = \sqrt{(-3)^2} = \sqrt{9} = 3$ (NOT -3)

$= |-3| = |x|$

SO $\sqrt{(y+2)^2} = |y+2|$

LIKEWISE, $\sqrt[4]{x^4} = |x|$

$\sqrt[6]{x^6} = |x|$ $\sqrt[8]{x^8} = |x|$, ... etc.

DEFINITION: 4-39

0. RATIONAL (FRACTIONAL) EXPONENTS

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(a^m\right)^{1/n}$$ PROVIDED THESE ARE DEFINED. THE TOP (m) IS THE POWER. THE BOTTOM (n) IS THE ROOT. (I. MUCIO)

1. $8^{2/3} = \left(8^{1/3}\right)^2 = 2^2 = 4$

2. $(-27)^{-4/3} = \dfrac{1}{(-27)^{4/3}} = \dfrac{1}{\left[(-27)^{1/3}\right]^4}$

$= \dfrac{1}{(-3)^4} = \dfrac{1}{81}$

3. BAD $(-16)^{3/4}$ $\quad(-16)^{1/4}$ NOT DEFINED

4. ALTERNATE WAY OF WORKING $8^{2/3}$ (NOT RECOMMENDED)

$8^{2/3} = \left(8^2\right)^{1/3} = 64^{1/3} = 4$

THIS <u>EXPLODES</u> THE NUMBER FIRST, THEN YOU <u>SHRINK</u> IT. THE $\left(8^{1/3}\right)^2$ METHOD 1ST <u>SHRINKS</u> THEN EXPLODES - GENERALLY EASIER

5. RADICAL NOTATION FOR $x^{m/n}$

INDEX → $\sqrt[n]{x^m} = (x^m)^{1/n} = x^{m/n}$

$$\left(\sqrt[n]{x}\right)^m = (x^{1/n})^m = x^{m/n}$$

6. SOME SIMPLIFICATION

ODD INDEX

a. $\sqrt[5]{x^5} = x^{\frac{5}{5}} = x^1 = x$

b. $\sqrt[5]{x^{10}} = x^{\frac{10}{5}} = x^2$

c $\sqrt[5]{x^{15}} = x^{\frac{15}{5}} = x^3$

EVEN INDEX

d. $\sqrt[4]{x^4} = |x|$

e. $\sqrt[4]{x^8} = \sqrt[4]{(x^2)^4} = |x^2| = x^2$ SINCE $x^2 \geq 0$

f. $\sqrt[4]{x^{12}} = \sqrt[4]{(x^3)^4} = |x^3|$

g. SUPPOSE $x \geq 0$ $\sqrt[4]{x^{20}} =$

$\sqrt[4]{(x^5)^4} = |x^5| = x^5$ SINCE $x^5 \geq 0$

7. <u>CAREFUL</u> : $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

<u>PROVIDED EACH IS DEFINED</u>

a. $\sqrt[4]{16 \cdot 81} = \sqrt[4]{16} \cdot \sqrt[4]{81} = 2 \cdot 3 = 6$

b. $6 = \sqrt[4]{(-16)\cdot(-81)} \neq \sqrt[4]{-16} \cdot \sqrt[4]{-81}$

↑

NOT DEFINED

c. NOTE THIS IS SAYING $(ab)^{1/n} = a^{1/n} b^{1/n}$

PROVIDED EACH IS DEFINED

RECALL, BAD $\Rightarrow$ $(\text{NEGATIVE})^{\frac{1}{\text{even}}}$

8. <u>CAREFUL</u> : $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ PROVIDED

EACH EXISTS

a. $\sqrt[4]{\dfrac{16}{81}} = \dfrac{\sqrt[4]{16}}{\sqrt[4]{81}} = \dfrac{2}{3}$.

b. $\dfrac{2}{3} = \sqrt[4]{\dfrac{-16}{-81}} \neq \dfrac{\sqrt[4]{-16}}{\sqrt[4]{-81}}$ ← NOT DEFINED

c. NOTE THIS IS SAYING $\left(\dfrac{a}{b}\right)^{1/n} = \dfrac{a^{1/n}}{b^{1/n}}$

PROVIDED EACH IS DEFINED

9. WHEN YOU HAVE $(x \cdot y)^{m/n}$ OR $\sqrt[n]{x \cdot y}$ AND $x, y > 0$ YOU CAN SAY $(xy)^{m/n} = x^{m/n} y^{m/n}$ AND $\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$

10. WHEN ALL IS DEFINED THE LAWS OF EXPONENTS WORK THE SAME AS FOR INTEGERS.

P. FRACTION ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION

1. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

2. $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

3. $\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d}{b \cdot d} + \frac{b \cdot c}{b \cdot d} = \frac{ad + bc}{bd}$

4. $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

5. $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a}{b} \div \frac{c}{d}$

6. EXAMPLES

a. $\frac{2}{5} + \frac{7}{5} = \frac{9}{5}$

b. $\frac{2}{3} \cdot \frac{5}{7} = \frac{10}{21}$

c. $\frac{2}{3} + \frac{4}{5} = \frac{2 \cdot 5 + 3 \cdot 4}{3 \cdot 5} = \frac{10 + 12}{15} = \frac{22}{15}$

$5\frac{2}{3} = 5 + \frac{2}{3} = \frac{5}{1} + \frac{2}{3} = \frac{15+2}{3} = \frac{17}{3}$

d. $\frac{2}{5} - \frac{3}{7} = \frac{14-15}{35} = \frac{-1}{35}$

e. $\frac{2/5}{8/7} = \frac{2}{5} \cdot \frac{7}{8} = \frac{\overset{1}{\cancel{2}}}{5} \cdot \frac{7}{\cancel{8}_4} = \frac{7}{20}$

7. JUGULAR #1 REVISITED WITH FRACTIONAL EXPONENTS

$$\left(\frac{32\, x^{1/3}}{x^{5/3}\, y^{-1/2}}\right)^{2/3} = \left(2^5 \cdot x^{1/3 - 5/3}\, y^{1/2}\right)^{2/3} =$$

$$2^{5 \cdot \frac{2}{3}}\, x^{-\frac{4}{3} \cdot \frac{2}{3}}\, y^{\frac{1}{2} \cdot \frac{2}{3}} =$$

$$2^{\boxed{\frac{10}{3}}}\, x^{\boxed{-\frac{8}{9}}}\, y^{\boxed{1/3}}$$

Q. HOMEWORK (OIS)

1. PUT IN SCIENTIFIC NOTATION

a. 0.000743

b. 3267000

c. 253.7×10^{6}

d. $\dfrac{.00036 \times 10^{7}}{12000 \times 10^{-3}}$

e. $\dfrac{(70000 \times 10^{5})(.00021 \times 10^{-4})\, 8}{(.00049 \times 10^{-3})(6000 \times 10^{5})}$

2. EVALUATE EACH OF THE FOLLOWING. SHOW THE STEP BY STEP EVALUATION PROCESS. DECLARE THOSE THAT ARE UNDEFINED.

a. $27^{1/3}$

b. $(-32)^{1/5}$

c. $\left(-\frac{1}{8}\right)^{-1/3}$

d. $(-64)^{\frac{1}{6}}$

e. $64^{1/6}$

f. $\sqrt[3]{-8}$

g. $\sqrt{64}$

h. $16^{-\frac{1}{4}}$

i. $8^{4/3}$

j. $-8^{4/3}$

k. $(-8)^{5/3}$

l. $(-8)^{-5/3}$

m. $16^{3/4}$

n. $-16^{3/4}$

o. $(-16)^{3/4}$

p. $\sqrt{m^2}$

q. $\sqrt[6]{x^{18}}$

r. $\sqrt[4]{16x^{20}}$

3. SUPPOSE $x<0$ AND $y>0$. WHICH OF THE FOLLOWING ARE TRUE AND WHY?

a. $\sqrt{xy} = \sqrt{x}\sqrt{y}$

b. $\sqrt{-xy} = \sqrt{-x}\sqrt{y}$

c. $\sqrt{-xy} = \sqrt{x}\sqrt{-y}$

d. $\sqrt{xy} = \sqrt{(-x)(-y)} = \sqrt{-x}\sqrt{-y}$

e. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$ f. $\sqrt{-\frac{x}{y}} = \frac{\sqrt{-x}}{\sqrt{y}}$

4. WRITE AS ONE FRACTION IN LOWEST TERMS

a. $\frac{3}{4}+\frac{2}{5}$ b. $\frac{3}{2}-\frac{4}{3}$ c. $2+\frac{1}{5}$ d. $3\frac{1}{8}$

e. $\frac{2}{x}-5$ f. $\frac{2}{x}+\frac{y}{3}$ g. $x-\frac{2}{3}$

5. WRITE WITH NO NEGATIVE EXPONENTS. $x, y > 0$

a. $\left(8x^{2/3}y^{5/6}\right)^{-1/3}$ b. $\left(\frac{32x^{-5}y^{1/4}}{2x^{1/3}y^{2/5}}\right)^{1/4}$

c. $\left(2x^{\frac{1}{5}}y^{-2/3}\right)^3 \cdot \left(8x^{1/6}y^{1/5}\right)^{2/3}$

6. $\left(\frac{27x^{3/5}y^{-2}}{3x^{-1/4}y^{2/3}}\right)^{1/2} = 3^{\square}x^{\square}y^{\square}$

$x>0,\ y>0$

R. PERFECT n^{th} POWER DEFINITION

1. THE EXPONENT UNDER THE RADICAL IS A MULTIPLE OF THE INDEX

2. $\sqrt[3]{x^6} = x^2 \qquad \sqrt[5]{x^{15}} = x^3$

$$\sqrt[3]{8y^9} = \sqrt[3]{2^3 y^9} = 2y^3$$

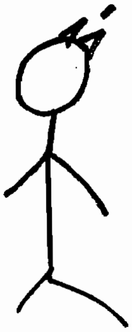
$$p \geq 0 \quad \sqrt[4]{p^{12}} = \sqrt[4]{(p^3)^4} = |p^3| = p^3$$

$$\sqrt[3]{5x^6y^7} = \sqrt[3]{5x^6y^6 \cdot y} = x^2y^2\sqrt[3]{5y}$$

S. RATIONALIZE DENOMINATOR DEF.

1. NO RADICAL IN DENOMINATOR (EVEN IN DISGUISE!)

2. $\dfrac{1}{\sqrt[3]{x}} = \dfrac{1}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x^2}}{x}$



<u>NOTE</u>: INDEX OF 3, SO YOU WANT MULTIPLES OF 3 AS POWERS UNDER THE RADICAL

I. SIMPLIFIED FORM: NO PERFECT n^{th} POWERS, DENOMINATOR IS RATIONALIZED, AND REDUCED AS FAR AS POSSIBLE

JUGULAR # 2

DEF.

1. PUT IN SIMPLIFIED FORM

$$\frac{2x^2}{\sqrt[5]{9x^2y^{18}}} = \frac{2x^2}{\sqrt[5]{3^2x^2y^{18}}} \cdot \frac{\sqrt[5]{3^3x^3y^2}}{\sqrt[5]{3^3x^3y^2}}$$

$$= \frac{2x^2\sqrt[5]{27x^3y^2}}{\sqrt[5]{3^5x^5y^{20}}} = \frac{2x^2\sqrt[5]{27x^3y^2}}{3xy^4} = \frac{2x\sqrt[5]{27x^3y^2}}{3y^4}$$

2. RADICALS IN THE DENOMINATOR IN DISGUISE

a. $\sqrt{\frac{3}{5}x} = \frac{\sqrt{3x}}{\sqrt{5}} = \frac{\sqrt{3x}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15x}}{5}$ FOR $x \geq 0$

b. $x^{-2/3} = \frac{1}{x^{2/3}} = \frac{1}{\sqrt[3]{x^2}} = \frac{1}{\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{x}$

3. FOR $x > 0$ AND $y > 0$, PUT IN SIMPLIFIED FORM

$$\frac{5x^2\sqrt[4]{y^2}}{\sqrt[4]{9xy^5}} = \frac{5x^2\sqrt[4]{y^2}}{\sqrt[4]{3^2xy^5}} \cdot \frac{\sqrt[4]{3^2x^3y^3}}{\sqrt[4]{3^2x^3y^3}} =$$

$$\frac{5x^2\sqrt[4]{3^2x^3y^5}}{\sqrt[4]{3^4x^4y^8}} = \frac{5x^2\sqrt[4]{9x^3y^4y}}{\sqrt[4]{3^4x^4(y^2)^4}} = \frac{5x^2|y|\sqrt[4]{9x^3y}}{3|x||y^2|}$$

$$\overset{\substack{x>0\\y>0}}{=} \frac{5x^2y\sqrt[4]{9x^3y}}{3xy^2} = \frac{5x\sqrt[4]{9x^3y}}{3y}$$

U. EXPRESSIONS WITH RADICALS $(x \geq 0)$

1. $\sqrt{9} + \sqrt{16} \neq \sqrt{9+16}$, SO IT IS NOT NECESSARILY TRUE THAT $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$

2. $\sqrt{27} - 5\sqrt{3} + x\sqrt{3} = \sqrt{9 \cdot 3} - 5\sqrt{3} + x\sqrt{3}$
$= 3\sqrt{3} - 5\sqrt{3} + x\sqrt{3} = (3-5+x)\sqrt{3} = (x-2)\sqrt{3}$

3. $2 + 3\sqrt{x} \neq (2+3)\sqrt{x}$

4. $5\sqrt{x} + \sqrt{18x^3} - 2\sqrt{y} = 5\sqrt{x} + \sqrt{9x^2 \cdot 2x} - 2\sqrt{y}$
$= 5\sqrt{x} + 3|x|\sqrt{2x} - 2\sqrt{y}$ since $x \geq 0$
$5\sqrt{x} + 3x(\sqrt{2})\sqrt{x} - 2\sqrt{y} =$
$(5 + 3\sqrt{2}\,x)\sqrt{x} - 2\sqrt{y}$

V. HOMEWORK (OIS) $(x>0,\ y>0)$

1. PUT EACH OF THE FOLLOWING IN SIMPLIFIED FORM.

a. $\sqrt[3]{x^5y^{10}}$ b. $\sqrt[3]{16x^8y^2}$

c. $\dfrac{2}{\sqrt[5]{x^7}}$ d. $\dfrac{5}{\sqrt{8xy}}$

e. $\dfrac{3x\sqrt[4]{y^3}}{\sqrt[4]{32x^5y^{10}}}$ f. $\dfrac{\sqrt[5]{\frac{2}{3}x^3}}{\sqrt[5]{4x^{11}y^3}}$

g. $5x^{-\frac{6}{7}}$ h. $\dfrac{\sqrt[3]{2x^{-1}}}{\sqrt[3]{7x^4y}}$

2. PERFORM THE INDICATED OPERATIONS

a. $5\sqrt{8}-x\sqrt{2}$ b. $\sqrt{75}-\sqrt{27}+\sqrt{3}$

c. $\sqrt{x^5}-2\sqrt{x}$ d. $\sqrt{y}-\sqrt{x^7y}+\sqrt{3y}$

[CHAPTER 5]

POLYNOMIALS

A. THIS COURSE WILL STUDY POLYNOMIAL EXPRESSIONS, FUNCTIONS, EQUATIONS.

B. <u>POLYNOMIAL EXPRESSIONS</u>

1. POLYNOMIAL EXPRESSION EXAMPLES

$3x^2+5x-\frac{1}{2}$, $\frac{2}{3}x^5+\sqrt{2}\,x^3+7x-\pi$,

$7x^4-\frac{2}{3}x^3+\frac{5}{6}x^2+7x-3$, $4x^5$, 2

2. EXAMPLES OF <u>NOT</u> POLYNOMIAL EXPRESSIONS

$5x^{-2}+7x-3$, $4x^{2/3}+7x-1$, $5\sqrt{x}+4$,

$\frac{2}{x^2+7x}$, $\frac{3}{x^{-1}+x^{-2}}$

3. FORM FOR A POLYNOMIAL EXPRESSION
<u>DEFINITION</u>:

$$a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\cdots+a_2x^2+a_1x+a_0$$

$a_nx^n,\ a_{n-1}x^{n-1},\ldots,a_1x,\ a_0$ TERMS

$a_n,\ a_{n-1},\ a_{n-2},\ldots,a_1,\ a_0$ COEFFICIENTS

$$7x^3+2x-4=a_3x^3+a_2x^2+a_1x+a_0$$

$a_3=7$ $a_2=0$ $a_1=2$ $a_0=-4$

EXPONENTS: NONNEGATIVE INTEGERS
WHOLE NUMBERS

4. NAMES FOR POLYNOMIAL EXPRESSIONS ACCORDING TO NUMBER OF TERMS

a. MONOMIALS: ONE-TERM

$3x^5$, $7x^3$, $-\pi x^{10}$

b. BINOMIAL: TWO-TERMED

$5x^4+2x$, $\frac{2}{3}x^3-7$

c. TRINOMIAL: THREE-TERMED

$5x^2+2x-1$, $\frac{2}{3}x^{10}-4x^3+x$

5. <u>NAMING</u> POLYNOMIAL EXPRESSIONS <u>BY</u> <u>DEGREE</u> INSTEAD OF NUMBER OF TERMS. (DEFINITIONS)

a. HIGHEST POWER OF THE TERMS WITH NONZERO COEFFICIENTS = DEGREE OF THE POLYNOMIAL

EXPRESSION	DEGREE
$-7x^3+4x^{10}-3$	10
$3x^7+100x^2-5$	7

b. <u>LINEAR</u> = DEGREE 1 $5x+2$

c. <u>QUADRATIC</u> = DEGREE 2 $5x^2-3x+7$

d. <u>CUBIC</u> = DEGREE 3 $4x^3-7x+6$

C. ADD/SUBTRACT POLYNOMIALS

a. JUST ADD/SUBTRACT LIKE TERMS

b. $(5x^2-7x)-(4x^2-3x+2)=$

$5x^2-7x-4x^2+3x-2=x^2-4x-2$

c. $(2x^3+6x)+(5x^3+4x^2-3x+6)=$

$7x^3+4x^2+3x+6$

D. MULTIPLYING POLYNOMIALS

a. BY A MONOMIAL

$3x^2(5x^3-2x+7)=15x^5-6x^3+21x^2$

OBVIOUS USE OF DISTRIBUTIVITY

b. GENERALLY, REPEATED USE OF DISTRIBUTIVITY

$(3x^2-2)(4x^2-3x+6)=(3x^2-2)4x^2+$

$(3x^2-2)(-3x)+(3x^2-2)(6)=12x^4-8x^2$

$-9x^3+6x+18x^2-12=12x^4-9x^3+10x^2+6x-12$

OBSERVE: EACH TERM IN ONE WAS MULTIPLIED BY EACH TERM IN THE OTHER. ANOTHER FORMAT FOR DOING THIS WILL NOW BE SHOWN.

$$
\begin{array}{r}
4x^2 - 3x + 6 \\
3x^2 - 2 \\
\hline
12x^4 - 9x^3 + 18x^2 \qquad\qquad\quad \\
- 8x^2 + 6x - 12 \\
\hline
12x^4 - 9x^3 + 10x^2 + 6x - 12
\end{array}
$$

PUT LIKE POWERS UNDER EACH OTHER

d. F O I L

First Outside Inside Last

Last

Inside

$$(a+b)(c+d) = ac + ad + bc + bd$$

First

Outside

$$(x+3)(2x-6) = 2x^2 - 6x + 6x - 18$$
$$= 2x^2 - 18$$

$$(2x-5)(x+4) = 2x^2 + 8x - 5x - 20$$
$$= 2x^2 + 3x - 20$$

e. $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

NOTE $(a+b)^2 \neq a^2+b^2$

$$(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2$$
$$= 4x^2 + 12x + 9$$

f. $\boxed{(a-b)^2 = a^2 - 2ab + b^2}$

$$(a-b)(a-b) = a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

$$(3x-4)^2 = (3x)^2 - 2(3x)(4) + 4^2$$
$$= 9x^2 - 24x + 16$$

g. $\boxed{(a+b)(a-b) = a^2 - b^2}$

$$(2y-5)(2y+5) = (2y)^2 - 5^2 = 4y^2 - 25$$

$$(a+b)(a-b) \overset{\text{FOIL}}{=} a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

$$(x^3-7)(x^3+7) = x^6 - 49$$

E. HOMEWORK (OIS)

1. WHICH OF THE FOLLOWING ARE POLYNOMIALS?

   a. $3x^5 - \sqrt{2}\, x^3 - 7$

   b. $5x^{-3} + 4x^{-2} + 6$

   c. $5\sqrt{x} + 3x^2 + 6$

   d. 7 e. π f. $\frac{1}{x^2+3}$

   g. $\frac{1}{2}x^3 + \pi x + 4$

2. NAME A 5^{TH} DEGREE MONOMIAL.

3. NAME A 3^{RD} DEGREE BINOMIAL.

4. NAME A QUADRATIC TRINOMIAL.

5. NAME A LINEAR MONOMIAL.

6. PERFORM THE INDICATED OPERATIONS

   a. $(3x^2 - 7x) - (4x^2 + 6x - 3)$

   b. $(2x^3 + 6x) + (5x^2 - 3x + 2)$

   c. $(2x - 7)(3x + 2)$

   d. $(5x - 1)(4x - 3)$

   e. $(5x + 2)(5x - 2)$

   f. $(x^{2/3} + 4)(x^{\frac{3}{5}} - 7)$

g. $(4x+7)^2$ h. $(4x-7)^2$

i. $(2x+5)^2$ j. $(2x-5)^2$

k. $(2x+5)(2x-5)$ l. $(4x+7)(4x-7)$

m. $(5x^{1/7}-3)(5x^{1/7}+3)$

n. $(\sqrt{x}-4)(\sqrt{x}+4)$ o. $(\sqrt{x}-4)^2$

p. $(\sqrt{x}+x)^2$ q. $(\sqrt{x}+4)^2$

7. PERFORM EACH OF THESE MULTIPLICATIONS THE TWO WAYS SHOWN IN SECTION D PART b OF THIS CHAPTER.

   a. $(2x^2+4x-7)(3x^2-5x+2)$

   b. $(4x^3-2x+3)(5x^2-2x+4)$

8. WHAT WAS SQUARED TO GET EACH OF THESE?

   a. $x^2-8x+16$ b. $4x^2+20x+25$

9. WHAT TWO POLYNOMIALS WERE MULTIPLIED TO GET EACH OF THESE

   a. $16x^2-9$ b. $25x^4-49$

F. AN APPLICATION OF $(a+b)(a-b)=a^2-b^2$ TO RATIONALIZING DENOMINATORS.

1. NOTE $(3+\sqrt{6})(3-\sqrt{6})=9-6=3$

   THE RADICAL IS GONE!

2. RATIONALIZE THE DENOMINATOR IN $\dfrac{5}{3+\sqrt{6}}$.

$$\frac{5}{3+\sqrt{6}}=\frac{5}{3+\sqrt{6}}\cdot\frac{3-\sqrt{6}}{3-\sqrt{6}}$$

$$=\frac{5(3-\sqrt{6})}{9-6}=\frac{15-5\sqrt{6}}{3}=\frac{15}{3}-\frac{5}{3}\sqrt{6}$$

$$=5-\frac{5}{3}\sqrt{6}.$$

3. $3+\sqrt{6}$ IS THE CONJUGATE OF $3-\sqrt{6}$.

   MULTIPLY BY $\dfrac{\text{CONJUGATE}}{\text{CONJUGATE}}$.

4. RATIONALIZE THE DENOMINATOR IN $\dfrac{3+\sqrt{5}}{\sqrt{2}-\sqrt{3}}$.

$$\frac{3+\sqrt{5}}{\sqrt{2}-\sqrt{3}}=\frac{(3+\sqrt{5})}{(\sqrt{2}-\sqrt{3})}\cdot\frac{(\sqrt{2}+\sqrt{3})}{(\sqrt{2}+\sqrt{3})}=\frac{3\sqrt{2}+3\sqrt{3}+\sqrt{5}\sqrt{2}+\sqrt{5}\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}$$

$$=\frac{3\sqrt{2}+3\sqrt{3}+\sqrt{10}+\sqrt{15}}{2-3}=\frac{3\sqrt{2}+3\sqrt{3}+\sqrt{10}+\sqrt{15}}{-1}$$

$$=-3\sqrt{2}-3\sqrt{3}-\sqrt{10}-\sqrt{15}$$

G. DIVISION OF POLYNOMIALS

a. $\dfrac{\text{MONOMIAL}}{\text{MONOMIAL}}$ $\qquad \dfrac{3x^8}{7x^{10}} = \dfrac{3}{7x^2}$

$$\frac{5x^{12}}{4x^5} = \frac{5x^7}{4} = \frac{5}{4}x^7$$

b. $\dfrac{\text{POLYNOMIAL}}{\text{MONOMIAL}}$ $\left(\text{HINT} \quad \dfrac{a+b}{d} = \dfrac{a}{d} + \dfrac{b}{d}\right)$

$$\frac{3x^7+4x^3-5}{2x^2} = \frac{3x^7}{2x^2} + \frac{4x^3}{2x^2} - \frac{5}{2x^2}$$

$$= \frac{3}{2}x^5 + 2x - \frac{5}{2x^2}$$

c. BUILD-UP FOR $\dfrac{\text{POLYNOMIAL}}{\text{POLYNOMIAL}}$

$\dfrac{271}{5} = \dfrac{T}{B}$ $\qquad$ T = TOP $\quad$ B = BOTTOM

Q = QUOTIENT

R = REMAINDER

$$\begin{array}{r|l} & \;\;\,54 \\ \hline 5 & 271 \\ & \ominus 25 \\ \hline & \;\;\,21 \\ & \ominus 20 \\ \hline & \quad\;\, 1 \end{array}$$

CHANGE SIGN AND ADD

BRING DOWN THE 1

$$\frac{271}{5} = 54 + \frac{1}{5}$$

$$\frac{T}{B} = Q + \frac{R}{B} \quad \text{FORM}$$

$$271 = 54(5) + 1$$

$$T = Q \cdot B + R \quad \text{FORM}$$

Q. LONG DIVISION $\dfrac{\text{POLYNOMIAL}}{\text{POLYNOMIAL}}$

$$\frac{8x^4-4x^3-20x^2+3x-11}{2x^2+3x-2}$$

PUT IN BOTH $\frac{T}{B} = Q + \frac{R}{B}$ AND

$T = Q \cdot B + R$ FORM

$$\begin{array}{r|l}
 & 4x^2 - 8x + 6 \\
\hline
2x^2+3x-2 & 8x^4 - 4x^3 - 20x^2 + 3x - 11 \\
 & 8x^4 + 12x^3 - 8x^2 \\
\hline
 & \quad -16x^3 - 12x^2 + 3x \\
 & \quad -16x^3 - 24x^2 + 16x \\
\hline
 & \qquad +12x^2 - 13x - 11 \\
 & \qquad 12x^2 + 18x - 12 \\
\hline
 & \qquad\qquad -31x + 1
\end{array}$$

STOP WHEN THE DEGREE OF THE REMAINDER < DEGREE OF THE BOTTOM

$$\frac{8x^4-4x^3-20x^2+3x-11}{2x^2+3x-2} =$$

$$4x^2 - 8x + 6 + \frac{(-31x+1)}{2x^2+3x-2}$$

$\frac{T}{B} = Q + \frac{R}{B}$ FORM

$$8x^4 - 4x^3 - 2x^2 + 3x - 11 =$$

$$(4x^2-8x+6)(2x^2+3x-2) + (-31x+1)$$

$T = Q \cdot B + R$ FORM

e. LONG DIVISION WHEN SOME POWERS OF x ARE MISSING.

$\dfrac{x^3-8}{x-2}$ PUT IN BOTH $\dfrac{T}{B} = Q + \dfrac{R}{B}$ AND $T = Q \cdot B + R$ FORM

$$\begin{array}{r|l} & \;\; x^2 + 2x + 4 \\ x-2 & \overline{x^3 + 0x^2 + 0x - 8} \\ & \ominus\, x^3 \oplus\!\!-2x^2 \\ \hline & \quad 2x^2 + 0x \\ & \quad \ominus\, 2x^2 \oplus\!\!-4x \\ \hline & \qquad 4x - 8 \\ & \qquad \ominus\, 4x \oplus\!\!-8 \\ \hline & \qquad\qquad 0 \end{array}$$

PUT IN MISSING TERMS WITH 0 COEFFICIENTS

$$\frac{x^3-8}{x-2} = x^2+2x+4 \qquad \frac{T}{B} = Q + \frac{R}{B} \text{ FORM}$$

$$x^3-8 = (x^2+2x+4)(x-2) \qquad T = Q \cdot B + R \text{ FORM}$$

<u>NOTE</u>: IN THIS CASE WE HAD 0 REMAINDER. THE BOTTOM IN THIS SITUATION IS SAID TO BE A <u>FACTOR</u> OF THE TOP. $x-2$ IS A FACTOR OF x^3-8. IT SHOULD REMIND ONE OF

$$\begin{array}{r|l} & \;\;5 \\ 7 & \overline{35} \\ & \ominus 35 \\ \hline & \;\;0 \end{array}$$

7 IS A FACTOR OF 35

H. HOMEWORK (OIS)

1. RATIONALIZE THE DENOMINATORS

a. $\dfrac{5}{\sqrt{3}-2}$ b. $\dfrac{2+\sqrt{3}}{\sqrt{5}+\sqrt{2}}$

c. $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

2. WRITE EACH OF THE FOLLOWING IN BOTH $\frac{T}{B}=Q+\frac{R}{B}$ AND $T=Q\cdot B+R$ FORM. IF NECESSARY, USE LONG DIVISION

a. $\dfrac{5x^7}{3x^2}$ b. $\dfrac{4x^3+7x}{3x^2}$

c. $\dfrac{2x^5-9x^3+7x}{3x^2}$

d. $\dfrac{9x^4+12x^3-6x^2+10x+5}{3x^2+6x-4}$ e. $\dfrac{x^4+16}{x+2}$

f. $\dfrac{3x^3+5x^2+\frac{9}{2}x+\frac{15}{2}}{2x^2+3}$

I. FACTORING: THE OPPOSITE OF MULTIPLYING

1. FACTORING OUT A MONOMIAL

$$9x^4 - 3x^3 + 6x^2 = 3x^2(3x^2 - x + 2)$$

2. GROUP THEN FACTOR

a. $(x^2+3)x - (x^2+3)5 = (x^2+3)(x-5)$

b. $(x^2+3)x + (x^2+3) = (x^2+3)x + (x^2+3)\cdot 1$
$= (x^2+3)(x+1)$

c. $x^3 + 3x + 4x^2 + 12 = (x^3 + 3x) + (4x^2 + 12)$
$= (x^2+3)x + (x^2+3)4 = (x^2+3)(x+4)$

d. $x^3 + 6x^2 + 3x + 18 = (x^3 + 3x) + (6x^2 + 18)$
$= (x^2+3)x + (x^2+3)6 = (x^2+3)(x+6)$

OR

$x^3 + 6x^2 + 3x + 18 = (x^3 + 6x^2) + (3x + 18)$
$= x^2(x+6) + 3(x+6) = (x^2+3)(x+6)$

e. $pw + qb - qw - pb = (pw - qw) + (qb - pb)$
$= (p-q)w + (q-p)b = (p-q)w - (p-q)b$
$= (p-q)(w-b)$

J. HOMEWORK (OIS)

1. FACTOR OUT ax^n

a. $5x^7 - 10x^2$

b. $4x^5 + 12x^8 - x^4$

c. $7x^{5/2} + 4x^{3/2} + x^{1/2}$

d. $12\sqrt{x} + 6x^3 + 2x$

2. GROUP THEN FACTOR

a. $x^4 + 6x + 3x^3 + 18$

b. $x^4 + 5x^2 - x^2 - 5$

c. $2x^6 - 3x^5 + 4x^2 - 6x$

d. $3x^3 - 2x - 12x^2 + 8$

e. $6x^3 + 8x^8 - 15x^2 - 20x^7$

f. $pq^2 + 2p^2q + 3q + 6p$

g. $3wb + 6qb - 3wp - 6pq$

K FACTORING A QUADRATIC BY LIMITED TRIAL AND ERROR.

1. THE FIRST 3 CASES LOOK AT A POSITIVE x^2 COEFFICIENT

2. <u>CASE 1</u> FACTOR $6x^2 + 13x + 5$

$$6x^2 \underset{\substack{\uparrow \\ \text{POS.}}}{+} 13x + \underset{\substack{\uparrow \\ \text{POS.}}}{5}$$

HENCE <u>LAST</u> <u>TERMS</u> ARE BOTH POSITIVE.
LAST TERMS MUST HAVE <u>A PRODUCT OF +5</u>
TRY 1 and 5 , 5 and 1
<u>FIRST</u> <u>TERMS</u> : POSITIVE FACTORS WHOSE PRODUCT IS 6 : TRY 6 <u>and 1,</u> 3 and 2, 2 and 3 , 1 and 6

$(6x + 1)(x + 5)$ BAD
$(6x + 5)(x + 1)$ BAD
$(3x + 1)(2x + 5)$ BAD
$(3x + 5)(2x + 1)$ YES THE ANSWER
$(2x + 1)(3x + 5)$
$(2x + 5)(3x + 1)$
$(x + 1)(6x + 5)$
$(x + 5)(6x + 1)$

REPEATS

3. <u>CASE 2</u> : FACTOR $6x^2 - 11x + 5$

-11 ↑ NEG, $+5$ ↑ POS.

<u>LAST TERMS</u>: BOTH NEGATIVE, PRODUCT OF 5
TRY -1 AND -5, -5 AND -1
<u>FIRST TERMS</u>: POSITIVE FACTORS WHOSE PRODUCT IS 6 : TRY 6 AND 1, 3 AND 2, 2 AND 3, 1 AND 6

$(6x-1)(x-5)$ BAD

$(6x-5)(x-1)$ YES ← ANSWER

$(3x-1)(2x-5)$

$(3x-5)(2x-1)$

$(2x-1)(3x-5)$

$(2x-5)(3x-1)$

$(x-1)(6x-5)$

$(x-5)(6x-1)$

REPEATS

REPEATS BELOW
NO NEED TO CHECK
SO YOU LIMIT TRIAL AND ERROR

$$6x^2 - 11x + 5 = (6x-5)(x-1)$$

TRIAL AND ERROR ALSO LIMITED BY DEDUCING THE FACTORS WHOSE PRODUCT IS <u>5</u> MUST <u>BOTH</u> BE NEGATIVE.

4. CASE 3: FACTOR $6x^2 - 13x - 5$

NEG. NEG.

LAST TERMS: 1 POS., 1 NEG., PRODUCT OF -5

TRY -1 AND 5, 1 AND -5, 5 AND -1, -5 AND 1

FIRST TERMS: POSITIVE FACTORS WHOSE PRODUCT IS 6 TRY 6 AND 1, 3 AND 2, 2 AND 3, 1 AND 6

$(6x-1)(x+5)$ BAD

$(6x+1)(x-5)$ BAD

$(6x+5)(x-1)$ BAD

$(6x-5)(x+1)$ BAD

$(3x-1)(2x+5)$ BAD

$(3x+1)(2x-5)$ YES ← ANSWER

$(3x+5)(2x-1)$

$(3x-5)(2x+1)$

$(2x-1)(3x+5)$

$(2x+1)(3x-5)$

... REST IS REPEATED NO NEED TO CHECK

$$6x^2 - 13x - 5 = (3x+1)(2x-5)$$

TRIAL AND ERROR LIMITED BY NOT CHECKING REPEATS AND DEDUCING THE FACTORS WHOSE PRODUCT IS -5 MUST HAVE OPPOSITE SIGNS.

5. WHAT TO DO WHEN THE x^2 COEFFICIENT IS NEGATIVE: FIRST FACTOR OUT THE MINUS SIGN, THEN PROCEED AS PREVIOUSLY.

FACTOR $-6x^2+13x+5$

FIRST $-6x^2+13x+5 = -(6x^2-13x-5)$

OBSERVE, THE LAST EXAMPLE FACTORED $6x^2-13x-5$ INTO $(3x+1)(2x-5)$

SO, $-6x^2+13x+5 = -(3x+1)(2x-5)$

$= (-3x-1)(2x-5) = (3x+1)(-2x+5)$

6. IRREDUCIBLE OVER THE INTEGERS (DOES NOT FACTOR WITH INTEGER COEFFICIENTS) (DEFINITION)

NOTE: TO FACTOR x^2+x+1 WITH ONLY INTEGER COEFFICIENTS, THE ONLY POSSIBILITY WOULD BE

$(x+1)(x+1)$, BUT THIS EQUALS x^2+2x+1, NOT x^2+x+1.

L. HOMEWORK (OIS) IF POSSIBLE, FACTOR BY LIMITED TRIAL AND ERROR SO THAT THE FACTORS HAVE INTEGER COEFFICIENTS. OTHERWISE, WRITE "IRREDUCIBLE"

a. $2x^2-2x-4$

b. $3x^2-x-10$

c. x^2-9

d. $10x^2-9x+1$

e. x^2+x+4

f. $8x^2+26x+15$

g. $5x^2+x+7$

h. x^6+29x^3-30

i $6x^{10}-17x^5+7$

j $3x^4-10x^2-8$

k $16y^2-24y+9$

l $-8x^2+19x+15$

MINI-JUGULAR #3

M. ALTERNATE METHOD: REDUCE TO "GROUP, THEN FACTOR"

$$ax^2+bx+c$$

1. FIND 2 NUMBERS WHOSE SUM IS b AND WHOSE PRODUCT IS ac
2. REPLACE b BY THE SUM
3. GROUP, THEN FACTOR

FACTOR $6x^2-17x+5$

1. 2 NUMBERS WHOSE SUM IS -17 AND WHOSE PRODUCT IS $6(5)=30$

   $-15, -2$

2. REPLACE -17 BY $-15-2$

   $$6x^2-17x+5=$$
   $$6x^2+(-15-2)x+5=$$

3. GROUP, THEN FACTOR

   $$6x^2-15x-2x+5=$$
   $$(6x^2-15x)-(2x-5)=$$
   $$3x(2x-5)-1\cdot(2x-5)=$$
   $$(2x-5)(3x-1)$$

FACTOR $6x^2 - 11x + 5$

1. FIND 2 NUMBERS WHOSE SUM IS -11 AND WHOSE PRODUCT IS $6(5) = 30$

$$-6, -5$$

2. REPLACE -11 with $-6-5$

$$6x^2 - 11x + 5 =$$

$$6x^2 + (-6-5)x + 5 =$$

3. GROUP, THEN FACTOR

$$6x^2 - 6x - 5x + 5 =$$

$$(6x^2 - 6x) - (5x - 5) =$$

$$6x(x-1) - 5(x-1) =$$

$$(x-1)(6x-5)$$

NOTE: WE JUST REWORKED A COUPLE OF PROBLEMS WE HAD PREVIOUSLY WORKED USING LIMITED TRIAL AND ERROR.

N. FACTORING USING FORMULAS

1. $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

$$4x^2 + 12x + 9 = (2x+3)^2$$

2. $\boxed{(a-b)^2 = a^2 - 2ab + b^2}$

$$4x^2 - 12x + 9 = (2x-3)^2$$

$$25x^6 - 30x^3y + 9y^2 =$$

$$(5x^3)^2 - 2(5x^3)(3y) + (3y)^2 = (5x^3 - 3y)^2$$

3. $\boxed{(a^2 - b^2) = (a-b)(a+b)}$

$$4x^2 - 9 = (2x+3)(2x-3)$$

$$25x^8 - 36y^6 = (5x^4 + 6y^3)(5x^4 - 6y^3)$$

4. $\boxed{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$

PROOF: $(a-b)(a^2 + ab + b^2) =$

$a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3 =$

$a^3 - b^3$

a. $8x^3-27y^3=(2x)^3-[3y]^3=$

$(2x-3y)((2x)^2+(2x)[3y]+[3y]^2)$

$=(2x-3y)(4x^2+6xy+9y^2)$

b. $x^{12}-y^{12}=(x^4)^3-[y^4]^3=$

$(x^4-y^4)((x^4)^2+(x^4)[y^4]+[y^4]^2)=$

$(x^2-y^2)(x^2+y^2)(x^8+x^4y^4+y^8)=$

$(x-y)(x+y)(x^2+y^2)(x^8+x^4y^4+y^8)$

5. $\boxed{a^3+b^3=(a+b)(a^2-ab+b^2)}$

a. $8x^3+27y^3=(2x)^3+[3y]^3=$

$((2x)+[3y])((2x)^2-(2x)[3y]+[3y]^2)=$

$(2x+3y)(4x^2-6xy+9y^2)$

b. $x^6+y^6=(x^2)^3+[y^2]^3=$

$((x^2)+[y^2])((x^2)^2-(x^2)[y^2]+[y^2]^2)=$

$(x^2+y^2)(x^4-x^2y^2+y^4)$

O. HOMEWORK (OIS)

1. REDUCE TO "GROUP, THEN FACTOR"
   a. $8x^2+26x+15$
   b. $4x^2-16x+15$
   c. $16y^2-24y+9$
   d. $6x^2-5x-6$
2. FACTOR USING FORMULAS
   a. $36y^6-4x^2$
   b. a^2b^4-9
   c. $9x^2+30x+25$
   d. $9x^2-30x+25$
   e. x^3+27
   f. x^3-27
   g. x^4-81
   h. x^4+18x^2+81
   i. $27x^6y^3-125$
   j. $16x^8y^4-36z^2$
   k. $25y^2+60y+36$
   l. $25y^2-70y+49$
3. DERIVE A FORMULA FOR $(a+b)^3$
4. DERIVE A FORMULA FOR $(a-b)^3$
5. FACTOR EACH OF THE FOLLOWING
   a. $x^3+6x^2+12x+8$
   b. $125x^3+150x^2+60x+8$
   c. $8x^3-36x^2+54x-27$

P. FACTOR COMPLETELY (UNTIL EACH EXPRESSION IS IRREDUCIBLE OVER THE INTEGERS)

$$-8x^7+8x = -8x(x^6-1) =$$

$$-8x(x^3-1)(x^3+1) =$$

$$-8x(x-1)(x^2+x+1)(x+1)(x^2-x+1)$$

Q USING LONG DIVISION TO HELP YOU FACTOR.

1. IS $x-2$ A FACTOR OF x^3-7x+6? IF SO, FACTOR COMPLETELY.

$$
\begin{array}{r|l}
 & x^2+2x-3 \\
\hline
x-2 & x^3+0x^2-7x+6 \\
 & \underline{x^3-2x^2} \\
 & \quad 2x^2-7x \\
 & \quad \underline{2x^2-4x} \\
 & \qquad -3x+6 \\
 & \qquad \underline{-3x+6}
\end{array}
$$

$$x^3-7x+6 = (x-2)(x^2+2x-3)$$

$$= (x-2)(x+3)(x-1)$$

2. SUPPOSE YOU FORGET THE FORMULA FOR a^3-b^3 BUT REMEMBER $(a-b)(\ ?\ )$

FACTOR $27x^3-125 = (3x-5)(\ ?\ )$

$$
\begin{array}{rl}
 & 9x^2+15x+25 \\
3x-5 & \overline{)\,27x^3+0x^2+0x-125} \\
 & \ominus 27x^3 \overset{\oplus}{-} 45x^2 \\
\hline
 & 45x^2+0x \\
 & \ominus 45x^2 \overset{\oplus}{-} 75x \\
\hline
 & 75x-125 \\
 & \ominus 75x \overset{\oplus}{-} 125 \\
\hline
\end{array}
$$

$$27x^3-125=(3x-5)(9x^2+15x+25)$$

R. RATIONAL EXPRESSIONS

1. RATIONAL NUMBER = FRACTION OF INTEGERS
2. RATIONAL EXPRESSION = FRACTION OF POLYNOMIALS (DEFINITION)

$$\frac{\text{POLYNOMIAL}}{\text{POLYNOMIAL}}$$

3. EXAMPLES:

$$\frac{x^2-9}{x^2-2x-3}\ ,\qquad \frac{2x^2+4x+5}{x^6-1}$$

S. REDUCING RATIONAL EXPRESSIONS TO THEIR LOWEST TERMS

1. $$\frac{x^2-4}{x^3-8} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}$$

$$= \frac{x+2}{x^2+2x+4}$$

2. $$\frac{x^2-9}{x^2-25} \cdot \frac{x^2-6x+5}{x^2-4x+3} =$$

$$\frac{(x-3)(x+3)}{(x-5)(x+5)} \cdot \frac{(x-5)(x-1)}{(x-3)(x-1)} =$$

$$\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-5)}(x+5)} \cdot \frac{\cancel{(x-5)}\cancel{(x-1)}}{\cancel{(x-3)}\cancel{(x-1)}} = \frac{x+3}{x+5}$$

3. $$\frac{x^2-3x+2}{x^2-4} \div \frac{5x^2-5}{x^2+3x+2} =$$

$$\frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+2)} \div \frac{5(x^2-1)}{(x+1)(x+2)} =$$

$$\frac{x-1}{x+2} \cdot \frac{\cancel{(x+1)}\cancel{(x+2)}}{5\cancel{(x-1)}\cancel{(x+1)}} = \frac{1}{5}$$

T. HOMEWORK (OIS)

1. FACTOR COMPLETELY

a. $-3x^4+48$ b. x^6-9x^3+8

c. $x^{12}-1$ d. x^6+7x^3-8

e. $x^4-3x^3-2x^2+12x-8$ HINT: x^2-4 IS A FACTOR OF THIS.

f. $x^4-5x^3-5x^2+45x-36$ HINT: x^2-9 IS A FACTOR OF THIS

2. REDUCE TO LOWEST TERMS

a. $\dfrac{x^2+2x-3}{x^2-9}$ b. $\dfrac{x^2-4}{x^2+x-6}\cdot\dfrac{x^2-9}{x^2-x-6}$

c. $\dfrac{x^2-2x-15}{x^2-25}\div\dfrac{x^2+2x-3}{x^2+9x+20}$

d. $\dfrac{x^3-8}{x^2-x-2}\div\dfrac{x^2+2x+4}{x^3+1}$

e. $\dfrac{x^3+3x^2+3x+1}{x^2-1}\div\dfrac{x^4+x^3+x+1}{2x-2}$

U. LEAST COMMON MULTIPLE (lcm)

1. FACTOR COMPLETELY
   EACH FACTOR PRESENT
   HIGHEST EXPONENT FOR EACH FACTOR

2. $\text{lcm}\left(2^3 \cdot 5^7 \cdot 11^5,\ 2^6 \cdot 5^4 \cdot 11^3\right) = 2^6 \cdot 5^7 \cdot 11^5$

3. $\text{lcm}\left(2^6 \cdot 5^{10} \cdot 7^{20},\ 3^8 \cdot 5^6 \cdot 7^4 \cdot 11^8\right)$
   $= 2^6 \cdot 3^8 \cdot 5^{10} \cdot 7^{20} \cdot 11^8$

4. $\text{lcm}(36, 45) = \text{lcm}(4 \cdot 9, 5 \cdot 9) =$
   $\text{lcm}(2^2 \cdot 3^2, 5^1 \cdot 3^2) = 2^2 \cdot 3^2 \cdot 5^1$

5. $\text{lcm}\left(x^5(x+1)^3,\ y(x+1)^4 x^2\right) = x^5(x+1)^4 y$

6. $\text{lcm}\left(x^5 + 3x^4 + 3x^3 + x^2,\ x^3 - x + x^2 - 1\right) =$
   $\text{lcm}\left(x^2(x^3 + 3x^2 + 3x + 1),\ x(x^2-1) + (x^2-1)\cdot 1\right) =$
   $\text{lcm}\left(x^2(x+1)^3,\ (x^2-1)(x+1)\right) =$
   $\text{lcm}\left(x^2(x+1)^3,\ (x+1)(x-1)(x+1)\right) =$
   $\text{lcm}\left(x^2(x+1)^3,\ (x+1)^2(x-1)\right) =$
   $x^2(x+1)^3(x-1)$

V ADDING RATIONAL EXPRESSIONS

1. USE THE lcm OF THE DENOMINATORS TO GET SMALLEST COMMON DENOMINATOR (TOP = NUMERATOR BOTTOM = DENOMINATOR)

2. $\frac{a}{b}+\frac{c}{d} = \frac{ad}{bd}+\frac{bc}{bd} = \frac{ad+bc}{bd}$

   MULTIPLY BY $\frac{d}{d}=1$ $\frac{b}{b}=1$

3. FORMER WAY

$$\frac{3}{x^5y^2}+\frac{4}{x^3y^7} = \frac{3x^3y^7+4x^5y^2}{x^8y^9}$$

$$= \frac{x^3y^2(3y^5+4x^2)}{x^8y^9} = \frac{3y^5+4x^2}{x^5y^7}$$

4. USING SMALLEST COMMON DENOMINATOR

$$\text{lcm}(x^5y^2, x^3y^7) = x^5y^7$$

$$\frac{3}{x^5y^2}+\frac{4}{x^3y^7} = \frac{3y^5}{x^5y^2y^5}+\frac{4x^2}{x^3y^7x^2}$$

$$= \frac{3y^5+4x^2}{x^5y^7}$$

THIS WAY USUALLY SIMPLER

5. $$\frac{3z}{x^5y^2} - \frac{2m}{x^3y^6} + \frac{4}{x^4y^7} =$$

NOTE: $\operatorname{lcm}(x^5y^2, x^3y^6, x^4y^7) = x^5y^7$

$$\frac{3zy^5}{x^5y^2y^5} - \frac{2mx^2y}{x^3y^6x^2y} + \frac{4x}{x^4y^7x} =$$

$$\frac{3zy^5 - 2mx^2y + 4x}{x^5y^7}$$

NOTE: $\operatorname{lcm}\left((x+3)^2, (x+2)(x+3)\right) = (x+3)^2(x+2)$

6. $$\frac{x}{x^2+6x+9} - \frac{(x-3)}{x^2+5x+6} =$$

$$\frac{x}{(x+3)^2} - \frac{(x-3)}{(x+2)(x+3)} = \frac{x(x+2)}{(x+3)^2(x+2)} - \frac{(x-3)(x+3)}{(x+2)(x+3)(x+3)}$$

$$= \frac{x(x+2)-(x-3)(x+3)}{(x+3)^2(x+2)} = \frac{x^2+2x-(x^2-9)}{(x+3)^2(x+2)}$$

$$= \frac{x^2+2x-x^2+9}{(x+3)^2(x+2)} = \frac{2x+9}{(x+3)^2(x+2)}$$

7. $$5 - \frac{4}{x} = \frac{5x-4}{x}$$

8. $$3 + \frac{x}{2} = \frac{6+x}{2}$$

W. COMPLEX FRACTIONS (FRACTION IN THE NUMERATOR OR DENOMINATOR) DEF.

1. WAY 1 TO SOLVE: MULTIPLY TOP AND BOTTOM BY THE LEAST COMMON MULTIPLE OF THE DENOMINATORS

$$\frac{\frac{2}{3}+\frac{8}{x+1}}{5-\frac{2}{x+2}} = \frac{3(x+1)(x+2)\left[\frac{2}{3}+\frac{8}{x+1}\right]}{3(x+1)(x+2)\left[5-\frac{2}{x+2}\right]} =$$

$$\frac{3(x+1)(x+2)\frac{2}{3} + \frac{3(x+1)(x+2)8}{x+1}}{3(x+1)(x+2)5 - 3(x+1)(x+2)\frac{2}{x+2}} =$$

$$\frac{(x+1)(x+2)2 + 3(x+2)8}{15(x+1)(x+2) - 3(x+1)2} =$$

$$\frac{(x+2)\left[(x+1)2 + 3(8)\right]}{(x+1)\left[15(x+2) - 3(2)\right]} =$$

$$\frac{(x+2)\left[2x+2+24\right]}{(x+1)\left[15x+30-6\right]} = \frac{(x+2)(2x+26)}{(x+1)(15x+24)}$$

$$= \frac{2(x+2)(x+13)}{3(x+1)(5x+8)}$$

2. WAY 2 TO SOLVE: SIMPLIFY IN PIECES; PERFORM OPERATIONS.

$$\frac{\frac{2}{3}+\frac{8}{x+1}}{5-\frac{2}{x+2}} = \frac{\frac{2(x+1)+24}{3(x+1)}}{\frac{5(x+2)-2}{x+2}} = \frac{\frac{2x+2+24}{3(x+1)}}{\frac{5x+10-2}{x+2}}$$

$$= \frac{\frac{2x+26}{3(x+1)}}{\frac{5x+8}{x+2}} = \frac{2x+26}{3(x+1)}\cdot\frac{x+2}{5x+8} = \frac{2(x+13)(x+2)}{3(x+1)(5x+8)}$$

X. WARNING!!! ONLY CANCEL WHEN THERE IS A PRODUCT IN TOP AND BOTTOM

1. BAD $\frac{\cancel{x^4}+y^5}{5\cancel{x^4}}$ SUM IN TOP

2. BAD $\frac{3\cancel{(x+1)}(x+2)-5(x+7)}{4\cancel{(x+1)}(x+2)}$

NOT GOOD

3. GOOD $\frac{\cancel{x^4}y^5}{5\cancel{x^4}} = \frac{y^5}{5}$ PRODUCT IN TOP

Y. JUGULAR PROBLEM #4: PUT IN SIMPLEST FORM WITH NO NEGATIVE EXPONENTS.

$$\frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} \div \frac{y^{-1}-x^{-1}}{3x} = \frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}} \cdot \frac{3x}{y^{-1}-x^{-1}}$$

$$= \frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}} \cdot \frac{3x}{\frac{1}{y}-\frac{1}{x}} = \frac{\frac{y^2-x^2}{x^2y^2}}{\frac{y+x}{xy}} \cdot \frac{\frac{3x}{1}}{\frac{x-y}{xy}}$$

$$= \frac{y^2-x^2}{x^2y^2} \cdot \frac{xy}{y+x} \cdot \frac{3x}{1} \cdot \frac{xy}{x-y}$$

$$= \frac{\cancel{(y-x)}\cancel{(y+x)}}{\cancel{x^2y^2}} \cdot \frac{\cancel{xy}}{\cancel{y+x}} \cdot \frac{3x}{1} \cdot \frac{\cancel{xy}}{-\cancel{(y-x)}}$$

$$= \frac{3x}{-1} = -3x$$

Z. HOMEWORK (OIS)

1. $\operatorname{lcm}(5^3 \cdot 2^7 \cdot 3^8,\ 2^6 \cdot 3^{10} \cdot 7^9) =$

2. $\operatorname{lcm}(x^3y^5z^4,\ x^6y^2,\ xy^{10}z^5) =$

3. $\operatorname{lcm}((x+1)^2(2x-3),\ (x+1)(2x-3)^5) =$

4. WRITE AS A SINGLE FRACTION IN SIMPLEST FORM

   a. $\dfrac{3}{5x^3y^2} - \dfrac{2}{4xy^3} =$

   b. $\dfrac{6m}{4x^3z} - \dfrac{2p}{3xy^4} + \dfrac{4}{5y^5z^2} =$

   c. $\dfrac{3}{(x+2)^2(2x-1)} - \dfrac{4}{(x+2)(2x-1)^2} =$

   d. $\dfrac{5}{x^2-7x+12} + \dfrac{4}{x^2-6x+9} =$

   e $\dfrac{x}{x^3+3x^2+3x+1} + \dfrac{4}{(x^2+2x+1)(x-2)} =$

f. $$\frac{\frac{2}{3}+\frac{4}{5}}{\frac{3}{4}-\frac{1}{5}} =$$

g. $$\frac{4-\frac{5}{x}}{3-\frac{2}{x}} =$$

h. $$\frac{5-\frac{2}{x+4}}{\frac{3}{x+2}+\frac{4}{x-1}} =$$

i. $$\frac{\frac{3}{x-1}-\frac{2}{x+1}}{\frac{4}{x+2}-\frac{5}{x+3}} =$$

j. $$\frac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}} =$$

k. $$\frac{x^{-1}+y^{-1}}{x^{-1}-y^{-1}} \cdot \frac{x-y}{x^{-2}-y^{-2}} =$$

l. $$\frac{x^{-2}+y^{-2}}{x^{-1}-y^{-1}} \div \frac{x^{2}+y^{2}}{y^{-2}-x^{-2}} =$$

m. $$\frac{x^{-3}-y^{-3}}{x^{-1}+y^{-1}} \div \frac{x^{2}+xy+y^{2}}{y^{-2}x^{-2}} =$$

[CHAPTER 6]

COMPLEX NUMBERS

A. UP TO NOW WE HAVE STUDIED THE <u>REALS</u>

B. THE <u>COMPLEX</u> <u>NUMBERS</u> WILL CONTAIN THE <u>REALS</u> AND HAVE EXTRA NUMBERS

C. UP UNTIL NOW, BAD → $\sqrt{\text{NEGATIVE}}$ THIS IS ALLOWED WITH COMPLEX NUMBERS

D. THE COMPLEX NUMBER i : $i = \sqrt{-1}$ (DEF.)

E. $i = \sqrt{-1}$ $\quad i^2 = -1$ $\quad i^3 = i^2 \cdot i = -1 \cdot i = -i$ $\quad i^4 = i^2 \cdot i^2 = 1$

$i^5 = i^4 \cdot i = 1 \cdot i = i$ $\quad i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$ $\quad i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$

$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$ $\quad i^9 = i$ $\quad i^{10} = -1$ $\quad i^{11} = -i$ $\quad i^{12} = 1$

$i^{13} = i$ $\quad i^{14} = -1$ $\quad i^{15} = -i$ $\quad i^{16} = 1$

$i^{97} = i^{96} \cdot i = (i^4)^{24} \cdot i = 1^{24} \cdot i = 1 \cdot i = i$

$\boxed{\text{DEFINITION}}$

F. z IS A COMPLEX NUMBER IFF z CAN BE WRITTEN IN THE FORM

$$z = a + bi$$

WHERE a AND b ARE REAL

G. TERMINOLOGY

STANDARD FORM

$z = a + bi$

1. a IS THE REAL PART. b IS THE IMAGINARY PART

   a. $3+4i$ — 3 REAL PART; 4 IMAGINARY PART

   b. $5-\frac{1}{2}i$ — 5 REAL PART; $-\frac{1}{2}$ IMAGINARY PART

   c. $6 = 6+0i$ — 6 REAL PART; 0 IMAGINARY PART

   STANDARD FORM

2. EVERY REAL IS COMPLEX: $\frac{1}{2} = \frac{1}{2}+0i$

DEFINITIONS

3. IMAGINARY NUMBER: A COMPLEX NUMBER THAT IS NOT REAL (I.E $a+bi$ WHERE $b \neq 0$)

   $3+2i$, $\pi - \frac{2}{3}i$, $\frac{4}{5}i$ IMAGINARY

4. PURE IMAGINARY NUMBER: A COMPLEX NUMBER WHERE THE REAL PART IS ZERO AND THE IMAGINARY PART IS NOT ZERO

   $5i$, $\frac{2}{3}i$, πi, $-6i$ PURE IMAGINARY NUMBERS

5. ADVICE: WRITE $\sqrt{3}\, i$ AS $i\sqrt{3}$

   $$i\sqrt{3} = \sqrt{3}\, i \neq \sqrt{3i}$$

H. OPERATIONS FOR COMPLEX NUMBERS

1. $a+bi = c+di$ IFF $a=c$ AND $b=d$

   $2+3i = 2+pi$ IMPLIES $p=3$

2. FOR OPERATIONS TREAT i LIKE A VARIABLE (YOU CAN FACTOR IT OUT, DISTRIBUTE IT, SQUARE IT, ETC.) BUT REMEMBER $i^2=-1$

3. ADDITION

   $$(a+bi)+(c+di)=(a+c)+(b+d)i$$

   $$(4+3i)+\left(2-\tfrac{1}{3}i\right)= 4+2+3i-\tfrac{1}{3}i$$

   $$=6+\left(3-\tfrac{1}{3}\right)i = 6+\tfrac{8}{3}i$$

4. SUBTRACTION: $(a+bi)-(c+di) =$

   $a+bi-c-di = a-c+bi-di =$

   $(a-c)+(b-d)i$

   $$\left(\tfrac{4}{3}-\tfrac{5}{6}i\right)-\left(\tfrac{1}{4}-\tfrac{2}{3}i\right) = \tfrac{4}{3}-\tfrac{5}{6}i-\tfrac{1}{4}+\tfrac{2}{3}i$$

   $$\tfrac{4}{3}-\tfrac{1}{4}+\tfrac{2}{3}i-\tfrac{5}{6}i = \left(\tfrac{4}{3}-\tfrac{1}{4}\right)+\left(\tfrac{2}{3}-\tfrac{5}{6}\right)i =$$

   $$\frac{16-3}{12}+\frac{12-15}{18}i = \frac{13}{12}-\frac{3}{18}i = \frac{13}{12}-\frac{1}{6}i$$

5. <u>MULTIPLICATION</u> $(a+bi)\cdot(c+di) =$

$ac + adi + bci + bdi^2 = ac + (ad+bc)i + bd(-1)$

$(ac-bd) + (ad+bc)i$

a. $(3+2i)(5-3i) = 15 - 9i + 10i - 6i^2 =$

$15 + i - 6(-1) = 21 + i$ ← STANDARD FORM

b. $-i(3-2i) = -3i + 2i^2 = -3i + (2)(-1)$

$= -2 + (-3)i$ ← STANDARD FORM

$\boxed{\text{DEFINITION}}$

6. <u>COMPLEX</u> <u>CONJUGATE</u> OF $z = a+bi$ IS $a-bi$

<u>NOTATION</u> $\overline{z}$ IS READ "THE CONJUGATE OF z".

$\overline{a+bi}$ IS READ "THE CONJUGATE OF $a+bi$".

a. $\overline{3+2i} = 3-2i$ b. $\overline{4-3i} = 4+3i$

c. $\overline{5} = \overline{5+0i} = 5-0i = 5$

d. $\overline{-3i} = \overline{0-3i} = 0+3i = 3i$

e. $\overline{\overline{z}} = z$ LET $z = a+bi$

$\overline{z} = a-bi$ $\overline{\overline{z}} = \overline{a-bi} = a+bi = z$

f. <u>LET $z = 3+2i$</u> <u>AND $w = 4+5i$</u>

$\underline{z + \overline{z+w}} = 3+2i + \overline{3+2i+4+5i} =$

$3+2i + \overline{7+7i} = 3+2i+7-7i = \overline{10-5i} = 10+5i$

I HOMEWORK (OIS)

1. $i^{17} =$ 2. $i^{34} =$

3. NAME A PURE IMAGINARY NUMBER WHOSE IMAGINARY PART IS LESS THAN 1.

4. NAME A COMPLEX NUMBER THAT IS NOT A REAL NUMBER.

5. PROVE: IF $z = a+bi$ IS A COMPLEX NUMBER, THEN $z \cdot \overline{z}$ IS A REAL NUMBER.

6. WRITE EACH OF THE FOLLOWING IN STANDARD FORM (I.E. $a+bi$ FORM)

   a. $(2-\frac{1}{2}i)-(\frac{3}{4}+5i)$

   b. $(-4-\pi i)+(\sqrt{2}+i\sqrt{3})$

   c. $(\frac{1}{2}-5i)-[(2-3i)-(\frac{1}{5}-\frac{1}{6}i)]$

   d. $(2+3i)(2-3i)$

   e. $(\sqrt{2}+i\sqrt{3})(\sqrt{2}-i\sqrt{3})$

   f. $(5-\frac{1}{3}i)(\frac{1}{2}+\frac{1}{5}i)$

   g. $(2+3i)(5-4i)(6-2i)$

7. LET $z = 4-3i$ AND $w = 5-6i$. PUT EACH OF THE FOLLOWING IN $a+bi$ FORM

   a. $\overline{z} - \overline{w}$ b. $\overline{\overline{z} \cdot w}$

   c. $\overline{z \cdot w}$ d. $\overline{z} \cdot \overline{w}$

   e. $\overline{(3-2i) \cdot (\overline{z}+w)}$

J. DIVISION OF COMPLEX NUMBERS

1. $\dfrac{2+3i}{5+4i} \neq \dfrac{2}{5}+\dfrac{3i}{4i}$

2. TO PUT $\dfrac{z}{w}$ IN $a+bi$ FORM COMPUTE $\dfrac{z}{w}\cdot\dfrac{\overline{w}}{\overline{w}}$

(I.E. MULTIPLY TOP AND BOTTOM BY THE CONJUGATE OF THE BOTTOM).

a. $\dfrac{4+3i}{2+5i} = \dfrac{4+3i}{2+5i}\cdot\dfrac{2-5i}{2-5i} = \dfrac{8-20i+6i-15i^2}{4-25i^2}$

$= \dfrac{8-14i-15(-1)}{4-25(-1)} = \dfrac{23-14i}{29} = \dfrac{23}{29}+\left(\dfrac{-14}{29}\right)i$

b. $\dfrac{7-3i}{4i} = \dfrac{7-3i}{4i}\cdot\dfrac{(-4i)}{(-4i)} = \dfrac{-28i+12i^2}{-16i^2}$

$= \dfrac{12(-1)-28i}{-16(-1)} = \dfrac{-12-28i}{16} = \dfrac{-12}{16}-\dfrac{28}{16}i$

$= \dfrac{-3}{4}-\dfrac{7}{4}i$

c. <u>JUGULAR PROBLEM #5</u>: WRITE

$$\frac{\sqrt{3} - i\sqrt{2}}{\sqrt{5} + i\sqrt{3}} \text{ IN } a+bi \text{ FORM}$$

$$\frac{\sqrt{3} - i\sqrt{2}}{\sqrt{5} + i\sqrt{3}} = \frac{\sqrt{3} - i\sqrt{2}}{\sqrt{5} + i\sqrt{3}} \cdot \frac{\sqrt{5} - i\sqrt{3}}{\sqrt{5} - i\sqrt{3}} =$$

$$\frac{\sqrt{15} - 3i - i\sqrt{10} + i^2\sqrt{6}}{5 - 3i^2} =$$

$$\frac{\sqrt{15} + (-3 - \sqrt{10})i + (-1)\sqrt{6}}{5 - 3(-1)} =$$

$$\frac{(\sqrt{15} - \sqrt{6}) + (-3 - \sqrt{10})i}{8} =$$

$$\frac{\sqrt{15} - \sqrt{6}}{8} + \frac{(-3 - \sqrt{10})}{8}i$$

K WHEN USING $\sqrt{-b}$, CHANGE TO $i\sqrt{b}$, THEN PERFORM OPERATIONS. $(b>0)$

1. $\sqrt{-9} = \sqrt{(-1)9} = \sqrt{-1}\sqrt{9} = i3 = 3i$

2. $\sqrt{-4} = 2i$ 3. $\sqrt{-7} = i\sqrt{7}$

4. CAREFUL: $\sqrt{ab} = \sqrt{a}\sqrt{b}$ WHEN $(a<0$ AND $b>0)$ OR $(a>0$ AND $b>0)$ OR $(a>0$ AND $b<0)$ BUT NOT WHEN $(a<0$ AND $b<0)$!!

$$-1 = \sqrt{-1}\sqrt{-1} \overset{\text{BAD}}{=} \sqrt{(-1)(-1)} = \sqrt{1} = 1$$

DON'T DO THIS

5. CONVERT TO $a+bi$ FORM FIRST, THEN OPERATE.

a. $\sqrt{-8}\left(\sqrt{8} - \sqrt{-2}\right) = i\sqrt{8}\left(\sqrt{8} - i\sqrt{2}\right)$
$= 8i - i^2\sqrt{16} = 8i - (-1)4 = 4+8i$

b. $(\sqrt{3} - \sqrt{-2})(\sqrt{3} + \sqrt{-5}) =$
$(\sqrt{3} - i\sqrt{2})(\sqrt{3} + i\sqrt{5}) = 3 + i\sqrt{15} - i\sqrt{6} - i^2\sqrt{10}$
$= 3 + i\sqrt{15} - i\sqrt{6} - (-1)\sqrt{10}$
$= (3+\sqrt{10}) + (+\sqrt{15} - \sqrt{6})i$

L. HOMEWORK (OIS)

1. WRITE IN $a+bi$ FORM

a. $\dfrac{2+3i}{4+5i}$ b. $\dfrac{4-7i}{-2i}$ c. $\dfrac{4-7i}{-2}$

d. $\dfrac{\frac{1}{2}-\frac{2}{3}i}{\frac{1}{5}+\frac{1}{6}i}$ e. $\dfrac{\sqrt{3}-i\sqrt{7}}{\sqrt{2}+i\sqrt{3}}$

f. $\dfrac{\sqrt{5}+i\sqrt{6}}{\sqrt{3}+i\sqrt{2}}$ g. $\dfrac{\sqrt{2}-i\sqrt{6}}{\sqrt{3}-i\sqrt{7}}$

2. EVALUATE THE FOLLOWING

a. $\sqrt{-6}\left(\sqrt{2}-\sqrt{-8}\right)$

b. $\left(\sqrt{-5}-\sqrt{-6}\right)\left(\sqrt{-125}+\sqrt{-9}\right)$

c. $\left(\sqrt{-5}-\sqrt{-3}\right)^2\left(\sqrt{-5}+\sqrt{6}\right)$

3. DO EACH OF $\dfrac{-1+i\sqrt{3}}{2}$ AND $\dfrac{-1-i\sqrt{3}}{2}$ SATISFY THE EQUATION $x^2+x+1=0$?

[CHAPTER 7]
SOLVING EQUATIONS

A. DIFFERENCE BETWEEN AN <u>EXPRESSION</u> AND AN E<u>QUATION</u>

$2x-3$ ← EXPRESSION

$2x-3=0$ ← EQUATION

B. OPEN SENTENCE, SOLUTION, SOLUTION SET

1. $x+4=7$ IS AN OPEN SENTENCE. NEITHER TRUE NOR FALSE

2. REPLACE x WITH THE VALUE 3 IN THE OPEN SENTENCE $x+4=7$ TO GET $3+4=7$ ← <u>TRUE</u>

   SO 3 IS A <u>SOLUTION</u> TO $x+4=7$

DEFINITIONS

3. THE <u>SOLUTION SET</u> FOR AN EQUATION IS THE <u>SET</u> OF ALL SOLUTIONS FOR THE EQUATION

   $\{3\}$ IS THE SOLUTION SET FOR $x+4=7$

   $\{-2,+2\}$ IS THE SOLUTION SET FOR $x^2=4$

   <u>SOLVE</u> MEANS F<u>IND</u> <u>A</u> SO<u>LUTION</u> <u>SET</u>

DEFINITIONS: 7-96

C. TYPES OF EQUATIONS BY SOLUTIONS

1. IDENTITIES: TRUE FOR ALL VALUES FOR WHICH BOTH SIDES OF THE EQUATION ARE DEFINED

$$x + x = 2x$$

$$\frac{1}{x} + \frac{2}{x} = \frac{3}{x}$$

$$(x+1)^2 = x^2 + 2x + 1$$

2. INCONSISTENT: NO SOLUTION

$$\left.\begin{aligned} x^2 &= -4 \\ x^2 + 1 &= 0 \end{aligned}\right\}$$

WE ONLY CONSIDER THE REALS, NOT THE COMPLEX NUMBERS IN THIS CASE

3. CONDITIONAL: TRUE FOR AT LEAST ONE, BUT NOT ALL VALUES FOR WHICH BOTH SIDES OF THE EQUATION ARE DEFINED

$x^2 + 1 = 5$ SOLUTION SET $\{-2, 2\}$

$2x + 1 = 7$ SOLUTION SET $\{3\}$

NOTE: AT TIMES THE WORD ROOT IS USED TO MEAN THE SAME THING AS SOLUTION

D. EQUIVALENT EQUATIONS AND SOLVING

DEFINITION:

1. EQUIVALENT EQUATIONS : EQUATIONS THAT HAVE THE SAME SOLUTION SET.

$$\left.\begin{aligned} 2x+1&=7\\ 2x&=6\\ x&=3 \end{aligned}\right\}$$

EQUIVALENT EQUATIONS. ALL HAVE THE SAME SOLUTION SET, $\{3\}$

2. WAYS TO GET EQUIVALENT EQUATIONS

a. ADD/SUBTRACT THE SAME THING TO/FROM BOTH SIDES.

$$\left.\begin{aligned} 2x+1&=7\\ 2x+1-1&=7-1\\ 2x&=6 \end{aligned}\right\}$$

EQUIVALENT EQUATIONS (OK TO LEAVE MIDDLE STEP OUT)

$$\left.\begin{aligned} & x^2-6x=x-12\\ * \quad & x^2-6x-x=-x+x-12\\ & x^2-7x=-12\\ * \quad & x^2-7x+12=12-12\\ & x^2-7x+12=0 \end{aligned}\right\}$$

EQUIVALENT EQUATIONS. YOU CAN THINK OF THE SIGN CHANGING AS IT JUMPS OVER THE EQUALITY SIGN.

* OK TO LEAVE OUT THESE LINES

b. MULTIPLY/DIVIDE BOTH SIDES BY THE SAME NONZERO NUMBER

$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

EQUIVALENT EQUATIONS (OK TO LEAVE OUT MIDDLE LINE)

DIVIDED BOTH SIDES BY NONZERO 2 (I.E. MULTIPLIED BOTH SIDES BY $\frac{1}{2}$)

3. STRATEGY FOR SOLVING EQUATIONS:

START WITH ORIGINAL EQUATION. CREATIVELY PRODUCE A LIST OF EQUIVALENT EQUATIONS THAT HAS THE VARIABLE ISOLATED ON ONE SIDE OF THE EQUATION AND KNOWN VALUES ON THE OTHER SIDE OF THE EQUATION.

a. $5x - 3 = 2x + 7$ SOLVE

$$-2x + 5x - 3 = 7$$
$$3x - 3 = 7$$
$$3x = 3 + 7$$
$$3x = 10$$
$$x = \frac{10}{3}$$

← "WORLD'S EASIEST EQUATION TO SOLVE"

SOLUTION SET $\left\{\frac{10}{3}\right\}$

b. $5(2x-1)(x+4)=(10x-1)(x+3)$ SOLVE

$$5(2x^2+8x-x-4)=10x^2+30x-x-3$$

$$5(2x^2+7x-4)=10x^2+29x-3$$

$$10x^2+35x-20=10x^2+29x-3$$

$$35x-20=29x-3$$

$$-29x+35x-20=-3$$

$$6x-20=-3$$

$$6x=+20-3$$

$$6x=17$$

$$x=\frac{17}{6}$$

SOLUTION SET $\left\{\frac{17}{6}\right\}$

c. FIND THE SOLUTION SET FOR UNKNOWN x

$$3b-2x+7=5x-bx \qquad b\neq 7$$

$$-2x-5x+bx=-3b-7$$

$$-7x+bx=-3b-7$$

$$x(-7+b)=-3b-7$$

$$x=\frac{-3b-7}{-7+b}=\frac{-(3b+7)}{-(7-b)}=\frac{3b+7}{7-b}$$

SOLUTION SET $\left\{\frac{3b+7}{7-b}\right\}$

E. HOMEWORK (OIS)

1. IDENTIFY AS EITHER IDENTITY, INCONSISTENT, CONDITIONAL (REALS ONLY)

   a. $x(x+1)=x^2+x$ b. $x^2+6=-4$

   c. $x^3+6=-4$ d. $x^4+6=-4$

2. WHICH PAIRS OF EQUATIONS ARE EQUIVALENT

   a. $2x-4=0$, $2x=4$ b. $x^2=4$, $x^4=16$ c. $x^2=9$, $x=3$

3. FIND THE SOLUTION SET. DO A STEP BY STEP DEDUCTION.

   a. $3x-7=4$ b. $4(2x-3)=5x$

   c. $x(2x-4)=2x^2-3x$ d. $x^2-5x=x-9$

   e. $5\left(2x-\frac{3}{4}\right)+\frac{1}{3}=\frac{2}{3}\left(5x-\frac{1}{7}\right)$

   f. $(3x-5)(2x+1)=(6x+4)(x-1)$

   g. $\frac{2}{3}x-\frac{1}{5}=\frac{3}{5}\left(\frac{2}{7}x+5\right)$

   h. $x\sqrt{7}-3bx+\frac{1}{5}=4b-\frac{2}{3}x$

   $\left(\sqrt{7}-3b+\tfrac{2}{3}\neq 0\right)$

   i. $\dfrac{5x-2}{x^2+1}=\dfrac{2}{3}\left(\dfrac{7x-p}{\frac{2}{3}x^2+\frac{2}{3}}\right)$

F. WHEN MULTIPLYING OR DIVIDING BY AN EXPRESSION THAT COULD BE ZERO, CHECK YOUR ANSWER.

1. SOLVE: $\dfrac{x}{x-4}+2=\dfrac{4}{x-4}$

$$(x-4)\left[\frac{x}{x-4}+2\right]=(x-4)\frac{4}{x-4}$$

★ CHECK ANSWER $x-4$ COULD BE ZERO

$$\frac{(x-4)x}{x-4}+(x-4)2=4$$

$$x+2x-8=4$$

$$3x=12$$

$$x=4$$

★ CHECK IN ORIGINAL EQUATION

$\dfrac{4}{4-4}+2=$ BAD, DIVISION BY 0

NO SOLUTION. SOLUTION SET $=\{\ \}$

INCONSISTENT EQUATION

4 IS AN <u>EXTRANEOUS</u> SOLUTION

AN EXTRANEOUS SOLUTION IS NOT A SOLUTION.

2. FIND THE SOLUTION SET FOR

$$\frac{y}{y+3} + 4 = \frac{5}{y+3}$$

$$(y+3)\left[\frac{y}{y+3} + 4\right] = (y+3)\frac{5}{y+3}$$

★ CHECK FOR EXT. SOL.

$$(y+3)\frac{y}{y+3} + (y+3)4 = 5$$

$$y + 4y + 12 = 5$$

$$5y = -12 + 5$$

$$5y = -7$$

$$y = \frac{-7}{5}$$

★ CHECK FOR EXTRANEOUS SOLUTION IN ORIGINAL EQUATION. (SHOW LEFT SIDE = RIGHT SIDE OF EQUATION)

$$\frac{-\frac{7}{5}}{-\frac{7}{5}+3} + 4 = \frac{-\frac{7}{5}}{\frac{-7+15}{5}} + 4 = \frac{-\frac{7}{5}}{\frac{8}{5}} + 4$$

$$= -\frac{7}{5}\cdot\frac{5}{8} + 4 = -\frac{7}{8} + 4 = \frac{-7+32}{8} = \frac{25}{8}.$$

$$\frac{5}{-\frac{7}{5}+3} = \frac{5}{\frac{-7+15}{5}} = \frac{\frac{5}{1}}{\frac{8}{5}} = \frac{5}{1}\cdot\frac{5}{8} = \frac{25}{8}.$$

YES

SOLUTION SET $= \{-7/5\}$

A NOTE ABOUT CHECKING YOUR ANSWER.

DO NOT LET THE FIRST LINE OF YOUR CHECKING READ

$$\frac{\frac{-7}{5}}{-\frac{7}{5}+3} + 4 = \frac{5}{-\frac{7}{5}+3}$$

THIS IS ALREADY STATING AS PROVEN, WHAT YOU ARE TRYING TO PROVE!

TO PROVE THE ABOVE EQUATION, GET THE LEFT SIDE OF THE EQUATION EQUAL TO SOMETHING* AND THE RIGHT SIDE OF THE EQUATION EQUAL TO THE SAME SOMETHING*. THE LEFT SIDE AND RIGHT SIDE ARE THEREFORE EQUAL TO EACH OTHER.

* ON THE PREVIOUS PAGE THAT SOMETHING WAS $\frac{25}{8}$.

G. HOMEWORK (OIS) SOLVE THESE EQUATIONS. IF MULTIPLYING BY AN EXPRESSION THAT COULD BE ZERO, CHECK YOUR ANSWER.

1. $\dfrac{4}{x-2} - 4 = \dfrac{3}{x-2}$

2. $\dfrac{x}{x+5} + 3 = \dfrac{-5}{x+5}$

3. $\dfrac{4}{x-2} - \dfrac{4}{x+2} = 4$

4. $\dfrac{1}{x-3} + \dfrac{2}{x+3} = \dfrac{3}{(x-3)(x+3)}$

5. $\dfrac{1}{x-3} - \dfrac{6}{x^2-9} = \dfrac{-2}{x+3}$

6. $\dfrac{2}{3x-2} - \dfrac{4}{x+1} = \dfrac{3x+1}{3x^2+x-2}$

7. $\dfrac{2}{3x-2} - \dfrac{4}{x+1} = \dfrac{3x-16}{3x^2+x-2}$

H. SOLVING $(x-p)^2 = b$

1. $x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$|x| = 3$

<u>NOTE</u>: THE $\pm$ COMES FROM REMOVING ABSOLUTE VALUE SIGNS, <u>NOT</u> FROM TAKING SQUARE ROOTS.

$x = +3$ <u>OR</u> $x = -3$ (DENOTED $x = \pm 3$)

SOLUTION SET $\{3, -3\}$

(NOTE: THIS IS <u>NOT</u> SAYING $\sqrt{9} = \pm 3$)

2. $x^2 = 8$

$\sqrt{x^2} = \sqrt{8}$

$|x| = \sqrt{8} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$

$x = \pm 2\sqrt{2}$ SOLUTION SET $\{2\sqrt{2}, -2\sqrt{2}\}$

3. $(x-3)^2 = 5$

$$\left.\begin{aligned} \sqrt{(x-3)^2} &= \sqrt{5} \\ |x-3| &= \sqrt{5} \end{aligned}\right\} \text{OK TO LEAVE THESE LINES OUT}$$

$x - 3 = \pm\sqrt{5}$

$x = 3 \pm \sqrt{5}$

SOLUTION SET $\{3+\sqrt{5}, 3-\sqrt{5}\}$

4. $(x+2)^2 = 16$

$x+2 = \pm\sqrt{16} = \pm 4$

$x = -2 \pm 4$

$x = -2+4 = 2$ <u>OR</u> $x = -2-4 = -6$

SOLUTION SET $\{2, -6\}$

5. $(x+\frac{1}{2})^2 = -27$

$x + \frac{1}{2} = \pm\sqrt{-27} = \pm i\sqrt{27} = \pm 3i\sqrt{3}$

$x = -\frac{1}{2} \pm 3i\sqrt{3}$

SOLUTION SET $\{-\frac{1}{2}+3i\sqrt{3}, -\frac{1}{2}-3i\sqrt{3}\}$

I. SOLVING $ax^2+bx+c=0$ (QUADRATICS) BY <u>FACTORING</u>

1. $a \cdot b = 0$ IFF $a=0$ <u>OR</u> $b=0$

2. $(x-2)(x-3) = 0$

$x-2=0$ <u>OR</u> $x-3=0$

$x=2$ <u>OR</u> $x=3$

SOLUTION SET $\{2, 3\}$

3. SOLVE $8x^2-2x-15=0$

$$(2x-3)(4x+5)=0$$

$2x-3=0$ <u>OR</u> $4x+5=0$

$2x=3$ OR $4x=-5$

$x=\frac{3}{2}$ OR $x=-\frac{5}{4}$

SOLUTION SET $\left\{\frac{3}{2}, -\frac{5}{4}\right\}$

4. SOLVE $x^2+x+1=0$

★ $$\left(x-\left[\frac{-1+i\sqrt{3}}{2}\right]\right)\left(x-\left[\frac{-1-i\sqrt{3}}{2}\right]\right)=0$$

$x-\left[\frac{-1+i\sqrt{3}}{2}\right]=0$ <u>OR</u> $x-\left[\frac{-1-i\sqrt{3}}{2}\right]=0$

$x=\frac{-1+i\sqrt{3}}{2}$ OR $x=\frac{-1-i\sqrt{3}}{2}$

SOLUTION SET $\left\{\frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$

★ YOU ARE <u>NOT</u> EXPECTED TO LOOK AT x^2+x+1 AND KNOW IT FACTORS INTO THIS. THIS WAS TO BAIT YOU TO KNOW WE NEED TO LEARN ANOTHER WAY TO SOLVE QUADRATIC EQUATIONS.

J. SOLVING $ax^2+bx+c=0$ (QUADRATICS) BY COMPLETE THE SQUARE METHOD

1. NAME THAT SQUARE

$$a^2+2ab+b^2 = (a+b)^2$$

$$x^2+6x+9 = (x+3)^2$$

$$x^2+8x+16 = (\_\_\_\_)^2$$

$$x^2-8x+16 = (\_\_\_\_)^2$$

$$x^2-\tfrac{2}{3}x+\tfrac{1}{9} = (\_\_\_\_)^2$$

2. NOW COMPLETE THE LEFT SIDE OF THE EQUATION TO MAKE IT A PERFECT SQUARE

$$x^2+10x+\_\_\_\_ = (\_\_\_\_)^2$$

ANSWER 25 (NOTE $25 = (\tfrac{1}{2}\cdot 10)^2$)

$$x^2-\tfrac{5}{3}x+\_\_\_\_ = (\quad)^2$$

ANSWER $\frac{25}{36}$ (NOTE $\frac{25}{36} = \left[\frac{1}{2}\left(-\frac{5}{3}\right)\right]^2$)

$$x^2-\tfrac{5}{3}x+\tfrac{25}{36} = \left(x-\tfrac{5}{6}\right)^2$$

3. NOW THE ENTIRE METHOD.

SOLVE $2x^2+12x+4=0$ BY COMPLETE THE SQUARE METHOD.

a. GET x^2 COEFFICIENT TO 1

$$x^2+6x+2=0$$

b. GET ONLY TERMS WITH x ON LEFT

$$x^2+6x \qquad = -2$$

c. COMPLETE THE SQUARE ON LEFT SIDE <u>ADDING THE SAME THING TO BOTH SIDES.</u>

$$x^2+6x+9 = -2+9$$

$$(x+3)^2 \qquad = 7$$

d. SOLVE THE EQUATION

$$x+3 = \pm\sqrt{7}$$

$$x = -3 \pm \sqrt{7}$$

SOLUTION SET $\{-3+\sqrt{7}, -3-\sqrt{7}\}$

4. SOLVE $-3x^2+9x-5=0$ BY COMPLETE THE SQUARE METHOD

a. GET x^2 COEFFICIENT TO 1

$$x^2 - 3x + \frac{5}{3} = 0$$

b. GET ONLY TERMS WITH x ON LEFT

$$x^2 - 3x = -\frac{5}{3}$$

c. COMPLETE THE SQUARE ON LEFT SIDE ADDING THE SAME THING TO BOTH SIDES

$$x^2 - 3x + \frac{9}{4} = -\frac{5}{3} + \frac{9}{4} \qquad \left[\frac{1}{2}(-3)\right]^2 = \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{-20+27}{12} = \frac{7}{12}$$

d. SOLVE THE EQUATION

$$x - \frac{3}{2} = \pm\sqrt{\frac{7}{12}} = \pm\sqrt{\frac{7}{4\cdot 3}} = \pm\frac{1}{2}\sqrt{\frac{7}{3}}$$

$$x = \frac{3}{2} \pm \frac{1}{2}\sqrt{7/3}$$

SOLUTION SET $\left\{\frac{3}{2}+\frac{1}{2}\sqrt{7/3}, \frac{3}{2}-\frac{1}{2}\sqrt{7/3}\right\}$

5. A FACTORING MYSTERY REVEALED

SOLVE $x^2+x+1=0$ BY COMPLETE THE SQUARE METHOD.

$$x^2+x = -1$$

$$x^2+x+\frac{1}{4} = -1+\frac{1}{4} \qquad \left(\frac{1}{2}\cdot 1\right)^2 = \frac{1}{4}$$

$$\left(x+\frac{1}{2}\right)^2 = -\frac{3}{4}$$

$$x+\frac{1}{2} = \pm\sqrt{\frac{-3}{4}} = \pm i\sqrt{\frac{3}{4}} = \pm\frac{i\sqrt{3}}{2}$$

$$x = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

SOLUTION SET $\left\{\frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$

NOTE $\left(x-\left[\frac{-1+i\sqrt{3}}{2}\right]\right)\cdot\left(x-\left[\frac{-1-i\sqrt{3}}{2}\right]\right) =$

$$x^2-\left[\frac{-1-i\sqrt{3}}{2}\right]x - \left[\frac{-1+i\sqrt{3}}{2}\right]x + \frac{1-i^2 3}{4} =$$

$$x^2+\frac{1}{2}x+\frac{i\sqrt{3}}{2}x+\frac{1}{2}x-\frac{i\sqrt{3}}{2}x+\frac{1-(-1)3}{4} =$$

x^2+x+1. SO x^2+x+1 FACTORS INTO $\left(x-\left[\frac{-1+i\sqrt{3}}{2}\right]\right)\cdot\left(x-\left[\frac{-1-i\sqrt{3}}{2}\right]\right)$

EARLIER IN THE CHAPTER WE SOLVED $x^2+x+1=0$ BY THE FACTORING METHOD. NOW YOU SEE HOW TO FACTOR A QUADRATIC.

FACTOR $-3x^2+9x-5$

a. SOLVE $-3x^2+9x-5=0$

WE PREVIOUSLY FOUND THE SOLUTION SET OF $\left\{\frac{3}{2}+\frac{1}{2}\sqrt{\frac{7}{3}}, \frac{3}{2}-\frac{1}{2}\sqrt{\frac{7}{3}}\right\}$

b. FACTOR INTO THE PATTERN

$$\left(\begin{array}{c}x^2\\ \text{COEF-}\\ \text{FICIENT}\end{array}\right)\left(x-\begin{array}{c}\text{SOLU-}\\ \text{TION 1}\end{array}\right)\left(x-\begin{array}{c}\text{SOLU-}\\ \text{TION 2}\end{array}\right)$$

$$-3\left(x-\left[\frac{3}{2}+\frac{1}{2}\sqrt{\frac{7}{3}}\right]\right)\left(x-\left[\frac{3}{2}-\frac{1}{2}\sqrt{\frac{7}{3}}\right]\right)$$

$\uparrow$

DO NOT FORGET TO PUT THE x^2 COEFFICIENT IN FRONT SO THAT WHEN EVERYTHING IS MULTIPLIED OUT YOU GET $-3x^2+9x-5$

K. HOMEWORK (OIS)

1. $x^2 + 12x + \_\_\_ = (x + \_\_\_)^2$
2. $x^2 - 17x + \_\_\_ = (x + \_\_\_)^2$
3. SOLVE (FIND SOLUTION SET)

   a. $x^2 = 25$ b. $x^2 = 27$ c. $(x-2)^2 = 9$

   d. $(x+5)^2 = 7$ e. $(x+\frac{1}{2})^2 = -4$

   f. $(x-3)^2 = -5$

4. SOLVE (FIND SOLUTION SET) BY FACTORING

   a. $2x^2 + 11x - 6 = 0$ b. $6x^2 + 5x - 6 = 0$

   c. $x^2 - 9x + 20 = 0$ d. $4x^2 - 11x + 6 = 0$

   e. $x^2 + 2x - 15 = 0$ f. $10x^2 - 80x + 150 = 0$

5. SOLVE (FIND SOLUTION SET) BY COMPLETE THE SQUARE METHOD

   a. $x^2 + 2x - 15 = 0$ b. $x^2 - 9x + 20 = 0$

   c. $2x^2 + 11x - 6 = 0$ d. $x^2 + 4x + 8 = 0$

   e. $3x^2 + 2x + 7 = 0$ f. $2x^2 + 5 = x$

6. FACTOR INTO THE FORM $a(x-p)(x-q)$

   a. $x^2 + 4x + 8$ b. $3x^2 + 2x + 7$

   c. $2x^2 - 3x + 9$

L. QUADRATIC FORMULA

1. FIND A FORMULA FOR SOLVING $ax^2+bx+c=0$

a. CASE 1 $a>0$

COMPLETE THE SQUARE TO SOLVE $ax^2+bx+c=0$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{b^2}{4a^2}-\frac{c}{a} \qquad \left(\frac{1}{2}\cdot\frac{b}{a}\right)^2=\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{4ac}{4aa}=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\frac{\sqrt{b^2-4ac}}{2|a|}$$

SINCE $a>0$ $|a|=a$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$ QUADRATIC FORMULA

b. CASE 2 $a<0$ TO BE DONE FOR HOMEWORK.

2. SOLVE $-3x^2+9x-5=0$ BY THE QUADRATIC FORMULA. $ax^2+bx+c=0$

$a=-3 \quad b=9 \quad c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-9\pm\sqrt{9^2-4(-3)(-5)}}{2(-3)}$$

$$=\frac{-9\pm\sqrt{81-60}}{-6}=\frac{-9\pm\sqrt{21}}{-6}$$

SOLUTION SET $\left\{\frac{-9+\sqrt{21}}{-6}, \frac{-9-\sqrt{21}}{-6}\right\}$

NOTE IN SECTION J,4 OF THIS CHAPTER WE SOLVED THIS BY THE COMPLETE THE SQUARE METHOD. ONE ANSWER WAS

$$\frac{3}{2}+\frac{1}{2}\sqrt{\frac{7}{3}} \underset{\text{EQUALS}}{\overset{\text{WHICH}}{=}} \frac{3}{2}+\frac{1}{2}\sqrt{\frac{7\cdot 3}{3\cdot 3}}$$

$$=\frac{3}{2}+\frac{1}{2}\cdot\frac{\sqrt{21}}{3}=\frac{3\cdot 3}{2\cdot 3}+\frac{\sqrt{21}}{6}=$$

$$\frac{9+\sqrt{21}}{6}=\frac{-(9+\sqrt{21})}{-6}=\frac{-9-\sqrt{21}}{-6}.$$

ALL 3 METHODS (1) FACTOR, (2) COMPLETE THE SQUARE, (3) QUADRATIC FORMULA GIVE THE SAME ANSWERS

(NOTE OUR ANSWER WAS 2 REAL SOLUTIONS. NOTE $b^2-4ac=21>0$)

3. SOLVE $x^2-6x+9=0$ BY QUADRATIC FORMULA.

$$ax^2+bx+c=0$$

$a=1 \quad b=-6 \quad c=9$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)9}}{2(1)}$$

$$=\frac{6\pm\sqrt{36-36}}{2}=\frac{6\pm\sqrt{0}}{2}=\frac{6\pm0}{2}=\frac{6}{2}=3$$

(NOTE $b^2-4ac=0$ AND WE HAD ONE REAL SOLUTION, 3. WE COULD HAVE SOLVED BY FACTORING:

$$x^2-6x+9=0$$

$$(x-3)(x-3)=0$$

$$x-3=0 \quad \underline{\underline{OR}} \quad x-3=0$$

$$x=3 \quad \underline{\underline{OR}} \quad x=3$$

MORE TECHNICALLY WE SAY, ONE REAL ROOT OF <u>MULTIPLICITY</u> <u>TWO</u>)

4. SOLVE $x^2+x+1=0$ BY QUADRATIC FORMULA

$$x=\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2(1)}=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm i\sqrt{3}}{2}$$

SOLUTION SET $\left\{\frac{-1+i\sqrt{3}}{2}, \ \frac{-1-i\sqrt{3}}{2}\right\}$

(NOTE: $b^2-4ac=-3<0$. 2 IMAGINARY SOLUTIONS)

7-117

5. DISCRIMINANT b^2-4ac <u>DEF.</u>

a. A WAY TO PEEP AHEAD TO SEE IF YOU ARE GOING TO GET 2 REAL, 1 REAL, OR 2 IMAGINARY SOLUTIONS

b.

$b^2-4ac>0$	2 REAL SOLUTIONS
$b^2-4ac=0$	1 REAL SOLUTION OF MULTIPLICITY 2
$b^2-4ac<0$	2 IMAGINARY SOLUTIONS

c. WITHOUT SOLVING $-3x^2+9x-5=0$ TELL HOW MANY AND WHAT TYPE OF SOLUTIONS IT WILL HAVE BY EVALUATING THE DISCRIMINANT

$$b^2-4ac=9^2-4(-3)(-5)=81-60=21>0$$

THERE WILL BE 2 REAL SOLUTIONS <u>IF</u> WE WERE TO SOLVE IT.

(WELL, WE DID PREVIOUSLY SOLVE IT IN SECTION L,2 OF THIS CHAPTER AND IT DID HAVE 2 REAL SOLUTIONS, $\frac{-9+\sqrt{21}}{-6}$ AND $\frac{-9-\sqrt{21}}{-6}$.)

M. HOMEWORK (OIS)

1. DO NOT SOLVE THESE EQUATIONS NOW. USE THE DISCRIMINANT TO FIND OUT HOW MANY AND WHAT TYPE OF SOLUTIONS IT WILL HAVE

   a. $x^2+2x-15=0$ b. $x^2-9x+20=0$

   c. $2x^2+11x-6=0$ d. $x^2+4x+8=0$

   e. $3x^2+2x+7=0$ f. $2x^2+5=x$

   g. $x^2+4x+4=0$

2. NOW SOLVE EACH OF THE EQUATIONS IN PART 1 ABOVE BY THE QUADRATIC FORMULA.

3. FACTOR $3x^2+2x+7$ INTO THE FORM $a(x-p)(x-q)$

N. ADVANCED SOLVING BY FACTORING

1. $x^3 + 4x^2 + 2x + 8 = 0$

$x^2(x+4) + 2(x+4) = 0$

$(x+4)(x^2+2) = 0$

$x+4=0$ <u>OR</u> $x^2+2=0$

$x=-4$ OR $x^2=-2$

$x=-4$ OR $x = \pm\sqrt{-2} = \pm i\sqrt{2}$

SOLUTION SET $\{-4, i\sqrt{2}, -i\sqrt{2}\}$

2. FACTORING OUT *

FACTOR b^3 OUT OF b^7

$b^7 = b^3 \cdot b^4 = b^3 \cdot b^{7-3}$ NOTE: ADD THE EXPONENTS

<u>SUBTRACT</u> THE EXPONENT OF WHAT WAS FACTORED OUT.

FACTOR $b^{\frac{2}{3}}$ OUT OF $b^{7/3}$

$b^{\frac{7}{3}} = b^{\frac{2}{3}} \cdot b^{\frac{7}{3}-\frac{2}{3}} = b^{\frac{2}{3}} \cdot b^{\frac{5}{3}}$

FACTOR $b^{-\frac{1}{5}}$ OUT OF $b^{\frac{4}{5}}$

$b^{\frac{4}{5}} = b^{-\frac{1}{5}} \cdot b^{\frac{4}{5}-(-\frac{1}{5})} = b^{-\frac{1}{5}} \cdot b^{\frac{4}{5}+\frac{1}{5}} = b^{-\frac{1}{5}} \cdot b^{\frac{5}{5}}$

$= b^{-\frac{1}{5}} \cdot b$

* METHOD USED BY CLAUDE ANDERSON

7-119 A

FACTOR $(2x+1)^{-\frac{1}{3}}$ OUT OF $(2x+1)^{\frac{2}{3}}$.

$$(2x+1)^{\frac{2}{3}} = (2x+1)^{-\frac{1}{3}}(2x+1)^{\frac{2}{3}-(-\frac{1}{3})}$$

$$= (2x+1)^{-\frac{1}{3}}(2x+1)^{\frac{2}{3}+\frac{1}{3}} = (2x+1)^{-\frac{1}{3}}(2x+1)$$

---

3. JUGULAR #6 : SOLVE

$$\frac{7}{3}(x+3)^{\frac{5}{3}}(2x-1)^{\frac{1}{6}} + \frac{5}{3}(2x-1)^{\frac{7}{6}}(x+3)^{\frac{2}{3}} = 0$$

FACTOR OUT PARTS WITH <u>LESSER</u> EXPONENTS.

$$(x+3)^{\frac{2}{3}}(2x-1)^{\frac{1}{6}}\left[\frac{7}{3}(x+3)^{\frac{5}{3}-\frac{2}{3}} + \frac{5}{3}(2x-1)^{\frac{7}{6}-\frac{1}{6}}\right] = 0$$

$$(x+3)^{\frac{2}{3}}(2x-1)^{\frac{1}{6}}\left[\frac{7}{3}(x+3)^{\frac{3}{3}} + \frac{5}{3}(2x-1)^{\frac{6}{6}}\right] = 0$$

$$(x+3)^{\frac{2}{3}}(2x-1)^{\frac{1}{6}}\left[\frac{7}{3}(x+3) + \frac{5}{3}(2x-1)\right] = 0$$

$$(x+3)^{\frac{2}{3}}(2x-1)^{\frac{1}{6}}\left[\frac{7}{3}x + 7 + \frac{10}{3}x - \frac{5}{3}\right] = 0$$

$$(x+3)^{\frac{2}{3}}(2x-1)^{\frac{1}{6}}\left[\frac{17}{3}x + \frac{16}{3}\right] = 0$$

$(x+3)^{\frac{2}{3}} = 0$ OR $(2x-1)^{\frac{1}{6}} = 0$ OR $\frac{17}{3}x + \frac{16}{3} = 0$

$x+3 = 0$ OR $2x-1 = 0$ OR $\frac{17}{3}x = -\frac{16}{3}$

$x = -3$ OR $2x = 1$ OR $x = -\frac{16}{3} \cdot \frac{3}{17}$

$x = -3$ OR $x = \frac{1}{2}$ OR $x = -\frac{16}{17}$

SOLUTION SET $\rightarrow$ $\{-3, -\frac{16}{17}, \frac{1}{2}\}$

4. JUGULAR #6 TYPE AGAIN. SOLVE

$$\frac{6}{5}(2x+1)^{\frac{2}{3}}(x+5)^{\frac{1}{5}}+\frac{4}{3}(x+5)^{\frac{6}{5}}(2x+1)^{-\frac{1}{3}}=0$$

$$(2x+1)^{-\frac{1}{3}}(x+5)^{\frac{1}{5}}\left[\frac{6}{5}(2x+1)^{\frac{2}{3}-(-\frac{1}{3})}+\frac{4}{3}(x+5)^{\frac{6}{5}-\frac{1}{5}}\right]=0$$

$$\frac{(x+5)^{\frac{1}{5}}}{(2x+1)^{\frac{1}{3}}}\left[\frac{6}{5}(2x+1)+\frac{4}{3}(x+5)\right]=0$$

$$\frac{(x+5)^{\frac{1}{5}}}{(2x+1)^{\frac{1}{3}}}\left[\frac{12}{5}x+\frac{6}{5}+\frac{4}{3}x+\frac{20}{3}\right]=0$$

$$\frac{(x+5)^{\frac{1}{5}}}{(2x+1)^{\frac{1}{3}}}\left[\frac{12\cdot 3}{5\cdot 3}x+\frac{4\cdot 5}{3\cdot 5}x+\frac{6\cdot 3}{5\cdot 3}+\frac{20\cdot 5}{3\cdot 5}\right]=0$$

$$\frac{(x+5)^{\frac{1}{5}}}{(2x+1)^{\frac{1}{3}}}\left[\frac{56}{15}x+\frac{118}{15}\right]=0$$

[ZERO ON BOTTOM: BAD

NOT $(2x+1)^{\frac{1}{3}}=0$]

$(x+5)^{\frac{1}{5}}=0$ OR $\frac{56}{15}x+\frac{118}{15}=0$

$x+5=0$ OR $\frac{56}{15}x=-\frac{118}{15}$

$x=-5$ OR $x=-\frac{118}{15}\cdot\frac{15}{56}=\frac{-118}{56}=\frac{-59}{28}$

SOLUTION SET $\left\{-5,-\frac{59}{28}\right\}$

a. <u>NOTE:</u> ON THE PREVIOUS PAGE WE DID NOT ALLOW $(2x+1)^{1/3} = 0$ SINCE THAT WOULD HAVE CAUSED <u>DIVISION BY ZERO</u>.

b. <u>NOTE</u>: ONE REASON TO TAKE ALGEBRA IS TO BE PREPARED FOR CALCULUS. JUGULAR #6 TYPES OF PROBLEMS ARE SOMEWHAT SIMILAR TO PARTS OF BIGGER CALCULUS PROBLEMS.

O. SOLVING EQUATIONS WITH A $\sqrt{}$

1. FIND ALL REAL SOLUTIONS TO $\sqrt{x} = -3$.

   SQUARE BOTH SIDES: $x = 9$

   CHECK YOUR SOLUTION $\sqrt{9} = 3$.

   NO, NOT A SOLUTION

   <u>MORAL</u>: SQUARING BOTH SIDES CAN CAUSE AN EXTRANEOUS SOLUTION, SO CHECK YOUR ANSWER IF YOU SQUARE BOTH SIDES. (RECALL EARLIER IT WAS STUDIED: EXTRANEOUS ROOTS CAN ALSO BE INTRODUCED BY MULT/DIVIDING BY AN EXPRESSION INVOLVING VARIABLE THAT COULD BE <u>ZERO</u>. SOLUTIONS NEED TO BE CHECKED IF EITHER OF THESE SITUATIONS OCCUR.)

2. FIND ALL REAL SOLUTIONS TO

JUGULAR #7

$$x + \sqrt{x+3} = 9$$

a. ISOLATE THE RADICAL

$$\sqrt{x+3} = 9 - x$$

b. SQUARE BOTH SIDES (REMEMBER TO CHECK YOUR ANSWER)

$$x+3 = (9-x)^2 = 81 - 18x + x^2$$

$$0 = x^2 - 19x + 78$$

$$0 = (x-6)(x-13)$$

$x-6=0$ <u>OR</u> $x-13=0$

$x=6$ OR $x=13$

CHECK $x=6$ (IN ORIGINAL EQUATION)

$$6+\sqrt{6+3} = 6+\sqrt{9} = 6+3 = 9 \quad \text{OK}$$

CHECK $x=13$ (IN ORIGINAL EQUATION)

$$13+\sqrt{13+3} = 13+\sqrt{16} = 13+4 = 17 \neq 9$$

NOT A SOLUTION (EXTRANEOUS)

SOLUTION SET $= \{6\}$

JUGULAR PROBLEM #7

7-123

3. FIND ALL REAL SOLUTIONS TO

$$\sqrt{8+x}+\sqrt{1+x}-\sqrt{41+x}=0$$

a. ISOLATE A RADICAL

$$\sqrt{8+x}+\sqrt{1+x}=\sqrt{41+x}$$

b. SQUARE BOTH SIDES (REMEMBER TO CHECK YOUR ANSWER)

$$\left(\sqrt{8+x}+\sqrt{1+x}\right)^2=\left(\sqrt{41+x}\right)^2$$

$$8+x+2\sqrt{8+x}\sqrt{1+x}+1+x=41+x$$

c. THIS IS AN EQUATION INVOLVING A RADICAL SO, ISOLATE A RADICAL

$$2\sqrt{8+x}\sqrt{1+x}=32-x$$

d. SQUARE BOTH SIDES (CHECK)

$$4(8+x)(1+x)=(32-x)^2$$

$$4(8+9x+x^2)=1024-64x+x^2$$

$$32+36x+4x^2=1024-64x+x^2$$

$$3x^2+100x-992=0$$

$$(x-8)(3x+124)=0$$

$x - 8 = 0$ <u>OR</u> $3x + 124 = 0$

$x = 8$ OR $3x = -124$

$x = 8$ OR $x = -\frac{124}{3}$

CHECK $x = 8$ (IN ORIGINAL EQUATION)

$\sqrt{8+8} + \sqrt{1+8} - \sqrt{41+8} = \sqrt{16} + \sqrt{9} - \sqrt{49}$

$= 4 + 3 - 7 = 0$ YES

CHECK $x = -\frac{124}{3}$ (IN ORIGINAL EQUATION)

$\sqrt{8 - \frac{124}{3}} + \sqrt{1 - \frac{124}{3}} - \sqrt{41 - \frac{124}{3}} =$

BAD. NEGATIVE UNDER RADICAL

SOLUTION SET $\{8\}$

NOTE: JUGULAR #7 TYPE PROBLEMS ASK FOR <u>REAL</u> SOLUTIONS. YOU NEED TO CHECK FOR EXTRANEOUS SOLUTIONS

P. HOMEWORK (OIS)

1. SOLVE: FIND ALL REAL AND COMPLEX SOLUTIONS

   a. $2x^3+x^2-8x-4=0$

   b. $x^5+4x^3-8x^2-32=0$

   c. $2x^3+18x-3x^2-27=0$

   d. $\frac{5}{4}(2x+1)^{7/3}(x+2)^{1/4}+\frac{14}{3}(x+2)^{5/4}(2x+1)^{4/3}=0$

   e. $\frac{1}{5}(3x+2)^{4/3}(x+1)^{-4/5}+4(x+1)^{1/5}(3x+2)^{1/3}=0$

   f. $\frac{1}{5}(2x-3)^{1/3}(3x+2)^{-4/5}+\frac{2}{3}(3x+2)^{1/5}(2x-3)^{-2/3}=0$

2. FIND ALL REAL SOLUTIONS FOR

   a. $x+\sqrt{x-1}=7$

   b. $2x-\sqrt{x-5}=11$

   c. $\sqrt{x+3}+\sqrt{x+8}=5$

   d. $\sqrt{x+2}-\sqrt{x+9}+\sqrt{x-6}=0$

   e. $\sqrt{x+3}+\sqrt{x+8}=1$

Q EQUATIONS IN QUADRATIC FORM

1. $x^4 - x^2 - 12 = 0$ FIND ALL REAL AND COMPLEX SOLUTIONS

$(x^2)^2 - x^2 - 12 = 0$ LET $w = x^2$

$w^2 - w - 12 = 0$

$(w-4)(w+3) = 0$

$w - 4 = 0$ OR $w + 3 = 0$

$w = 4$ OR $w = -3$

TO PROBLEM STARTED IN x
SO SUBSTITUTE BACK TO GET
x ANSWERS

$x^2 = 4$ OR $x^2 = -3$

$x = \pm 2$ OR $x = \pm\sqrt{-3} = \pm i\sqrt{3}$

SOLUTION SET $\{-2, +2, -i\sqrt{3}, +i\sqrt{3}\}$

2. FIND ALL REAL SOLUTIONS

$(x^3-5)^2 + 3(x^3-5) - 18 = 0$

LET $w = x^3 - 5$

$w^2 + 3w - 18 = 0$

$(w-3)(w+6) = 0$

$w-3=0$ OR $w+6=0$

$w=3$ OR $w=-6$

SUBSTITUTE BACK FOR x

$x^3-5=3$ OR $x^3-5=-6$

$x^3=8$ OR $x^3=-1$

$x=2$ OR $x=-1$

SOLUTION SET $\{-1, 2\}$

3. $x^{4/3}-7x^{2/3}-8=0$ FIND ALL REAL AND COMPLEX SOLUTIONS

$(x^{2/3})^2-7x^{2/3}-8=0$ LET $w=x^{2/3}$

$w^2-7w-8=0$

$(w-8)(w+1)=0$

$w-8=0$ OR $w+1=0$

$w=8$ OR $w=-1$ SUBSTITUTE BACK

$x^{2/3}=8$ OR $x^{2/3}=-1$

$x^2=8^3=512$ OR $x^2=-1$ CUBE BOTH SIDES

$x=\pm\sqrt{512}=\pm 16\sqrt{2}$ OR $x=\pm\sqrt{-1}=\pm i$

SOLUTION SET $\{-16\sqrt{2}, +16\sqrt{2}, -i, +i\}$

R. SOLVING $x^{2/3}=8$ (NOT $8^{2/3}=x$)

1. LAST PROBLEM: CUBE BOTH SIDES, THEN TOOK THE SQUARE ROOT.

   $x^{2/3}=8$. $x^2=512$. $x=\pm\sqrt{512}=\pm 16\sqrt{2}$

2. $x^{3/2}=8$ ← FIND ALL REAL SOLUTIONS

   1st SQUARE, THEN CUBE ROOT

   $x^3=64$. $x=4$. SINCE SQUARED BOTH SIDES, CHECK YOUR ANSWER

   $4^{3/2}=(4^{1/2})^3=(\sqrt{4})^3=2^3=8$ YES

   SOLUTION SET $\{4\}$

3. $(x^2+12)^{3/4}=x^3$

   $(x^2+12)^3=x^{12}$ RAISE BOTH SIDES TO 4TH POWER. CHECK ANSWERS

   $x^2+12=x^4$ TAKE CUBE ROOT OF BOTH SIDES

   $0=x^4-x^2-12$ LET $w=x^2$

   $0=w^2-w-12$

   $0=(w-4)(w+3)$

   $w-4=0$ OR $w+3=0$

   $w=4$ OR $w=-3$ SUBSTITUTE BACK TO x

   $x^2=4$ OR $x^2=-3$

$x = \pm 2$ OR $x = \pm\sqrt{-3} = \pm i\sqrt{3}$

CHECK $x = +2$ IN ORIGINAL EQUATION

$$(2^2+12)^{3/4} = (4+12)^{3/4} \stackrel{?}{=} 16^{3/4} = \left(16^{1/4}\right)^3 = 2^3 \quad \text{OK}$$

CHECK $x = -2$

$$((-2)^2+12)^{3/4} = (4+12)^{3/4} \stackrel{?}{=} 16^{3/4} = \left(16^{1/4}\right)^3 = 2^3 \neq (-2)^3 \quad \text{NO}$$

CHECK $x = +i\sqrt{3}$

$$\left((i\sqrt{3})^2+12\right)^{3/4} \stackrel{?}{=} \left(i^2 3 + 12\right)^{3/4} \stackrel{?}{=} (-3+12)^{3/4} = 9^{3/4}$$

$$(i\sqrt{3})^3 = i^3 3\sqrt{3} = +i^2 i\, 3\sqrt{3} = -i\,3\sqrt{3} \quad \text{NO}$$

CHECK $x = -i\sqrt{3}$

$$\left((-i\sqrt{3})^2+12\right)^{3/4} = \left(i^2 3 + 12\right)^{3/4} = (-3+12)^{3/4} \stackrel{?}{=} 9^{3/4}$$

$$(-i\sqrt{3})^3 = -i^3 3\sqrt{3} = -i^2 \cdot i\, 3\sqrt{3} = i\,3\sqrt{3} \quad \text{NO}$$

SOLUTION SET $\{2\}$

Note: The original equation is

$$\left(x^2+12\right)^{3/4} = x^3$$

S. HOMEWORK (OIS)

SOLVE: FIND ALL REAL SOLUTIONS

1. $x^6 - 9x^3 + 8 = 0$ 2. $x^8 - 17x^4 + 16 = 0$

3. $x^{2/3} - 5x^{1/3} + 4 = 0$

4. $x^{6/5} - 7x^{3/5} - 8 = 0$

5. $(x^2-6)^2 - 13(x^2-6) + 30 = 0$

6. $(x-3) - \sqrt{x-3} - 12 = 0$

7. $x^{3/4} = 2$ 8. $x^{4/5} = 16$

9. $(x^2-4)^{2/3} = 4$ 10. $(x^3-5)^{3/5} = -8$

11. $(x^2+6)^{2/3} = x^{8/3}$

12. $(7x^3+8)^{4/3} = x^8$

13. FOR PROBLEMS 1 AND 11 FIND ALL REAL AND COMPLEX SOLUTIONS

[CHAPTER 8]

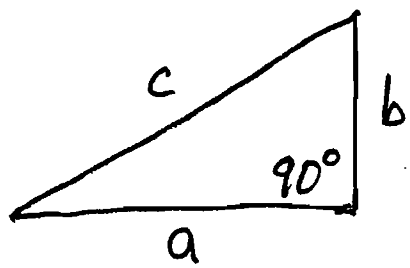
WORD PROBLEMS

A. THIS CHAPTER WILL SHOW APPLICATIONS OF WHAT YOU HAVE LEARNED.

B. THIS WILL BE HARDER. IT IS NOT ABNORMAL TO HAVE A HARD TIME WITH HARD THINGS.

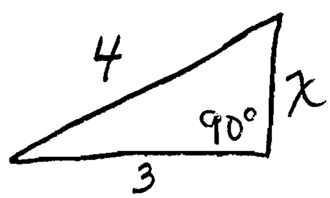
C. WE FIRST STUDY FORMULAS THAT WILL BE HELPFUL IN WORD PROBLEMS

D. PYTHAGOREAN THEOREM



$$\boxed{a^2+b^2=c^2}$$

HYPOTENUSE LENGTH = c
LENGTH OF LEGS a, b

1. 

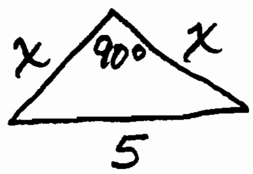
FIND x

$$3^2+x^2=4^2$$
$$9+x^2=16$$
$$x^2=7$$
$$x=\pm\sqrt{7}$$

NO NEGATIVE SIDE LENGTHS, SO

$$x=+\sqrt{7}$$

2.



FIND x

$$x^2 + x^2 = 5^2$$

$$2x^2 = 25$$

$$x^2 = \frac{25}{2}$$

$$x = \pm \frac{5}{\sqrt{2}} = \pm \frac{5\sqrt{2}}{\sqrt{2}\sqrt{2}} = \pm \frac{5\sqrt{2}}{2}$$

NO NEGATIVE SIDE LENGTH, SO

$$x = +\frac{5\sqrt{2}}{2}$$

E. DISTANCE = RATE • TIME $\boxed{D=RT}$

YOU TRAVEL 150 MILES IN $2\frac{1}{2}$ HOURS.
WHAT IS YOUR RATE

$$D = RT$$

$$150 = R\left(2\tfrac{1}{2}\right) = R\left(\tfrac{5}{2}\right)$$

$$\frac{2}{5}(150) = R = 60$$

F. AREA FORMULAS

CIRCLE πr^2

RECTANGLE → bh ← PARALLELOGRAM

TRIANGLE $\frac{1}{2}bh$

G. STEPS TO SOLVE WORD PROBLEMS

1. READ THE PROBLEM TO UNDERSTAND IT
2. DRAW A PICTURE
3. NAME YOUR VARIABLES
4. GET AN EQUATION
5. SOLVE THE EQUATION
6. CHECK YOUR ANSWER
7. ANSWER THE QUESTION

H. WHINING, STEADY, AND WISDOM

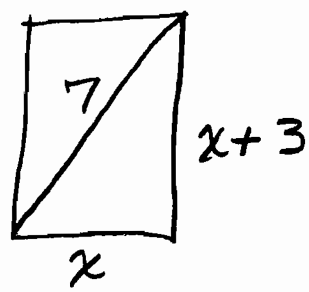
1. NO WHINING BY SAYING, "IF I COULD GET THE EQUATION, I COULD SOLVE THE PROBLEM". GETTING THE EQUATION IS THE MAJOR DIFFICULTY OF THE PROBLEM; THIS IS WHAT YOU ARE BEING LOVINGLY FORCED TO ENCOUNTER.
2. WISDOM : WHEN WORKING THESE PROBLEMS BE SURE TO PUT IN AS MANY OF THESE STEPS AS APPLY. THIS IS A DISCIPLINE TO HELP WORK THE PROBLEM.
3. STEADY : YOU WILL BE SO EXCITED WHEN YOU SOLVE THE EQUATION YOU MAY FORGET TO ANSWER THE QUESTION THE PROBLEM IS ASKING.

I WORD PROBLEMS

A RECTANGULAR DOOR HAS ITS HEIGHT 3 FEET MORE THAN ITS WIDTH. THE DIAGONAL LENGTH OF THE DOOR IS 7 FEET. WHAT IS THE AREA OF THE DOOR?

WISDOM

1. READ PROBLEM (DONE)
2. PICTURE



3. NAME VARIABLE: x = WIDTH
4. EQUATION: $x^2+(x+3)^2=7^2$
5. SOLVE EQUATION:

$$x^2+x^2+6x+9=49$$

$$2x^2+6x-40=0$$

$$x^2+3x-20=0$$

$$x=\frac{-3\pm\sqrt{3^2-4(1)(-20)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9+80}}{2} = \frac{-3 \pm \sqrt{89}}{2}$$

6. CHECK ANSWER:

$\frac{-3-\sqrt{89}}{2}$ <u>NOT</u> AN ANSWER. NO NEGATIVE WIDTH ALLOWED.

CHECK $x = \frac{-3+\sqrt{89}}{2}$ WIDTH

HEIGHT $x+3 = \frac{-3+\sqrt{89}}{2} + 3 =$

$$\frac{-3+\sqrt{89}+6}{2} = \frac{3+\sqrt{89}}{2}.$$

$$\left(\frac{-3+\sqrt{89}}{2}\right)^2 + \left(\frac{3+\sqrt{89}}{2}\right)^2 =$$

$$\frac{9-6\sqrt{89}+89}{4} + \frac{9+6\sqrt{89}+89}{4} = \frac{196}{4} = 49 = 7^2$$

OK

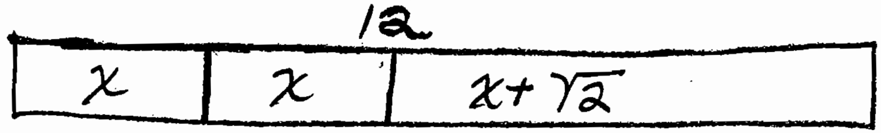
7. ANSWER QUESTION: AREA $= \left(\frac{-3+\sqrt{89}}{2}\right)\left(\frac{3+\sqrt{89}}{2}\right)$

$$= \frac{-9-3\sqrt{89}+3\sqrt{89}+89}{4} = \frac{80}{4} = \boxed{20 \text{ SQ. FT}}$$

J. A LOVER OF SQUARE ROOTS WANTS TO CUT A 12 FOOT BOARD INTO 3 PIECES, 2 SHORTER PIECES OF EQUAL LENGTH AND 1 LONGER PIECE. THE LONGER PIECE IS TO BE $\sqrt{2}$ FEET LONGER THAN A SHORTER PIECE. FIND THE LENGTHS OF THE PIECES.

WISDOM

1. READ PROBLEM: (DONE)
2. PICTURE:



3. NAME VARIABLE: x SHORTER PIECE LENGTH
4. EQUATION: $x + x + (x+\sqrt{2}) = 12$
5. SOLVE EQUATION: $3x + \sqrt{2} = 12$

$$3x = 12 - \sqrt{2}$$

$$x = \frac{12-\sqrt{2}}{3}$$

6. CHECK ANSWER: $x + x + (x+\sqrt{2}) =$

$$\frac{12-\sqrt{2}}{3} + \frac{12-\sqrt{2}}{3} + \left(\frac{12-\sqrt{2}}{3} + \sqrt{2}\right) =$$

$$\frac{12-\sqrt{2} + 12-\sqrt{2} + 12-\sqrt{2} + 3\sqrt{2}}{3} = \frac{36}{3} = 12 \text{ OK}$$

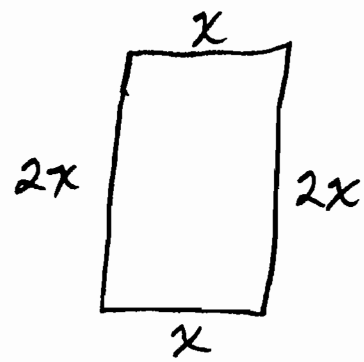
7. ANSWER: SHORTER LENGTH $= \frac{12-\sqrt{2}}{3}$

LONGER LENGTH $= \frac{12-\sqrt{2}}{3} + \sqrt{2} = \frac{12-\sqrt{2}+3\sqrt{2}}{3} = \frac{12+2\sqrt{2}}{3}$

K. THE AREA OF A RECTANGLE HAS A NUMERICAL VALUE THAT IS 3 MORE THAN THE PERIMETER. THE HEIGHT IS TWICE THE WIDTH. WHAT IS THE HEIGHT OF THE RECTANGLE?

WISDOM

1. READ PROBLEM (DONE)
2. PICTURE:



3. NAME VARIABLE:
   x WIDTH
4. EQUATION: OOZE FROM ENGLISH TO MATH

   THE HEIGHT IS TWICE THE WIDTH

   $2x$ HEIGHT

   AREA 3 MORE THAN PERIMETER

   AREA = PERIMETER + 3

$$x(2x) = x + 2x + x + 2x + 3$$

$$2x^2 = 6x + 3$$

5. SOLVE EQUATION:

$$2x^2 - 6x - 3 = 0$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{36+24}}{4} = \frac{6 \pm \sqrt{60}}{4} = \frac{6 \pm \sqrt{4 \cdot 15}}{4}$$

$$= \frac{6 \pm 2\sqrt{15}}{4} = \frac{2(3 \pm \sqrt{15})}{4} = \frac{3 \pm \sqrt{15}}{2}$$

6. CHECK ANSWER:

CHECK $\frac{3-\sqrt{15}}{2}$. THIS IS NEGATIVE.

CANNOT BE A WIDTH.

CHECK $\frac{3+\sqrt{15}}{2}$

$$\text{AREA} = x(2x) = \left(\frac{3+\sqrt{15}}{2}\right)2\left(\frac{3+\sqrt{15}}{2}\right) =$$

$$\frac{9+6\sqrt{15}+15}{2} = \frac{24+6\sqrt{15}}{2} = \frac{2(12+3\sqrt{15})}{2}$$

$$= 12+3\sqrt{15}$$

$$\text{PERIMETER} + 3 = x+2x+x+2x+3$$

$$= 6x+3 = 6\left(\frac{3+\sqrt{15}}{2}\right)+3 = 3(3+\sqrt{15})+3$$

$$= 9+3\sqrt{15}+3 = 12+3\sqrt{15} \quad \text{OK}$$

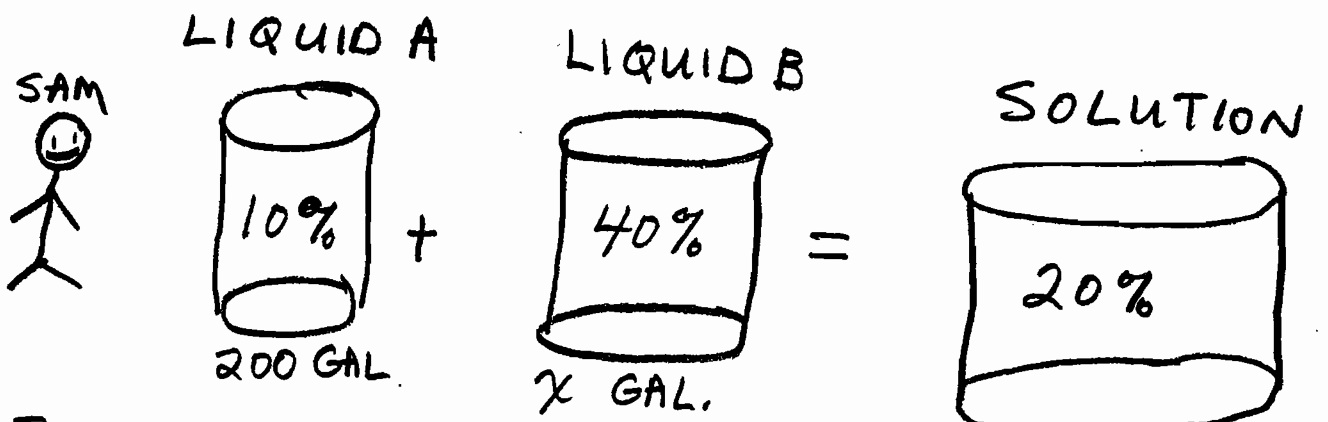
7. ANSWER: $\text{HEIGHT} = 2x = 2\left(\frac{3+\sqrt{15}}{2}\right)$

$$= 3+\sqrt{15}$$

L. MIXTURE PROBLEM. SAM HAS 200 GALLONS OF LIQUID A THAT IS 10% SUGAR. HOW MANY GALLONS OF LIQUID B, WHICH IS 40% SUGAR, SHOULD SAM MIX WITH LIQUID A TO MAKE A SOLUTION THAT IS 20% SUGAR.

WISDOM:

1. READ PROBLEM (DONE)
2. PICTURE



3. NAME VARIABLE:

   x NUMBER OF GALLONS OF 40% SOLUTION TO BE MIXED

4. EQUATION: OOZE FROM ENGLISH TO MATH.

$$\begin{pmatrix}\text{AMOUNT}\\ \text{OF SUGAR}\\ \text{IN LIQUID}\\ \text{A}\end{pmatrix} + \begin{pmatrix}\text{AMOUNT}\\ \text{OF SUGAR}\\ \text{IN LIQUID}\\ \text{B}\end{pmatrix} = \begin{pmatrix}\text{AMOUNT}\\ \text{OF SUGAR}\\ \text{IN FINAL}\\ \text{SOLUTION}\end{pmatrix}$$

$$200(.10) + x(.40) = (200 + x)(.20)$$

5. SOLVE EQUATION

$$20 + .4x = 40 + .2x$$

$$.2x = 20$$

$$2x = 200$$

$$x = \frac{200}{2} = 100$$

6. CHECK ANSWER

LEFT SIDE OF EQUATION: $200(.10) + 100(.40)$
$= 20 + 40 = 60$

RIGHT SIDE OF EQUATION: $(200 + 100)(.20)$
$= 300(.20) = 60$ OK

7. ANSWER: 100 GALLONS

NOTE: A BREAKTHROUGH FOR THIS PROBLEM IS RECOGNIZING THE NEED TO SET UP THE EQUATION IN TERMS OF <u>AMOUNT</u> OF SUGAR.

M. HOMEWORK (OIS) SHOW THE 7 STEP PROCESS

1. A LOT IS TWICE AS LONG AS IT IS WIDE. THE LOT IS TO BE PARTITIONED INTO 2 LOTS WITH THE PARTITION BEING A FENCE DOWN THE MIDDLE PARALLEL TO ITS WIDTH SIDE. THE LOT HAS AN AREA OF 200 SQUARE FEET. HOW MUCH FENCING IS REQUIRED TO ENCLOSE THE ENTIRE LOT INCLUDING THE PARTITION DOWN THE MIDDLE? THE LOT IS A RECTANGLE
2. A RIGHT TRIANGLE WITH 2 SIDES EQUAL HAS AN AREA OF 5 SQUARE FEET. WHAT IS THE PERIMETER OF THE TRIANGLE?
3. A LOT IS ORIGINALLY A SQUARE. A NEW LOT IS FORMED BY EXPANDING EACH SIDE TO WHERE THE NEW LOT HAS EACH SIDE 2 FEET LONGER THAN A SIDE OF THE ORIGINAL LOT. THE NEW LOT IS ALSO A SQUARE. THE NEW LOT HAS AN AREA OF 50 SQUARE FEET MORE THAN THE ORIGINAL LOT. WHAT IS THE AREA OF THE NEW LOT?

4. A WIRE 10 FEET LONG IS TO BE CUT INTO TWO PIECES. THE FIRST PIECE IS TO BE FORMED INTO A CIRCLE. THE SECOND PIECE IS TO BE FORMED INTO A SQUARE. THE SUM OF THE AREAS OF THE SQUARE AND CIRCLE IS 5 SQUARE FEET. WHAT ARE THE LENGTHS OF THE 2 PIECES OF WIRE.

(YOU WILL BE EXCUSED FROM CHECKING YOUR ANSWER ON PROBLEM 4)

5. SUE HAS 50 GALLONS OF LIQUID E THAT IS A 15% SOLUTION OF SUGAR. SUE HAS A LARGE SUPPLY OF A 50% SOLUTION OF SUGAR HOW MANY GALLONS OF THE 50% SOLUTION DOES SHE NEED TO MIX WITH THE 50 GALLONS OF LIQUID E TO MAKE A LIQUID THAT IS A 35% SOLUTION OF SUGAR?

N. WORKING TOGETHER TO GET A JOB DONE.

1. GENERALLY TOLD THE TIME IT TAKES EACH PERSON WORKING ALONE TO DO THE JOB, THEN ASKED THE TIME FOR THEM TO DO IT WORKING TOGETHER

2. THE RATE OF JOB/HOUR

   a. SAM TAKES 5 HOURS TO DO A JOB.
   SAM RATE: $\frac{1}{5}$ JOB/HOUR

   b. SUE TAKES 7 HOURS TO DO A JOB.
   SUE RATE: $\frac{1}{7}$ JOB/HOUR

3. SAM TAKES 5 HOURS TO MOW A YARD. SUE TAKES 7 HOURS TO MOW THE YARD. HOW MANY HOURS DOES IT TAKE FOR THEM TO MOW THE YARD WORKING TOGETHER?

   WISDOM:

   a. READ PROBLEM (DONE)

   b. PICTURE

(DOES NOT APPLY)

c. NAME VARIABLE: x THE TIME IT TAKES TO GET THE JOB DONE WITH SAM AND SUE WORKING TOGETHER.

d. EQUATION:

IN FUTURE PROBLEMS THIS CAN BE LEFT OUT

BUILDUP TO EQUATION

SAM RATE: $\frac{1}{5}$ JOB/HOUR

SAM WORKS 2 HOURS. WHAT FRACTION OF THE JOB DOES SAM GET DONE? $2\left(\frac{1}{5}\right)$

SAM WORKS 3 HOURS. WHAT FRACTION OF THE JOB DOES SAM GET DONE?

$3\left(\frac{1}{5}\right)$

SAM WORKS x HOURS. WHAT FRACTION OF THE JOB DOES SAM GET DONE?

$x\left(\frac{1}{5}\right)$

SUE RATE: $\frac{1}{7}$ JOB/HOUR

SUE WORKS 2 HOURS. WHAT FRACTION OF THE JOB DOES SUE GET DONE?

$2\left(\frac{1}{7}\right)$

SUE WORKS x HOURS, WHAT FRACTION OF THE JOB DOES SUE GET DONE?

$x\left(\frac{1}{7}\right)$

$$\begin{pmatrix}\text{FRACTION}\\ \text{OF JOB}\\ \text{DONE BY}\\ \text{SAM}\end{pmatrix} + \begin{pmatrix}\text{FRACTION}\\ \text{OF JOB}\\ \text{DONE BY}\\ \text{SUE}\end{pmatrix} = 1 \text{ JOB DONE}$$

$$x\left(\frac{1}{5}\right) + x\left(\frac{1}{7}\right) = 1$$

e. SOLVE EQUATION

$$\frac{x}{5} + \frac{x}{7} = 1$$

$$\frac{7x + 5x}{35} = 1$$

$$\frac{12x}{35} = 1$$

$$x = \frac{35}{12}$$

f. CHECK ANSWER

$$\frac{\frac{35}{12}}{5} + \frac{\frac{35}{12}}{7} = \frac{\left(\frac{35}{12}\right)7 + \left(\frac{35}{12}\right)5}{35} = \frac{\left(\frac{35}{12}\right)(7+5)}{35}$$

$$= \frac{\left(\frac{35}{12}\right)12}{35} = \frac{35}{35} = 1$$

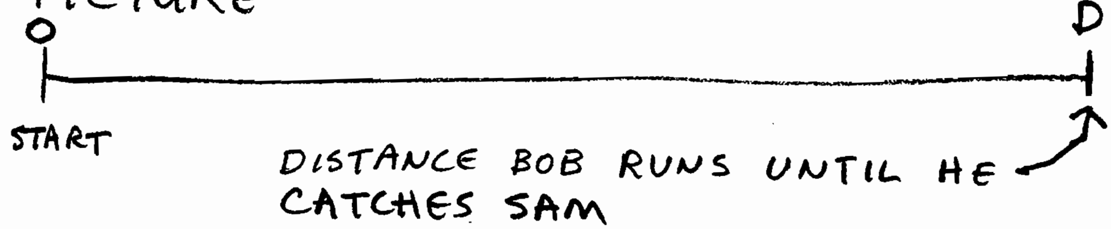
g. ANSWER $\frac{35}{12}$ HOURS (2 HOURS, 55 MINUTES)

θ DISTANCE = (RATE)$\cdot$(TIME)

SAM CAN RUN AT A RATE OF 4 MILES/HR.
BOB CAN RUN AT A RATE OF 7 MILES/HR.
SAM STARTS RUNNING AWAY IN A STRAIGHT LINE 90 MINUTES BEFORE BOB STARTS.
HOW LONG WILL IT TAKE BOB TO CATCH SAM?

WISDOM:

1. READ PROBLEM (DONE)
2. PICTURE



3. VARIABLES:

   D DISTANCE BOB RUNS UNTIL HE CATCHES SAM (ALSO, IT IS THE TOTAL DISTANCE SAM RUNS).

   T TIME BOB RUNS UNTIL HE CATCHES SAM

4. EQUATION:

$$\begin{pmatrix}\text{DISTANCE}\\ \text{SAM RUNS}\end{pmatrix} = \begin{pmatrix}\text{DISTANCE}\\ \text{BOB RUNS}\end{pmatrix}$$

$$(4)\left(\overset{*}{\tfrac{3}{2}}+T\right) = 7T$$

* 90 MINUTES = $\frac{3}{2}$ HOURS. ALSO EACH SIDE OF THE EQUATION IS RATE$\cdot$TIME

5. SOLVE EQUATION:

$$\frac{12}{2} + 4T = 7T$$

$$6 = 3T$$

$$2 = T$$

6. CHECK ANSWER

$$4\left(\frac{3}{2}+2\right) = \frac{12}{2} + 8 = 6 + 8 = 14$$

$$7(2) = 14$$

OK

7. ANSWER QUESTION: 2 HOURS

P. SIMPLE INTEREST: $I = P \cdot R \cdot T$

INTEREST = PRINCIPAL · RATE · TIME

TO COMPOUND INTEREST, ADD THE INTEREST TO THE PRINCIPAL AT THE END OF THE COMPOUNDING PERIOD. THIS WILL BE THE NEW PRINCIPLE.

HOW MUCH MONEY DOES SAM HAVE TO DEPOSIT IN AN ACCOUNT NOW SO THAT 2 YEARS FROM NOW SAM WILL HAVE \$2000 (THE ACCOUNT YIELDS 4% COMPOUNDED ANNUALLY)?

WISDOM:

1. READ PROBLEM: (DONE)
2. PICTURE: (DOES NOT APPLY)
3. VARIABLE: P THE ORIGINAL PRINCIPAL TO BE DEPOSITED.
4. EQUATION:

$$\begin{pmatrix}\text{PRINCIPAL}\\ \text{AFTER ONE}\\ \text{YEAR}\end{pmatrix} + \begin{pmatrix}\text{ONE YEAR'S}\\ \text{INTEREST ON}\\ \text{PRINCIPAL}\\ \text{AFTER ONE}\\ \text{YEAR}\end{pmatrix} = 2000$$

$$\begin{pmatrix}\text{PRINCIPAL}\\ \text{AFTER ONE}\\ \text{YEAR}\end{pmatrix} + \begin{pmatrix}\text{PRINCIPAL}\\ \text{AFTER ONE}\\ \text{YEAR}\end{pmatrix}(.04)(1) = 2000$$

$$\left(P+P(.04)(1)\right) + \left(P+P(.04)(1)\right)(.04) = 2000$$

$$\left(P+P(.04)\right)(1+.04) = 2000$$

$$P(1+.04)(1+.04) = 2000$$

5. SOLVE EQUATION

$$P(1.04)^2 = 2000$$

$$P = \frac{2000}{(1.04)^2}$$

6. CHECK ANSWER: $\frac{2000}{(1.04)^2} + \frac{2000}{(1.04)^2}(.04) + \left(\frac{2000}{(1.04)^2} + \frac{2000}{(1.04)^2}(.04)\right).04$

$$= \frac{1}{(1.04)^2}(2000)\left[(1+.04) + (1+.04).04\right] = \frac{2000}{(1.04)^2}(1+.04)(1.04)$$

$= 2000$ OK

7. ANSWER: $P = \frac{2000}{(1.04)^2}$

Q HOMEWORK (OIS)

1. JANE CAN DO A JOB IN 3 HOURS WORKING ALONE. BOB CAN DO THE JOB IN 4 HOURS WORKING ALONE. HOW LONG DOES IT TAKE THEM TO DO THE JOB WORKING TOGETHER?

2. JANE CAN DO A JOB IN 3 HOURS WORKING ALONE. BOB CAN DO THE JOB IN 4 HOURS WORKING ALONE. BOB STARTS THE JOB AT 12 NOON AND WORKS ALONE UNTIL 1:30 PM, THEN JANE AND BOB WORK TOGETHER UNTIL THE JOB IS COMPLETED. AT WHAT TIME DO THEY COMPLETE THE JOB?

3. AT 1 PM BOB STARTS DRIVING NORTH AT 20 MILES/HOUR. AT 1:30 PM JOE STARTS DRIVING SOUTH AT 40 MILES/HOUR. HOW FAR APART ARE BOB AND JOE AT 5:30 PM?

4. AT 1 PM BOB STARTS DRIVING NORTH AT 20 MILES/HOUR. AT 1:30 PM JOE STARTS DRIVING SOUTH AT 40 MILES/HOUR. WHAT TIME IS IT WHEN THEY ARE 90 MILES APART?

5. AT 1 PM BOB STARTS DRIVING NORTH AT 20 MILES/HOUR. AT 1:30 PM JOE STARTS DRIVING SOUTH AT x MILES/HOUR. AT 4PM THEY ARE 127 MILES APART. HOW FAST IS JOE DRIVING?

6. HOW MUCH MONEY DOES SAM HAVE TO DEPOSIT IN AN ACCOUNT SO THAT 3 YEARS FROM NOW SAM WILL HAVE \$2500? THE ACCOUNT YIELDS 5% COMPOUNDED ANNUALLY.

7. SAM DEPOSITS \$200 IN AN ACCOUNT THAT YIELDS x PER CENT COMPOUNDED ANNUALLY. AT THE END OF 2 YEARS THE ACCUMULATED PRINCIPAL AND INTEREST IS \$300. WHAT IS x?

[CHAPTER 9]

INTERVALS, INEQUALITIES, ABSOLUTE VALUE

A. OPEN INTERVALS

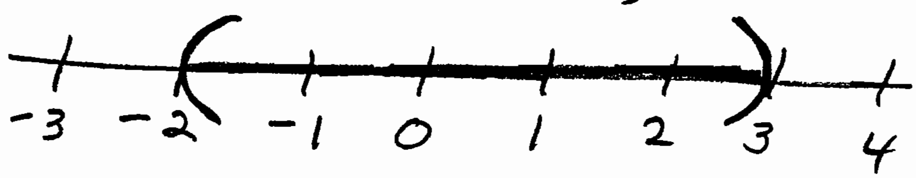
1. THE OPEN INTERVAL a, b, DENOTED (a,b), WHERE a AND b ARE REALS

   a. $(a,b) = \{x \mid a<x<b\}^*$ (DEF.)

   $(-2,3) = \{x \mid -2<x<3\}$

   $\frac{1}{2} \in (-2,3) \qquad 2 \in (-2,3) \qquad 3 \notin (-2,3)$

   b. PICTURE OF $(-2,3)$



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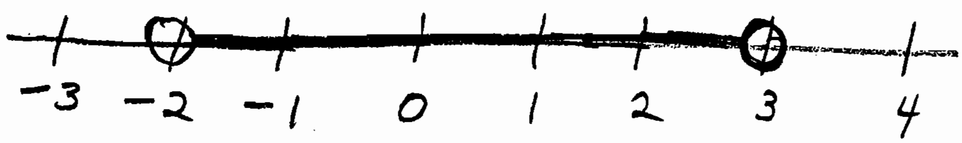


2. THE OPEN INTERVAL FROM a TO INFINITY, DENOTED (a, ∞), WHERE a IS A REAL.

   a. $(a,\infty) = \{x \mid x>a\}$ (DEF.)

   ∞ IS NOT A NUMBER. IT IS AN INTUITIVE CONCEPT OF BEING LARGE WITHOUT BOUND.

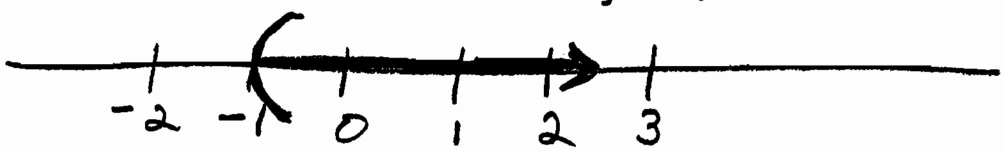
* $a<x<b$ MEANS $a<x$ AND $x<b$

b. $(-1, \infty) = \{x \mid x > -1\}$

$5 \in (-1, \infty)$, $-\frac{1}{2} \in (-1, \infty)$ $-1 \notin (-1, \infty)$

$-2 \notin (-1, \infty)$

c. PICTURE OF $(-1, \infty)$



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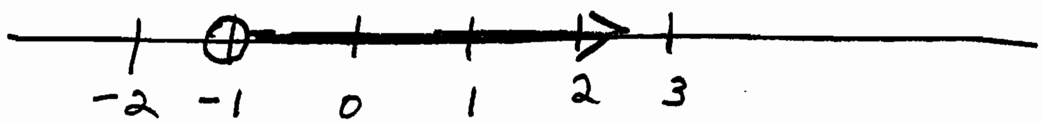


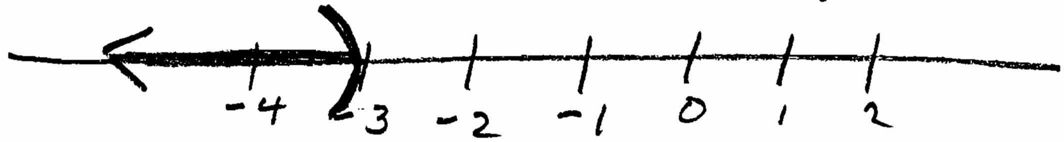
3. THE OPEN INTERVAL FROM $-\infty$ TO a, DENOTED $(-\infty, a)$, WHERE a IS REAL.

a. $(-\infty, a) = \{x \mid x < a\}$ (DEF.)

b. $(-\infty, -3) = \{x \mid x < -3\}$

$-4 \in (-\infty, -3)$, $-3 \notin (-\infty, -3)$ $2 \notin (-\infty, -3)$

c. PICTURE OF $(-\infty, -3)$



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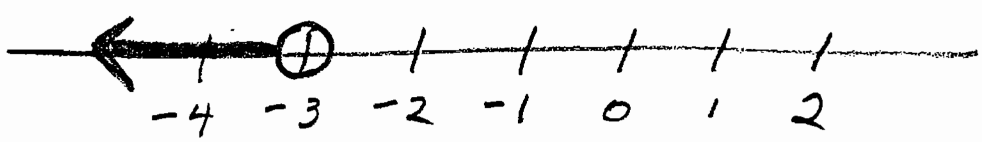


4. THE OPEN INTERVAL $(-\infty,\infty)$ IS JUST ANOTHER WAY OF DENOTING THE SET OF REAL NUMBERS.

B. CLOSED INTERVALS

1. THE CLOSED INTERVAL a, b, DENOTED $[a,b]$, WHERE a AND b ARE REALS

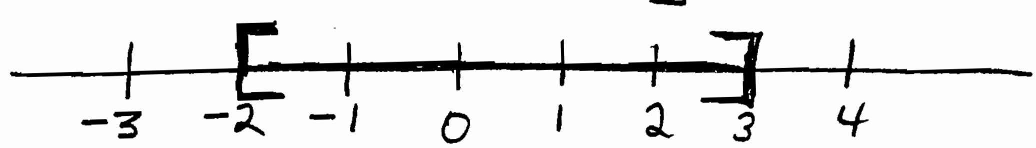
a. $[a,b] = \{x \mid a \le x \le b\}$ (DEF.)

b. $[-2,3] = \{x \mid -2 \le x \le 3\}$

$-\frac{1}{2} \in [-2,3]$. $-2 \in [-2,3]$ $3 \in [-2,3]$

$4 \notin [-2,3]$

c. PICTURE OF $[-2,3]$



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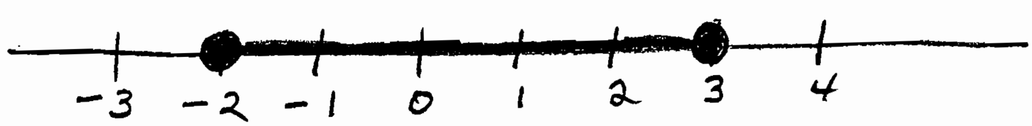


2. THE CLOSED INTERVAL FROM a TO INFINITY, DENOTED $[a,\infty)$, WHERE a IS A REAL.

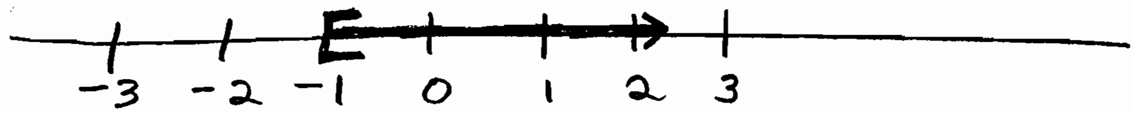
a. $[a,\infty) = \{x \mid x \ge a\}$ (DEF.)

DO NOT WRITE $[a,\infty]$. THIS WOULD IMPLY $\infty \in [a,\infty]$. ∞ IS NOT A NUMBER

b. $[-1,\infty) = \{x \mid x \geq -1\}$

$-1 \in [-1,\infty)$ $\quad 5 \in [-1,\infty)$ $\quad -\frac{3}{2} \notin [-1,\infty)$

c. PICTURE OF $[-1,\infty)$



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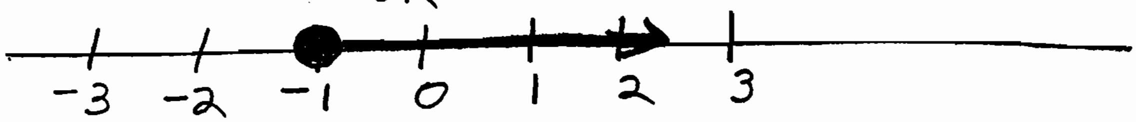


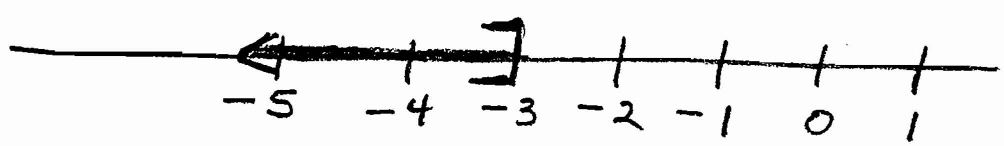
3. THE CLOSED INTERVAL FROM $-\infty$ TO a, DENOTED $(-\infty, a]$, WHERE a IS REAL.

a. $(-\infty, a] = \{x \mid x \leq a\}$ (<u>DEF.</u>)

b. $(-\infty,-3] = \{x \mid x \leq -3\}$

$-4 \in (-\infty,-3]$ $\quad -3 \in (-\infty,-3]$ $\quad -2 \notin (-\infty,-3]$

c. PICTURE OF $(-\infty,-3]$



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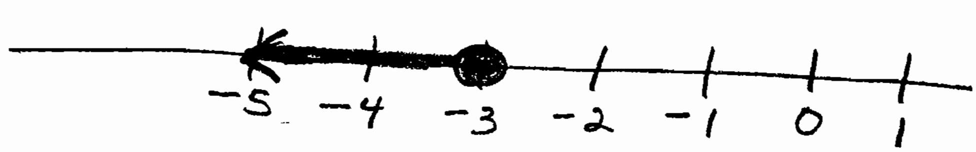


C. HALF OPEN INTERVALS

1. $[a,b)$ IS <u>READ</u> "THE HALF OPEN INTERVAL FROM a TO b, CLOSED AT a"

   OR IT IS READ

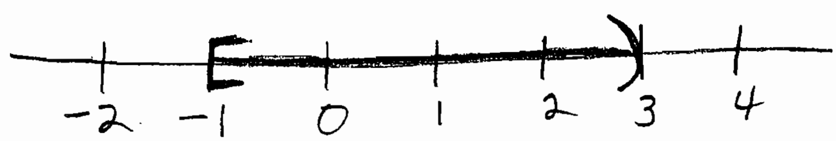
   "THE HALF OPEN INTERVAL FROM a TO b, OPEN AT b."

2. $[a,b) = \{x \mid a \le x < b\}$ (<u>DEF.</u>)

   a. $[-1,3) = \{x \mid -1 \le x < 3\}$

   $-1 \in [-1,3)$ $\quad 0 \in [-1,3)$ $\quad 3 \notin [-1,3)$

   b. PICTURE OF $[-1,3)$



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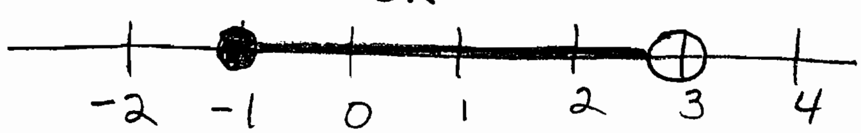


3. $(a, b]$ IS READ "THE HALF OPEN INTERVAL FROM a TO b, OPEN AT a"
OR IT IS READ
"THE HALF OPEN INTERVAL FROM a TO b, CLOSED AT b."

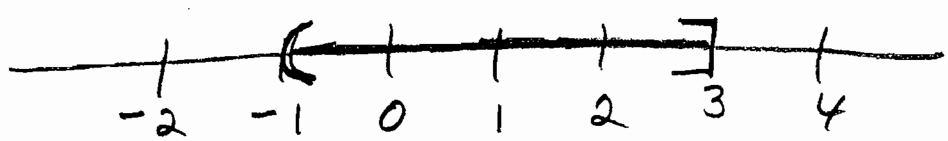
4. $(a, b] = \{x \mid a < x \leq b\}$ (DEF.)

a. $(-1, 3] = \{x \mid -1 < x \leq 3\}$

$-2 \notin (-1,3]$ $\quad -1 \notin (-1,3]$ $\quad \frac{1}{2} \in (-1,3]$

$3 \in (-1,3]$ $\quad 4.23 \notin (-1,3]$

b. PICTURE OF $(-1, 3]$



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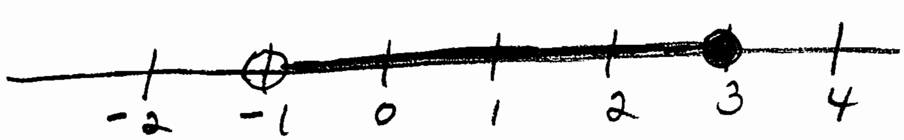


D. SOLVING INEQUALITIES

$\rightarrow$ $\boxed{\text{GIVEN} \quad A < B}$ $\leftarrow$

a. $A+C < B+C$ ADD SAME TO BOTH SIDES

b. $A-C < B-C$ SUBTRACT SAME FROM BOTH SIDES

c. FOR $C>0$
$AC < BC$
$\frac{A}{C} < \frac{B}{C}$
YOU CAN MULTIPLY OR DIVIDE BOTH SIDES BY THE SAME POSITIVE NUMBER.

d. FOR $C<0$
$AC > BC$
$\frac{A}{C} > \frac{B}{C}$
WHEN YOU MULTIPLY OR DIVIDE BOTH SIDES BY THE SAME NEGATIVE NUMBER, TURN THE INEQUALITY AROUND

SIMILAR STATEMENTS COULD BE MADE FOR $\leq$, $>$, $\geq$

e. SOLVE $3x > 6$

$$\frac{3x}{3} > \frac{6}{3}$$

$x > 2$ SOLUTION SET $(2, \infty)$

SOLVE MEANS FIND SOLUTION SET

f. SOLVE $-4x > 12$

$$\frac{-4x}{-4} < \frac{12}{-4}$$ TURN INEQUALITY AROUND

$x < -3$ SOLUTION SET $(-\infty, -3)$

DO NOT DO THE FOLLOWING

$-4x > 12$

$$\frac{-4x}{-4} > \frac{12}{-4}$$ ← INCORRECT

$x < -3$

g. SOLVE $\frac{1}{6}x - 2 \geq \frac{1}{2}x + 4$ AND SKETCH ANSWER ON A NUMBER LINE

$\frac{1}{6}x \geq \frac{1}{2}x + 4 + 2$

$\frac{1}{6}x - \frac{1}{2}x \geq 6$

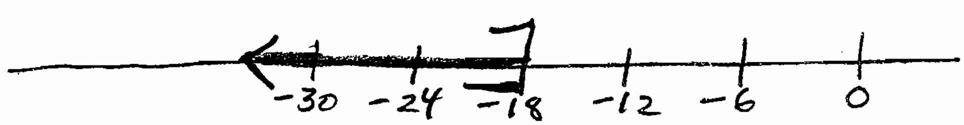
$\frac{1}{6}x - \frac{3}{6}x \geq 6$

$-\frac{2}{6}x \geq 6$

$-\frac{1}{3}x \geq 6$

$x \leq 6(-3)$

$x \leq -18$ SOLUTION SET $\{x \mid x \leq 18\} = (-\infty, -18]$



h. $-3 < 2 - \frac{3}{4}x < 5$ SOLVE AND SKETCH YOUR ANSWER ON A NUMBER LINE

$-3 < 2 - \frac{3}{4}x < 5$ MEANS

$-3 < 2 - \frac{3}{4}x$ AND $2 - \frac{3}{4}x < 5$

$-5 < -\frac{3}{4}x$ AND $-\frac{3}{4}x < 3$

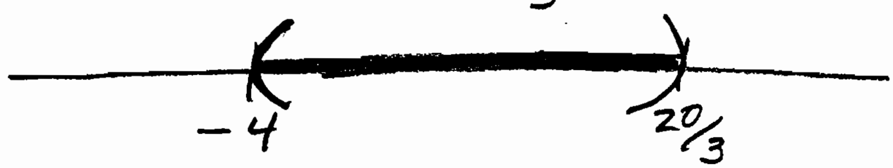
$\left(-\frac{4}{3}\right)(-5) > \left(-\frac{4}{3}\right)\left(-\frac{3}{4}\right)x$ AND $\left(-\frac{4}{3}\right)\left(-\frac{3}{4}x\right) > \left(-\frac{4}{3}\right)3$

$\frac{20}{3} > x$ AND $x > -4$

$-4 < x$ AND $x < \frac{20}{3}$

$-4 < x < \frac{20}{3}$

SOL. SET $\left(-4, \frac{20}{3}\right)$



NOTE: CAN KEEP IT WRITTEN TOGETHER AND SOLVE

$$-3 < 2 - \frac{3}{4}x < 5$$

$$-5 < -\frac{3}{4}x < 3$$

$$\left(-\frac{4}{3}\right)(-5) > x > \left(-\frac{4}{3}\right)3$$

$$\frac{20}{3} > x > -4$$

$$-4 < x < \frac{20}{3}$$

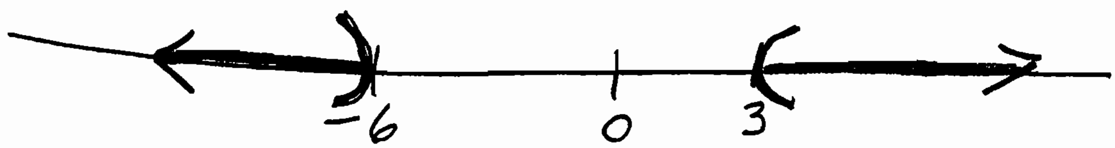
i SOLVE $3+2x>9$ OR $3+2x<-9$

SKETCH ANSWER ON NUMBER LINE.

$2x > 6$ OR $2x < -12$

$x > 3$ OR $x < -6$

SOLUTION SET $(-\infty,-6)\cup(3,\infty)$



IMPORTANT NOTE: WRITTEN TOGETHER MEANS **AND**.

DO **NOT** WRITE

$x > 3$ $\boxed{OR}$ $x<-6$

AS

$3 < x < -6$

↑

THIS MEANS

$3<x$ $\boxed{AND}$ $x<-6$

E. HOMEWORK (OIS) 9-160

1. EACH OF THE FOLLOWING IS GIVEN IN SET BUILDER NOTATION. GIVE INTERVAL FORM

a. $\{x \mid -2 \leq x \leq 3\}$ b. $\{x \mid x > -10\}$

c. $\{x \mid x < -10\}$ d. $\{x \mid 1 < x \leq 5\}$

2. EACH OF THE FOLLOWING IS IN INTERVAL FORM. GIVE THE SET-BUILDER NOTATION

a. $(-7,3)$ b. $(0,5)$ c. $[-4,\infty)$

d. $(-\infty,5)$ e. $(2,3]$ f. $[5,9)$

3. SOLVE EACH INEQUALITY. SKETCH THE ANSWER ON A NUMBER LINE.

a. $-3x \geq 7$ b. $-5 \leq 3x$

c. $5-3x < -2$ d. $\frac{4}{3} - \frac{2}{7}x \geq \frac{1}{5}$

e. $3x-2 \geq 7x-5$ f. $\frac{1}{2}x + \frac{4}{3} \leq \frac{7}{6}x - \frac{1}{2}$

g. $5 - \frac{2}{3}(4x-2) \leq -7(4-3x)$

h. $-7 < 5-2x < 4$ i. $\frac{2}{3} \leq \frac{4}{5} - \frac{2}{3}x < \frac{7}{6}$

j. $7-2x > 5$ OR $7-2x < -5$

k. $\frac{2}{3}x+5 > 3$ OR $\frac{2}{3}x+5 < -3$

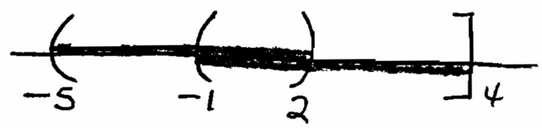
l. $-\frac{2}{3} \leq \frac{2-3x}{-5} \leq \frac{1}{7}$

F INTERSECTION AND UNION OF INTERVALS. (WRITE IN INTERVAL NOTATION AND SKETCH YOUR ANSWER ON A NUMBER LINE).

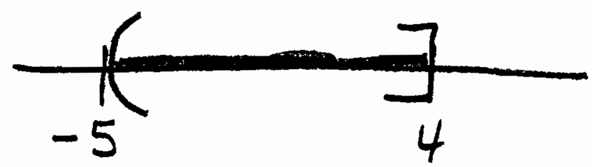
1. $(-5, 2) \cap (-1, 4]$

ANSWER: $(-1, 2)$

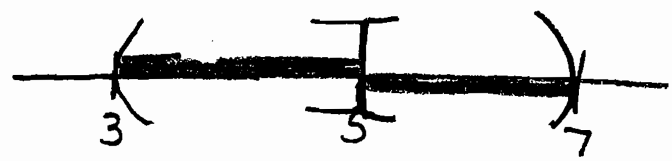
2. $(-5, 2) \cup (-1, 4]$



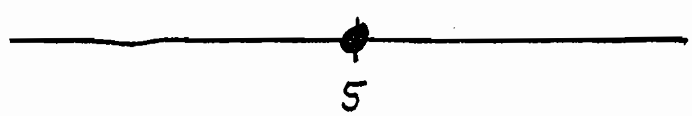
ANSWER $(-5, 4]$



3. $(3, 5] \cap [5, 7)$

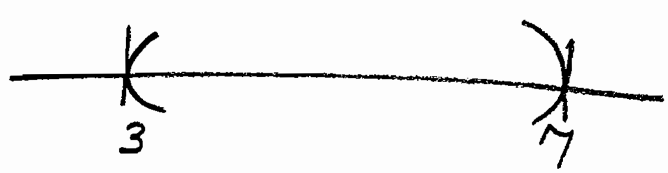


ANSWER $\{5\}$



4. $(3, 5] \cup [5, 7)$

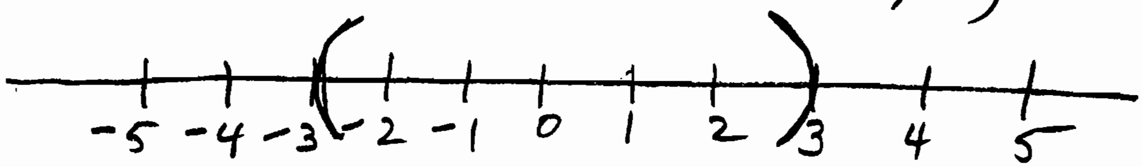
ANSWER $(3, 7)$



G. REMOVE ABSOLUTE VALUES $|x| < K$

1. $|x| < K$ IFF $-K < x < K$
2. $|x| < 3$

$-3 < x < 3$ SOLUTION $(-3, 3)$



3. SOLVE $|2 - 3x| < 7$

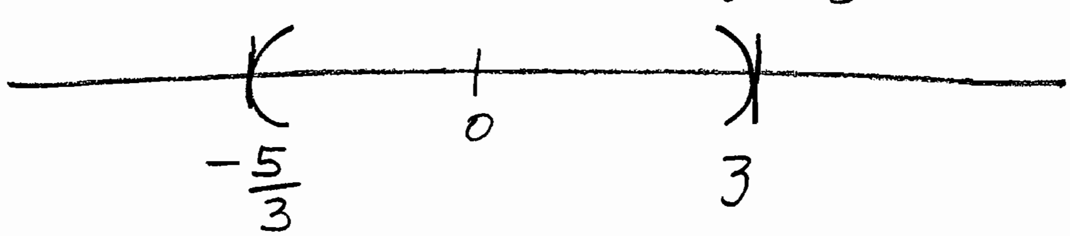
$$-7 < 2 - 3x < 7$$

$$-9 < -3x < 5$$

$$\frac{-9}{-3} > x > \frac{5}{-3}$$

$$-\frac{5}{3} < x < 3$$

SOLUTION $\left(-\frac{5}{3}, 3\right)$



MINI-JUGULAR #8 $\quad |\boxed{}| < K$

3. SOLVE $\quad \left|\frac{3}{4} - \frac{2}{3}x\right| \le \frac{1}{5}$

$$-\frac{1}{5} \le \frac{3}{4} - \frac{2}{3}x \le \frac{1}{5}$$

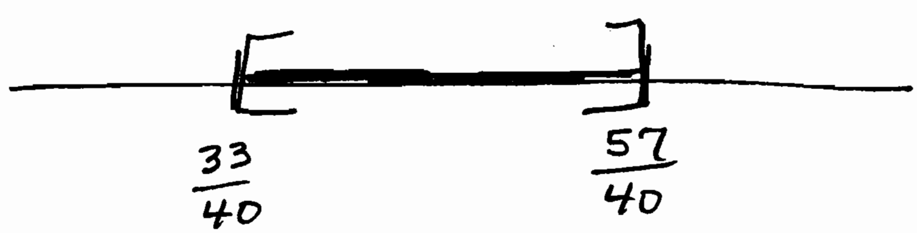
$$\frac{-4-15}{20} = -\frac{1}{5} - \frac{3}{4} \le -\frac{2}{3}x \le \frac{1}{5} - \frac{3}{4} = \frac{4-15}{20}$$

$$\frac{-19}{20} \le -\frac{2}{3}x \le \frac{-11}{20}$$

$$\left(-\frac{3}{2}\right)\left(-\frac{19}{20}\right) \ge \left(-\frac{3}{2}\right)\left(-\frac{2}{3}\right)x \ge \left(-\frac{3}{2}\right)\left(-\frac{11}{20}\right)$$

$$+\frac{57}{40} \ge x \ge \frac{33}{40}$$

$$\frac{33}{40} \le x \le \frac{57}{40}$$



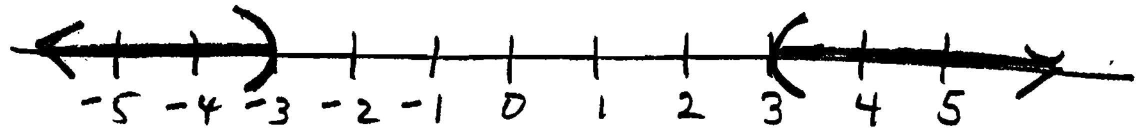
SOLUTION SET $\left[\frac{33}{40}, \frac{57}{40}\right]$

H REMOVE ABSOLUTE VALUES $|x| > K$

1. $|x| > K$ IFF $x > K$ <u>OR</u> $x < -K$

   (<u>OR</u> : DO NOT WRITE INEQUALITY TOGETHER)

2. $|x| > 3$ IFF $x > 3$ <u>OR</u> $x < -3$



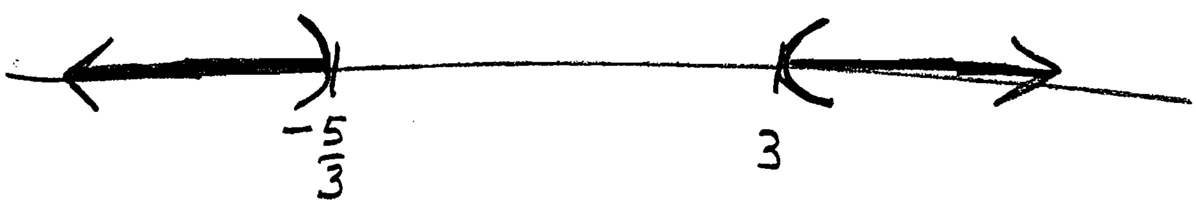
SOLUTION SET $(-\infty, -3) \cup (3, \infty)$

3. SOLVE $|2 - 3x| > 7$

$2 - 3x > 7$ <u>OR</u> $2 - 3x < -7$

$-3x > 5$ OR $-3x < -9$

$x < -\frac{5}{3}$ OR $x > 3$



SOLUTION SET

$(-\infty, -\frac{5}{3}) \cup (3, \infty)$

MINI-JUGULAR #8 $\left|\boxed{}\right| > K$

4. SOLVE $\left|\frac{3}{4} - \frac{2}{3}x\right| \geq \frac{1}{5}$

$\frac{3}{4} - \frac{2}{3}x \geq \frac{1}{5}$ OR $\frac{3}{4} - \frac{2}{3}x \leq -\frac{1}{5}$

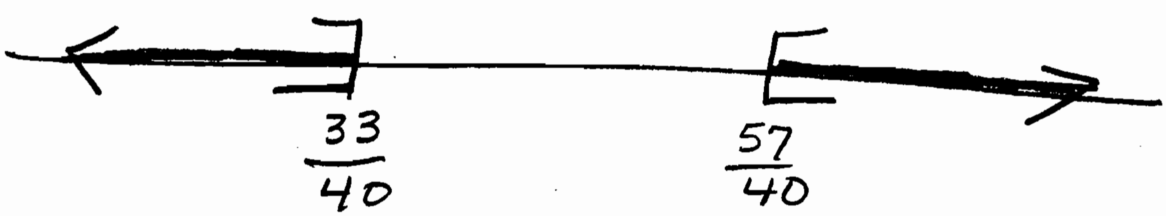
$-\frac{2}{3}x \geq \frac{1}{5} - \frac{3}{4}$ OR $-\frac{2}{3}x \leq -\frac{1}{5} - \frac{3}{4}$

$-\frac{2}{3}x \geq \frac{4-15}{20}$ OR $-\frac{2}{3}x \leq \frac{-4-15}{20}$

$-\frac{2}{3}x \geq \frac{-11}{20}$ OR $-\frac{2}{3}x \leq -\frac{19}{20}$

$\left(-\frac{3}{2}\right)\left(-\frac{2}{3}x\right) \leq \left(-\frac{3}{2}\right)\left(\frac{-11}{20}\right)$ OR $\left(-\frac{3}{2}\right)\left(-\frac{2}{3}\right)x \geq \left(-\frac{3}{2}\right)\left(-\frac{19}{20}\right)$

$x \leq \frac{33}{40}$ OR $x \geq \frac{57}{40}$



SOLUTION SET

$\left(-\infty, \frac{33}{40}\right] \cup \left[\frac{57}{40}, \infty\right)$

I. HOMEWORK (OIS)

1. WRITE YOUR ANSWER IN INTERVAL NOTATION AND SKETCH YOUR ANSWER ON A NUMBER LINE.

a. $(-2,5]\cap(3,6)$ b. $(-2,5]\cup(3,6)$

c. $(1,2)\cup[0,3]$ d. $(-\infty,4]\cap[4,\infty)$

2. REMOVE ABSOLUTE VALUES, SOLVE, SKETCH YOUR ANSWER ON A NUMBER LINE.

a. $|x|>5$ b. $|x|\leq 5$

c. $|-x|<3$ d. $|x-9|<4$

e. $|x+3|<2$ f. $|2x-5|\geq 7$

g. $|2x-5|<7$ h. $\left|\frac{2}{3}x-\frac{3}{5}\right|<\frac{1}{2}$

i $\left|\frac{3}{5}-\frac{2}{3}x\right|\geq\frac{1}{2}$ j. $|5-2x|<\frac{7}{2}$

K. $|4x-3|>0$ l. $|4x-3|<0$

m. $\left|\frac{2}{7}-\frac{3}{4}x\right|\leq\frac{1}{3}$ n. $\left|\frac{3}{7}+\frac{1}{5}x\right|>\frac{2}{3}$

JUGULAR #9

J QUADRATIC AND OTHER RATIONAL INEQUALITIES. (L=LEFT R=RIGHT)

1. SOLVE $x^2 < 8x - 15$ [L-R METHOD]

$\boxed{\text{GET 0 ON ONE SIDE OF INEQUALITY}}$

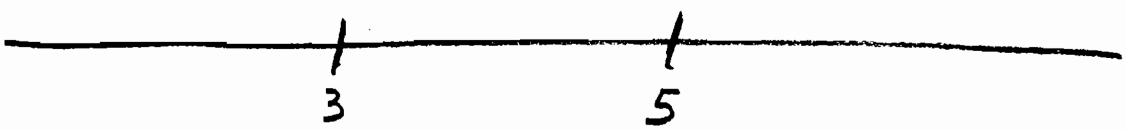
$$x^2 - 8x + 15 < 0$$

$\boxed{\text{IF POSSIBLE, FACTOR INTO } (x-p) \text{ FACTORS, WHERE } p \text{ IS REAL}}$

$$(x-3)(x-5) < 0$$

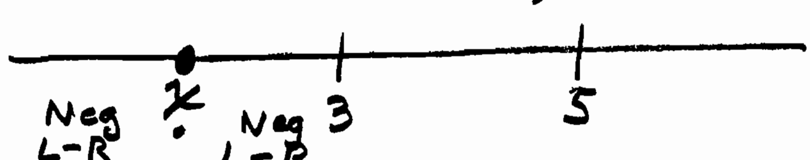
$\boxed{\text{FIND } x\text{-VALUES THAT MAKE THE FACTORS 0 AND MARK ON A NUMBER LINE}}$

$$x = 3, 5$$



PICK EACH POSSIBLE CASE FOR x AND GIVE L-R ANALYSIS (L=LEFT R=RIGHT) IF $L<R$, THEN $L-R<0$ (NEGATIVE) AND $R-L>0$ (POSITIVE)

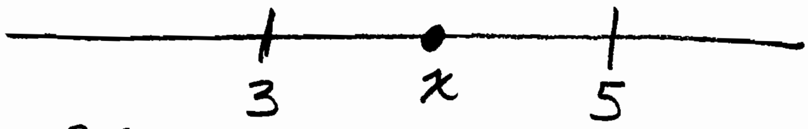
<u>CASE 1</u> $x \in (-\infty, 3)$



$$(x-3)(x-5) > 0 \qquad \text{NO}$$

NOTE: CONSIDER x AND 3. x IS THE <u>LEFT</u> NUMBER. 3 IS THE RIGHT NUMBER. $L-R<0$

CASE 2 $\quad x \in (3,5)$

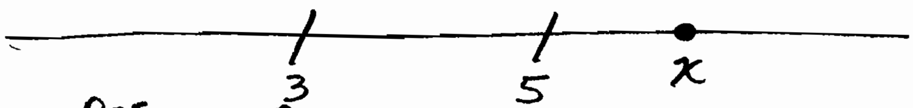


POS • NEG

R-L $\quad$ L-R

$$(x-3)(x-5) < 0$$ YES

CASE 3 $\quad x \in (5,\infty)$



POS • POS

R-L $\quad$ R-L

$$(x-3)(x-5) > 0$$ NO

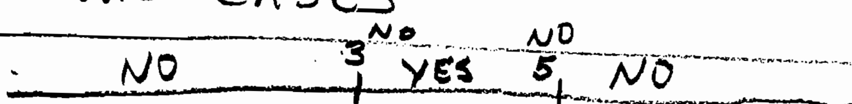
CASE 4 $\quad x = 3$

$$(x-3)(x-5) = (3-3)(3-5) = 0$$ NO

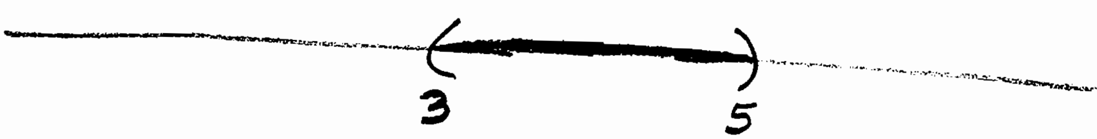
CASE 5 $\quad x = 5$

$$(x-3)(x-5) = (5-3)(5-5) = 0$$ NO

DEDUCE THE ANSWER FROM LOOKING AT THE CASES



SOLUTION SET $\quad (3,5)$



NOTE CASE 2 WAS THE ONLY CASE THAT GAVE $(x-3)(x-5) < 0$

JUGULAR #9

2. SOLVE $2x^3 + 3x^2 \geq 18x + 27$ [L-R METHOD]

$$\boxed{\text{GET 0 ON 1 SIDE OF THE INEQUALITY}}$$

$$2x^3 + 3x^2 - 18x - 27 \geq 0$$

$$\boxed{\text{IF POSSIBLE, FACTOR INTO } x-p \text{ FACTORS, WHERE } p \text{ IS REAL}}$$

$$x^2(2x+3) - 9(2x+3) \geq 0$$

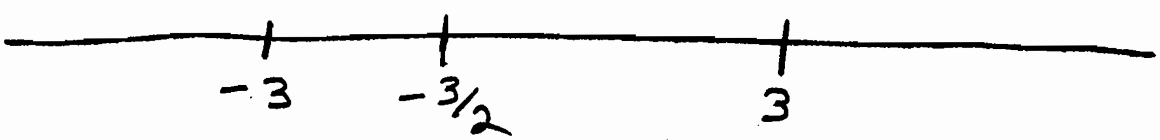
$$(2x+3)(x^2-9) \geq 0$$

$$2\left(x+\tfrac{3}{2}\right)(x+3)(x-3) \geq 0$$

$$2(x-[-3])\left(x-\left[-\tfrac{3}{2}\right]\right)(x-3) \geq 0$$

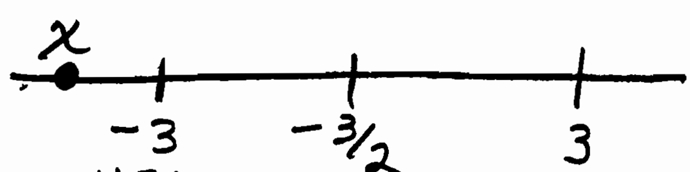
$$\boxed{\text{FIND } x\text{-VALUES THAT MAKE THE FACTORS 0 AND MARK ON A NUMBER LINE.}}$$

$$x = -3, -3/2, 3$$



$$\boxed{\text{PICK EACH POSSIBLE CASE FOR } x \text{ AND GIVE L-R ANALYSIS}}$$

<u>CASE 1</u> $x \in (-\infty, -3)$

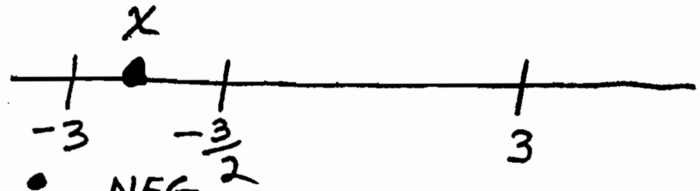


POS • NEG • NEG • NEG

L- R   L-R   L-R

$$2(x-[-3])\left(x-\left[-\tfrac{3}{2}\right]\right)(x-3) < 0$$ <u>NO</u>

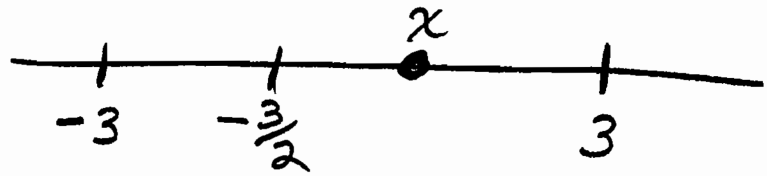
CASE 2 $x \in \left(-3, -\frac{3}{2}\right)$



$$\underset{\uparrow}{\text{POS}} \bullet \text{POS} \bullet \text{NEG} \bullet \text{NEG}$$

$$2\left(\overset{R-L}{x-[-3]}\right)\left(\overset{L-R}{x-\left[-\tfrac{3}{2}\right]}\right)\left(\overset{L-R}{x-3}\right) > 0 \quad \text{YES}$$

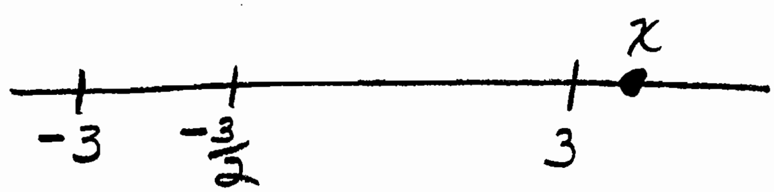
CASE 3 $x \in \left(-\frac{3}{2}, 3\right)$



$$\underset{\uparrow}{\text{POS}} \bullet \text{POS} \bullet \text{POS} \bullet \text{NEG}$$

$$2\left(\overset{R-L}{x-[-3]}\right)\left(\overset{R-L}{x-\left[-\tfrac{3}{2}\right]}\right)\left(\overset{L-R}{x-3}\right) < 0 \quad \underline{\text{NO}}$$

CASE 4 $x \in (3, \infty)$



$$\underset{\uparrow}{\text{POS}} \bullet \text{POS} \bullet \text{POS} \bullet \text{POS}$$

$$2\left(\overset{R-L}{x-[-3]}\right)\left(\overset{R-L}{x-\left[-\tfrac{3}{2}\right]}\right)\left(\overset{R-L}{x-3}\right) > 0 \quad \text{YES}$$

CASE 5 $x = -3$

$$2(x-[-3])\left(x-\left[-\tfrac{3}{2}\right]\right)(x-3) =$$

$$2(-3-[-3])\left(-3-\left[-\tfrac{3}{2}\right]\right)(-3-3) = 0 \geq 0 \quad \text{YES}$$

CASE 6 $x = -\frac{3}{2}$

$$2(x-[-3])\left(x-\left[-\tfrac{3}{2}\right]\right)(x-3) =$$

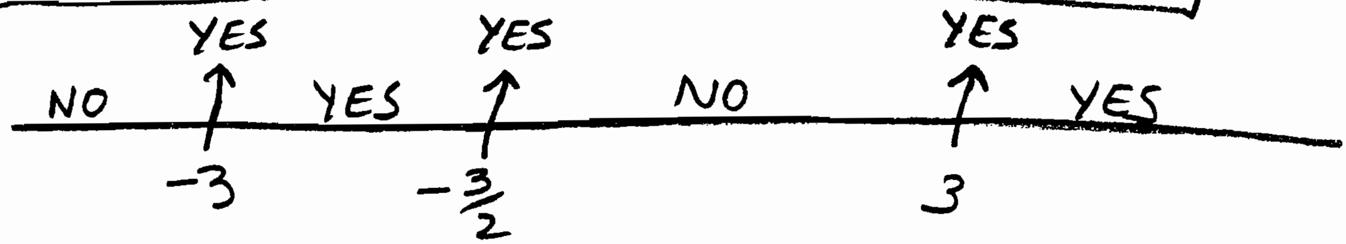
$$2\left(-\tfrac{3}{2}-[-3]\right)\left(-\tfrac{3}{2}-\left[-\tfrac{3}{2}\right]\right)\left(-\tfrac{3}{2}-3\right) = 0 \geq 0 \quad \text{YES}$$

CASE 7 $\quad x = 3$

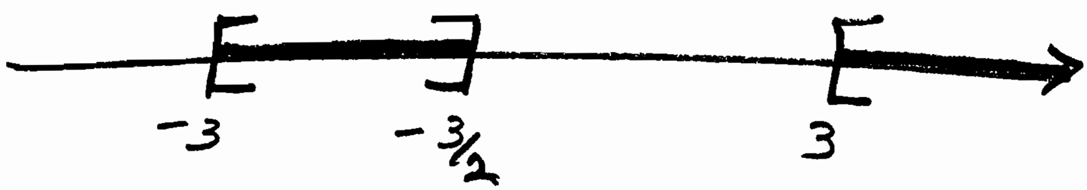
$$2(x-[-3])(x-[-\tfrac{3}{2}])(x-3) =$$

$$2(3-[-3])(3-[-\tfrac{3}{2}])(3-3) = 0 \geq 0 \quad \text{YES}$$

$$\boxed{\text{DEDUCE THE ANSWER FROM LOOKING AT THE CASES}}$$



SOLUTION SET $[-3, -\tfrac{3}{2}] \cup [3, \infty)$



3. $\boxed{\text{JUGULAR PROBLEM \#9}}$ L-R METHOD

FIND ALL VALUES FOR x SO THAT

$\sqrt{\dfrac{3x^2-4x-4}{x+1}}$ IS REAL.

SOLVE $\dfrac{3x^2-4x-4}{x+1} \geq 0$

$$\boxed{\text{NOTE: } 0 \text{ IS ALREADY ON ONE SIDE OF THE INEQUALITY}}$$

IF POSSIBLE, FACTOR INTO $x-p$ FACTORS, WHERE p IS REAL

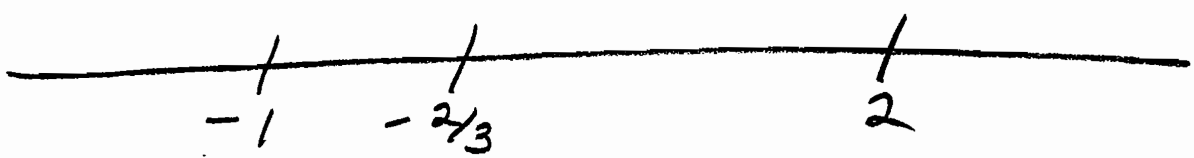
$$\frac{(3x+2)(x-2)}{x+1} \geq 0$$

$$\frac{3(x+\frac{2}{3})(x-2)}{x+1} \geq 0$$

$$\frac{3(x-[-\frac{2}{3}])(x-2)}{x-[-1]} \geq 0$$

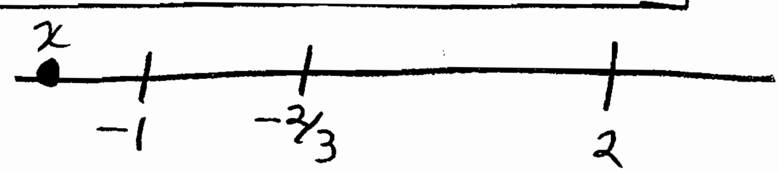
FIND THE x-VALUES THAT MAKE THE FACTORS 0 AND MARK ON A NUMBER LINE

$$x = -1,\ -2/3,\ 2$$



PICK EACH POSSIBLE CASE FOR x AND GIVE L-R ANALYSIS.

CASE 1 $\quad x \in (-\infty, -1)$



POS · NEG · NEG

↑ L−R L−R

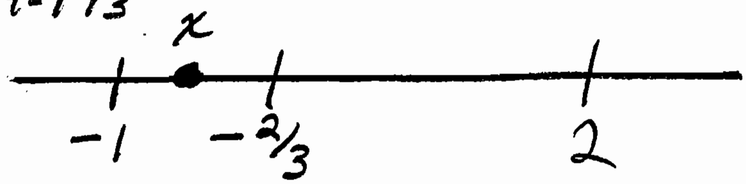
$$\frac{3(x-[-\frac{2}{3}])(x-2)}{x-[-1]} < 0 \qquad NO$$

L−R

NEG

9-173

CASE 2 $x \in \left(-1, -\frac{2}{3}\right)$



POS • NEG • NEG

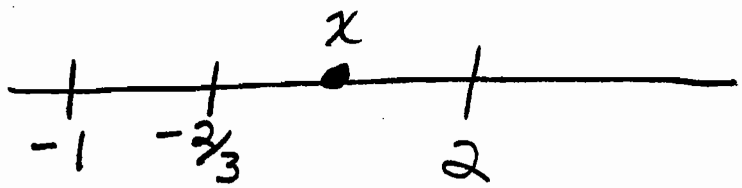
↑ L−R L−R

$$\frac{3\left(x-\left[-\frac{2}{3}\right]\right)(x-2)}{x-[-1]} > 0 \quad \text{YES}$$

R−L

POS

CASE 3 $x \in \left(-\frac{2}{3}, 2\right)$



POS • POS • NEG

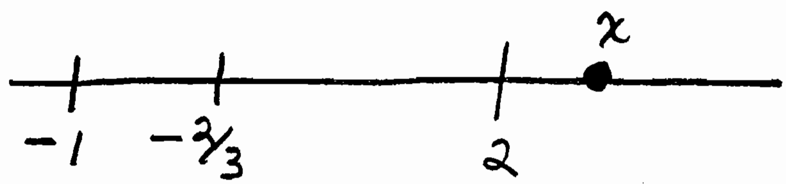
↑ R−L L−R

$$\frac{3\left(x-\left[-\frac{2}{3}\right]\right)(x-2)}{x-[-1]} < 0 \quad \text{NO}$$

R−L

POS

CASE 4 $x \in (2, \infty)$



POS • POS • POS

↑ R−L R−L

$$\frac{3\left(x-\left[-\frac{2}{3}\right]\right)(x-2)}{x-[-1]} > 0 \quad \text{YES}$$

R−L

POS

CASE 5 $x = -1$

$$\frac{3\left(x-\left[-\frac{2}{3}\right]\right)(x-2)}{x-[-1]} = \frac{3\left(-1-\left[-\frac{2}{3}\right]\right)(-1-2)}{-1-[-1]} \leftarrow 0 \quad \text{NO}$$

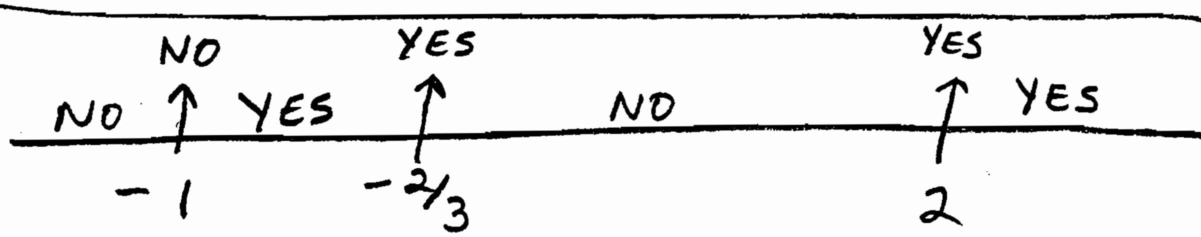
CASE 6 $x = -\frac{2}{3}$

$$\frac{3\left(x-\left[-\frac{2}{3}\right]\right)(x-2)}{x-[-1]} = \frac{3\left(-\frac{2}{3}-\left[-\frac{2}{3}\right]\right)\left(-\frac{2}{3}-2\right)}{-2/3-[-1]} = 0 \geq 0 \text{ YES}$$

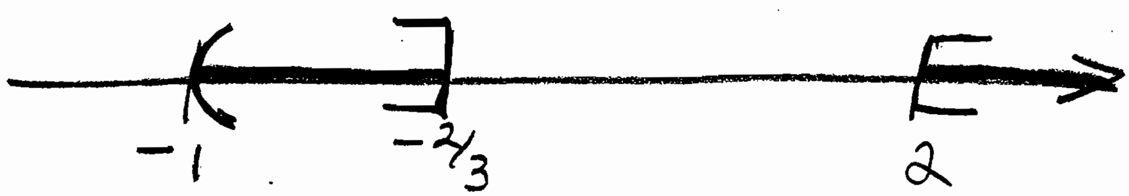
CASE 7 $x = 2$

$$\frac{3\left(x-\left[-\frac{2}{3}\right]\right)(x-2)}{x-[-1]} = \frac{3\left(2-\left[-\frac{2}{3}\right]\right)(2-2)}{2-[-1]} = 0 \geq 0 \text{ YES}$$

DEDUCE THE ANSWER FROM LOOKING AT THE CASES



SOLUTION SET $\left(-1, -\frac{2}{3}\right] \cup [2, \infty)$



4. AN ALTERNATE TEST POINT METHOD FOR SOLVING RATIONAL INEQUALITIES WILL NOW BE SHOWN. WE WILL REWORK A PROBLEM. SOME BOOKS USE THIS METHOD.

SOLVE $2x^3 + 3x^2 \geq 8x + 12$

$\boxed{\text{GET 0 ON ONE SIDE OF THE INEQUALITY}}$

$$2x^3 + 3x^2 - 8x - 12 \geq 0$$

$\boxed{\text{IF POSSIBLE, FACTOR INTO } x-p \text{ FACTORS WHERE } p \text{ IS REAL.}}$

$$x^2(2x+3) - 4(2x+3) \geq 0$$

$$(2x+3)(x^2-4) \geq 0$$

$$2\left(x+\tfrac{3}{2}\right)(x+2)(x-2) \geq 0$$

$$2\left(x-\left[-\tfrac{3}{2}\right]\right)\left(x-[-2]\right)(x-2) \geq 0$$

BEGIN TO MAKE A TABLE: MARK ON A NUMBER LINE THE x-VALUES THAT MAKE A FACTOR 0. MAKE ROWS FOR EACH FACTOR.

2				
$x-2$				
$x-\left[-\frac{3}{2}\right]$				
$x-[-2]$				

$-2 \qquad -\tfrac{3}{2} \qquad 2$

PICK TEST POINTS IN EACH OPEN INTERVAL. FILL IN THE TABLE (THE SPACE UNDER THE NUMBER LINE IS FOR THE SIGN OF THE WHOLE RATIONAL EXPRESSION)

	$x=-3$	$x=-7/4$	$x=0$	$x=3$
2	$+$	$+$	$+$	$+$
$x-2$	$-3-2$ $-$	$-\frac{7}{4}-2$ $-$	$0-2$ $-$	$3-2$ $+$
$x-[-\frac{3}{2}]$	$-3-[\frac{-3}{2}]$ $-$	$\frac{-7}{4}-[\frac{-3}{2}]$ $-$	$0-[\frac{-3}{2}]$ $+$	$3-[\frac{-3}{2}]$ $+$
$x-[-2]$	$-3-[-2]$ $-$	$\frac{-7}{4}-[-2]$ $+$	$0-[-2]$ $+$	$3-[-2]$ $+$
WHOLE EXPRESSION	$-$	$+$	$-$	$+$

WHOLE EXPRESSION: $-$ $\;-2\;$ $+$ $\;-3/2\;$ $-$ $\;2\;$ $+$

CHECK x-VALUES THAT MAKE FACTORS 0

CHECK $x=-2$

$$2\left(x-\left[\tfrac{-3}{2}\right]\right)\left(x-[-2]\right)(x-2)=2\left(-2-\left[\tfrac{-3}{2}\right]\right)\left(-2-[-2]\right)(-2-2)=0\geq 0 \quad \text{YES}$$

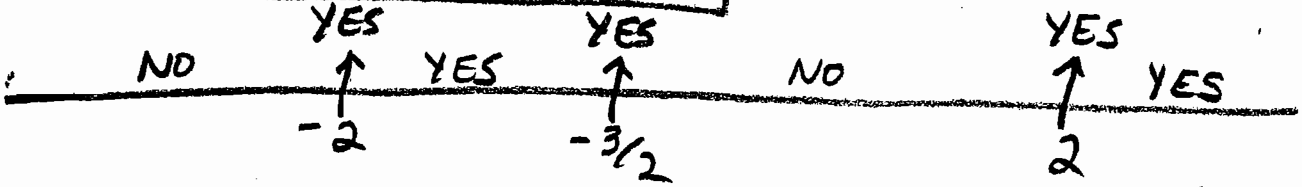
CHECK $x=-\frac{3}{2}$

$$2\left(x-\left[\tfrac{-3}{2}\right]\right)\left(x-[-2]\right)(x-2)=2\left(\tfrac{-3}{2}-\left[\tfrac{-3}{2}\right]\right)\left(\tfrac{-3}{2}-[-2]\right)\left(\tfrac{-3}{2}-2\right)=0\geq 0 \quad \text{YES}$$

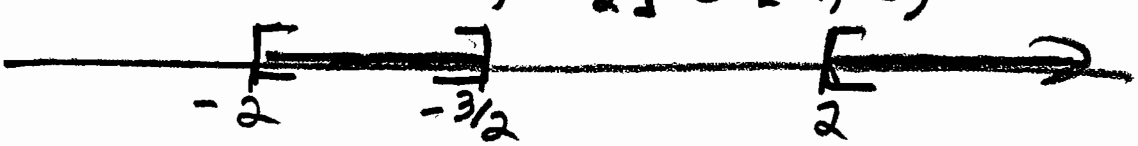
CHECK $x=2$

$$2\left(x-\left[\tfrac{-3}{2}\right]\right)\left(x-[-2]\right)(x-2)=2\left(2-\left[\tfrac{-3}{2}\right]\right)\left(2-[-2]\right)(2-2)=0\geq 0 \quad \text{YES}$$

DEDUCE THE ANSWER



SOLUTION SET $[-2,-3/2]\cup[2,\infty)$



K. HOMEWORK (OIS)

1. SOLVE EACH BY THE L-R METHOD

   a. $x^2 \geq 6x - 5$

   b. $\dfrac{2x+5}{x-3} \leq 0$

   c. $\dfrac{2}{x+2} < \dfrac{3}{2x-3}$

   d. FIND WHERE $\sqrt{\dfrac{x^2-x-6}{2x-5}}$ IS REAL

   e. FIND WHERE $\sqrt{\dfrac{x-3}{-2x-8}}$ IS REAL

2. SOLVE BY THE TEST POINT METHOD.

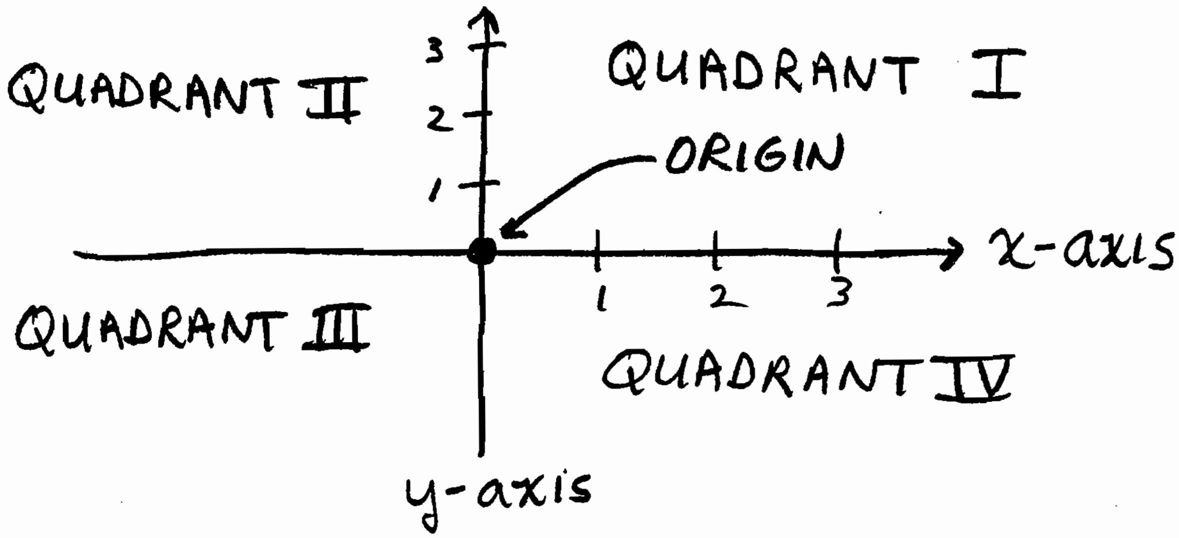
$$-2x^3+2x^2 < -18x+18$$

3. DO PROBLEM 1-b BY THE TEST POINT METHOD.

[CHAPTER 10]

CARTESIAN COORDINATE SYSTEM, GRAPHS, LINES

A. CARTESIAN COORDINATE SYSTEM = RECTANGULAR COORDINATE SYSTEM



B. ORDERED PAIR VS. A SET OF TWO ELEMENTS

1. ORDERED PAIR NOTATION & DEF.

a. $(2, 3)$

1^{st} TERM
x-COORDINATE
ABSCISSA

2^{nd} TERM
y-COORDINATE
ORDINATE

b. ORDER MAKES A DIFFERENCE

$$(2,3) \neq (3,2)$$

c. $(a,b) = (c,d)$ IFF $a=c$ AND $b=d$

2. SET OF 2 ELEMENTS NOTATION

a. $\{2,3\}$

b. ORDER MAKES <u>NO</u> DIFFERENCE

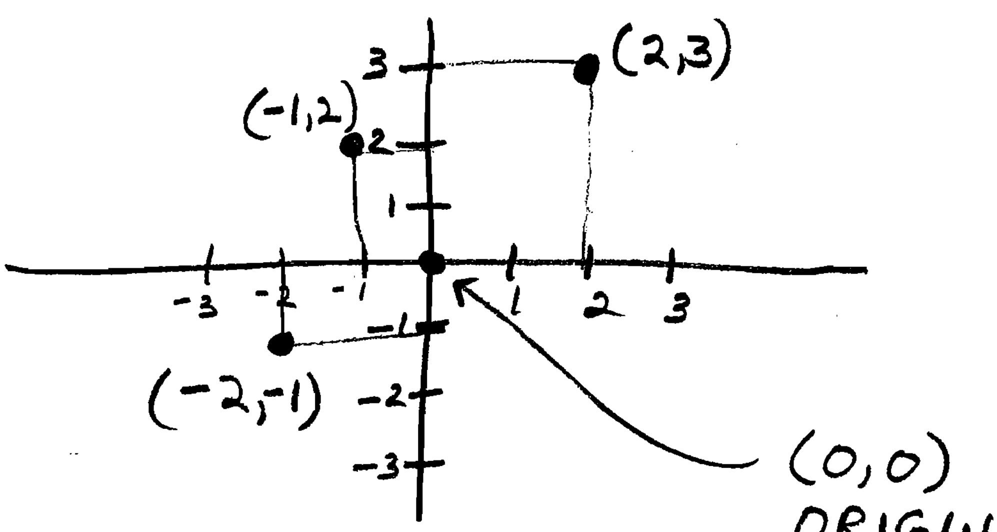
$$\{2,3\} = \{3,2\} = \{2,2,2,3\}$$

3. WHEN USING <u>ORDERED</u> PAIRS BE SURE TO USE PARENTHESES, NOT BRACES, BRACKETS OR NOTHING

$(2,3) \neq \{2,3\}$ $\quad$ $(2,3) \neq [2,3]$

$(2,3) \neq 2,3$

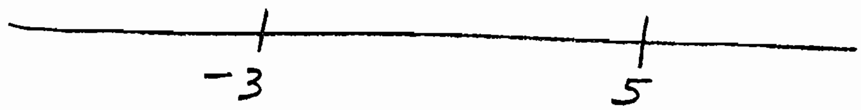
C. POINTS IN THE PLANE ARE ASSOCIATED WITH ORDERED PAIRS, SO THEY CAN BE <u>PLOTTED</u>: PLOT $(2,3)$, $(-1,2)$, $(-2,-1)$



D. DISTANCE BETWEEN POINTS

1. DIRECTED DISTANCE ON A LINE (<u>DEF.</u>)

THE DIRECTED DISTANCE FROM a TO b IS $b - a$



a. THE DIRECTED DISTANCE FROM 5 TO −3

$a = 5 \quad b = -3$

$\underbrace{b - a}_{\text{TO − FROM}} = -3 - 5 = -8$

b. THE DIRECTED DISTANCE FROM −3 TO 5

$b = 5 \quad a = -3 \qquad \underbrace{b - a}_{\text{TO − FROM}} = 5 - (-3) = 8$

2. (DEF.) DISTANCE (UNDIRECTED) ON A LINE

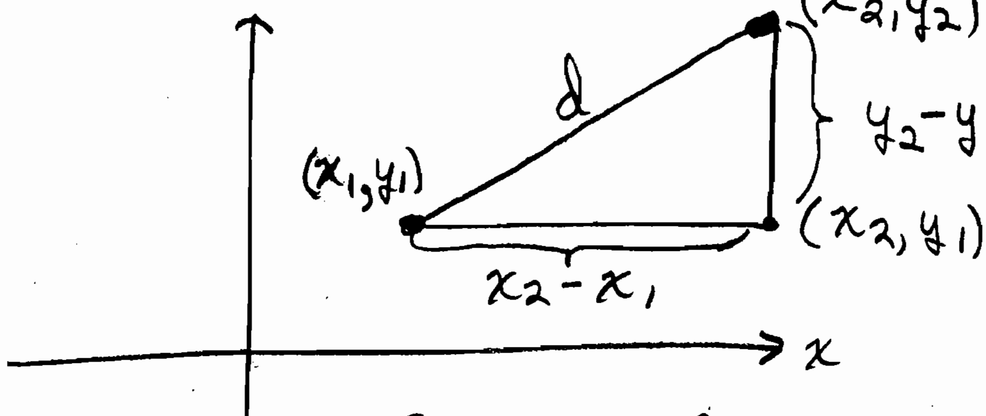
FROM a **TO** b IS $|a - b|$ (SAME AS $|b - a|$)

THE DISTANCE FROM 5 TO −3

$a = 5 \quad b = -3$

$|a - b| = |5 - (-3)| = |8| = 8$

3. DISTANCE (UNDIRECTED) BETWEEN POINTS IN THE PLANE

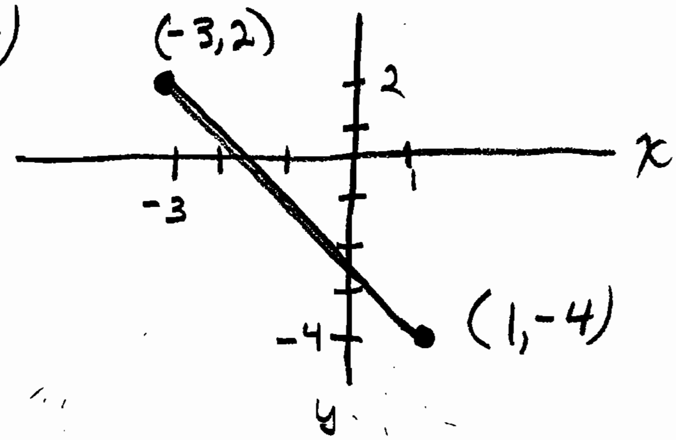


$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

DISTANCE BETWEEN (x_1, y_1) AND (x_2, y_2)
(DEFINITION)

1. FIND THE DISTANCE BETWEEN $(-3, 2)$ AND $(1, -4)$



LET $(x_1, y_1) = (-3, 2)$

LET $(x_2, y_2) = (1, -4)$

$x_1 = -3 \quad y_1 = 2$

$x_2 = 1 \quad y_2 = -4$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1-(-3))^2 + (-4-2)^2}$$

$$= \sqrt{4^2 + (-6)^2} = \sqrt{16 + 36} = \sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$$

2. IT DOES NOT MATTER WHICH POINT IS NAMED (x_1, y_1) AND WHICH POINT (x_2, y_2). REWORK PREVIOUS PROBLEM.

PREVIOUSLY $(x_1, y_1) = (-3, 2)$ $(x_2, y_2) = (1, -4)$

NOW $(x_1, y_1) = (1, -4)$ $(x_2, y_2) = (-3, 2)$

$x_1 = 1$ $y_1 = -4$ $x_2 = -3$ $y_2 = 2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$$

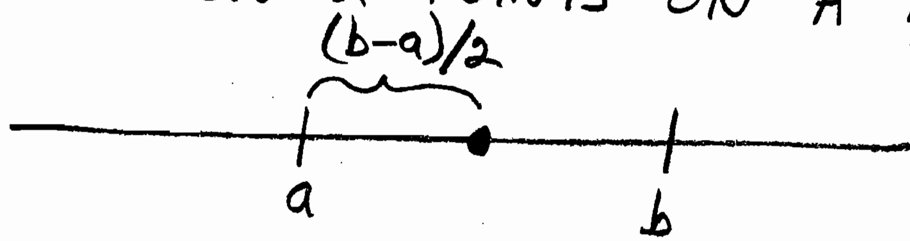
$$\sqrt{(-3-1)^2 + (2-(-4))^2} = \sqrt{(-4)^2 + 6^2} =$$

$$\sqrt{16+36} = \sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$$

NOTE: SAME ANSWER AS PREVIOUSLY

E. MIDPOINT FORMULAS

1. BETWEEN 2 POINTS ON A LINE

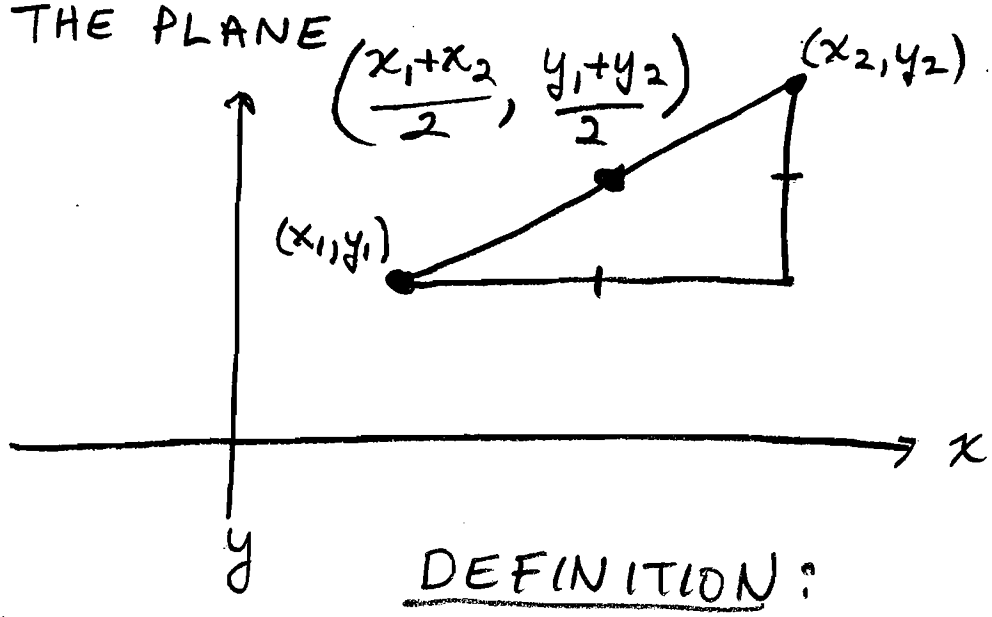


$$M = a + \frac{b-a}{2} = \frac{2a + b - a}{2} = \frac{a+b}{2}$$

$$\boxed{M = \frac{a+b}{2}}$$ DEFINITION

2. THE MIDPOINT BETWEEN -3 AND 5 IS $\frac{-3+5}{2} = \frac{2}{2} = 1$

3. MIDPOINT BETWEEN 2 POINTS IN THE PLANE



DEFINITION:

THE MIDPOINT BETWEEN (x_1,y_1) AND (x_2,y_2) IS $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

4. THE MIDPOINT BETWEEN $(-2,3)$ AND $(4,-6)$ IS $\left(\frac{-2+4}{2}, \frac{3+(-6)}{2}\right) = \left(\frac{2}{2}, \frac{-3}{2}\right) = \left(1, -\frac{3}{2}\right)$

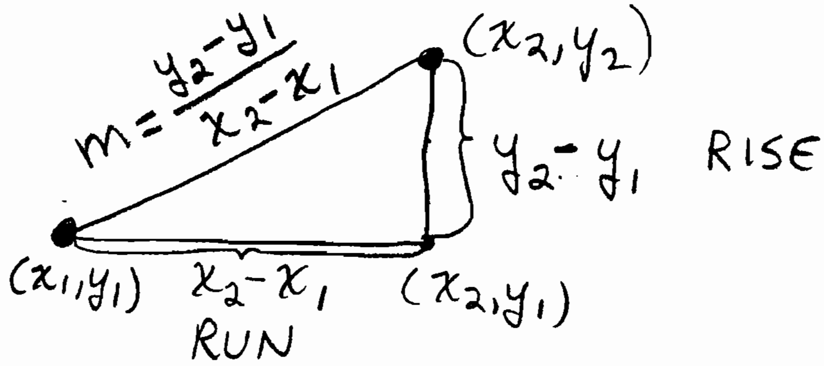
5. THE MIDPOINT BETWEEN THE ORIGIN, $(0,0)$ AND (x,y) IS $\left(\frac{0+x}{2}, \frac{0+y}{2}\right)$

$= \left(\frac{x}{2}, \frac{y}{2}\right)$

F. HOMEWORK (OIS)

1. NAME A POINT IN QUADRANT IV.
2. $(2,5)=(2,y)$. WHAT IS y ?
3. $\{a,b\}=\{2,5\}$. MUST $a=2$?
4. ON THE NUMBER LINE WHAT IS THE DIRECTED DISTANCE
   a. FROM 6 TO -1 b. FROM -1 TO 6
5. ON THE NUMBER LINE WHAT IS THE DISTANCE
   a. FROM 6 TO -1 b. FROM -1 TO 6
6. ON THE NUMBER LINE WHAT IS THE MIDPOINT BETWEEN
   a. -1 AND 6 b. -10 AND $-3/2$
7. IN THE PLANE, FIND THE DISTANCE BETWEEN, AND FIND THE MIDPOINT BETWEEN
   a. $(-2,5)$, $(7,-6)$ b. $(-1,-3)$, $(5,-2)$
   c. (x_1, y_1), $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
8. PROVE THAT THE 3 POINTS $(-2,1)$, $(1,10)$ AND $(4,-1)$ FORM A RIGHT TRIANGLE.

G SLOPE OF A STRAIGHT LINE



THE SLOPE OF THE LINE BETWEEN (x_1, y_1) AND (x_2, y_2) IS

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1} = \left(\frac{\text{RISE}}{\text{RUN}}\right)} \text{(DEF.)}$$

1. FIND THE SLOPE OF THE LINE BETWEEN $(-1, 2)$ AND $(3, -4)$

   a. LET $(x_1, y_1) = (-1, 2)$ AND $(x_2, y_2) = (3, -4)$

   $x_1 = -1 \quad y_1 = 2 \quad x_2 = 3 \quad y_2 = -4$

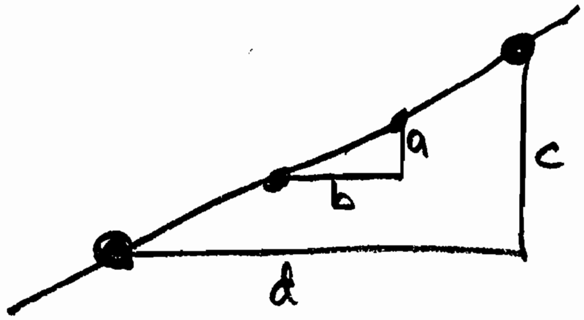
   $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{3 - (-1)} = \frac{-6}{4} = -\frac{3}{2}$$

   b. LET $(x_1, y_1) = (3, -4)$ AND $(x_2, y_2) = (-1, 2)$

   $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{-1 - 3} = \frac{6}{-4} = -\frac{3}{2}$$

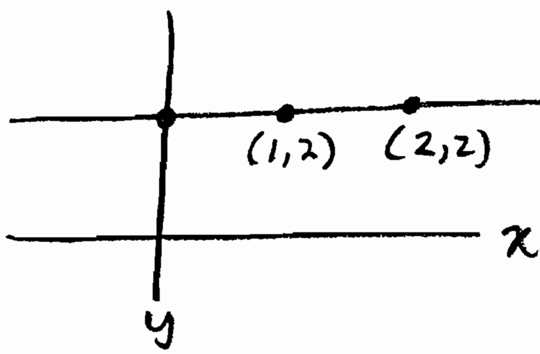
   c. NOTE: IT DOES NOT MATTER WHICH IS NAMED (x_1, y_1) AND WHICH IS NAMED (x_2, y_2)

2. IT DOES NOT MATTER WHICH 2 POINTS YOU PICK ON A LINE; YOU GET THE SAME SLOPE.



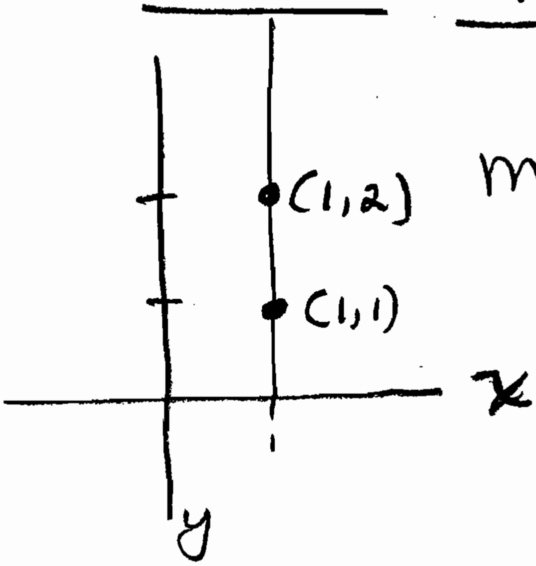
BY SIMILAR TRIANGLES $m = \frac{a}{b} = \frac{c}{d}$

3. <u>HORIZONTAL LINES</u> HAVE SLOPE <u>ZERO</u>.



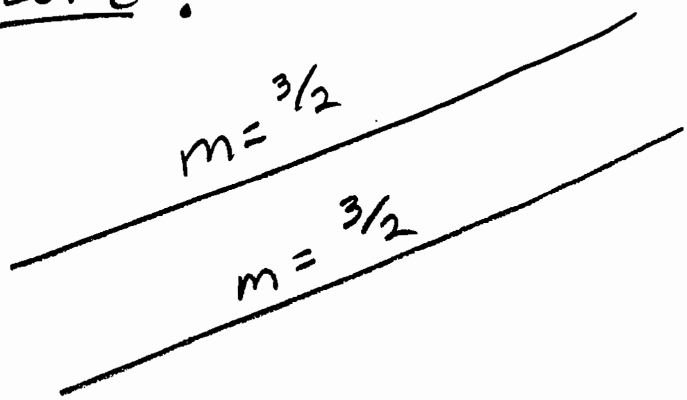
$$m = \frac{2-2}{2-1} = \frac{0}{1} = 0$$

4. THE SLOPE IS <u>UNDEFINED</u> FOR <u>VERTICAL LINES</u>.

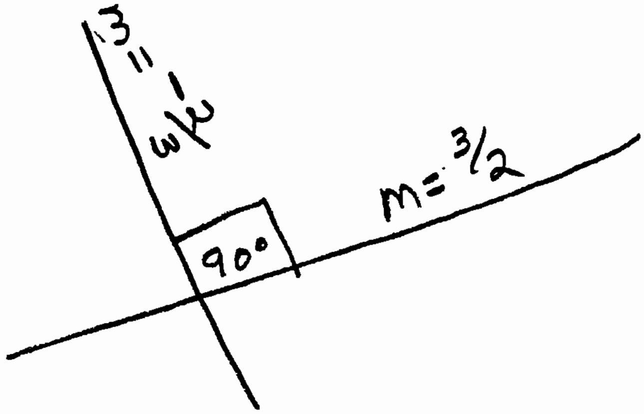


$$m = \frac{2-1}{1-1} = \frac{1}{0} \quad \text{UNDEFINED}$$

5. PARALLEL LINES HAVE THE SAME SLOPE.

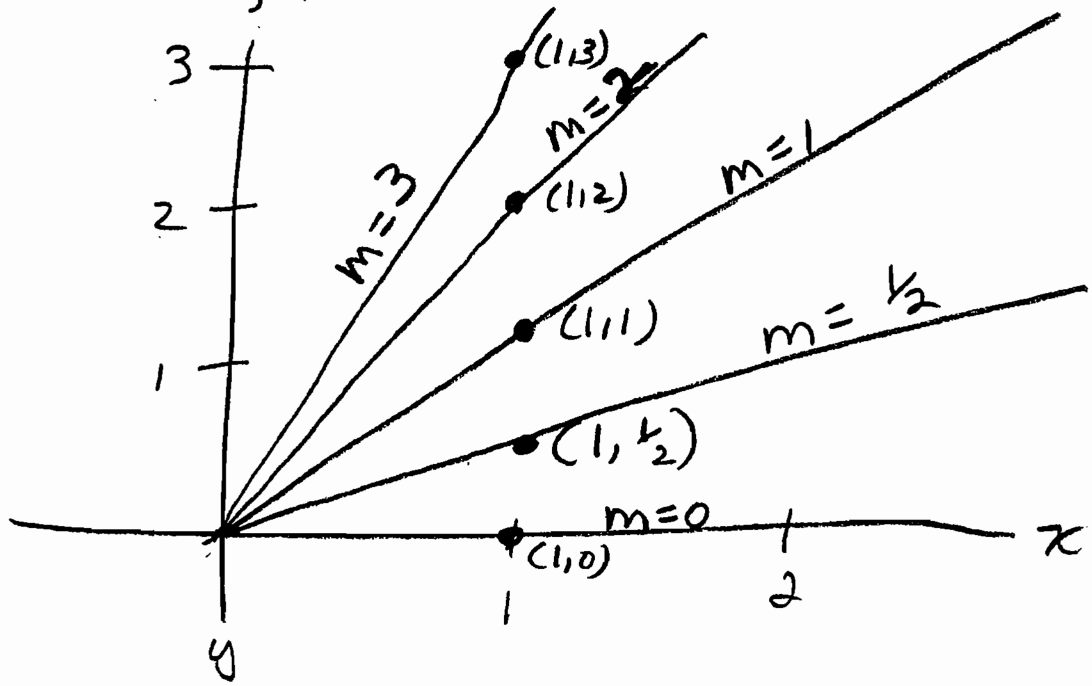


6. THE SLOPES OF PERPENDICULAR LINES ARE THE NEGATIVE RECIPROCAL OF EACH OTHER

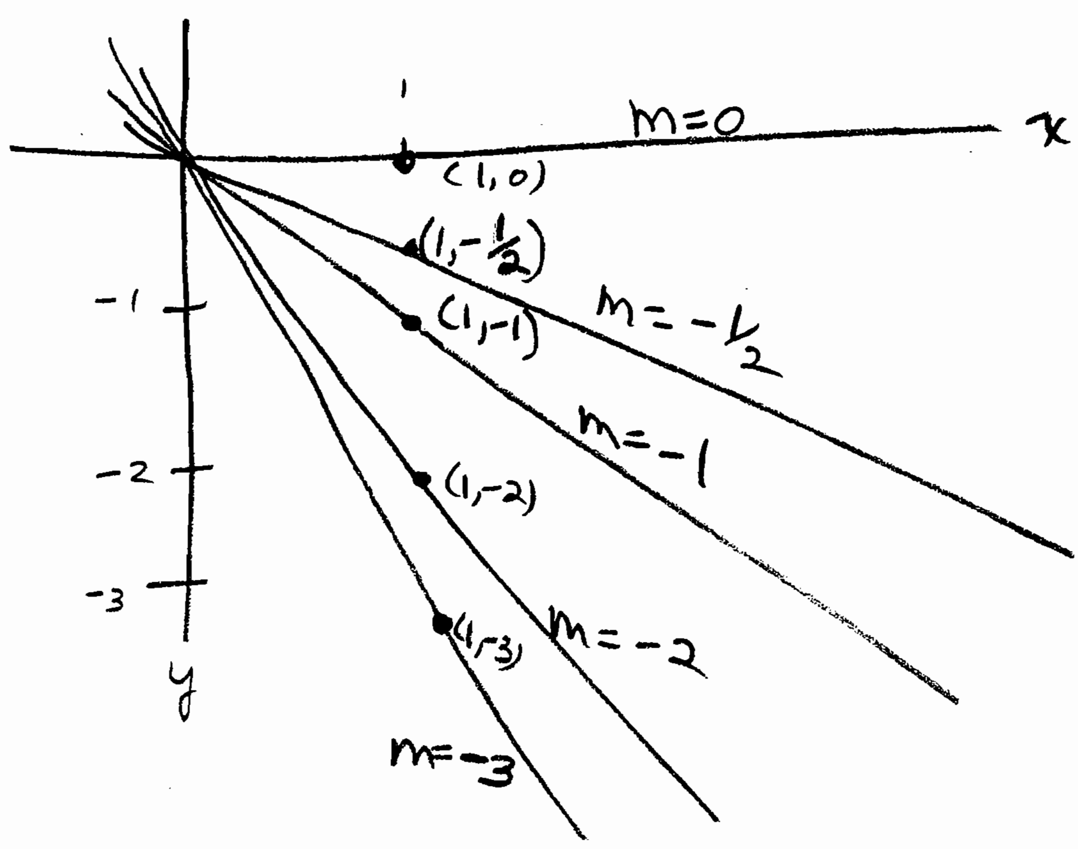


SLOPE OF GIVEN LINE	SLOPE OF LINE PERPENDICULAR
$-\frac{5}{7}$	$\frac{7}{5}$
3	$-\frac{1}{3}$
-2	$\frac{1}{2}$

7. <u>POSITIVE</u> <u>SLOPE</u>: AS YOU GO TO THE RIGHT, YOU GO UPHILL.

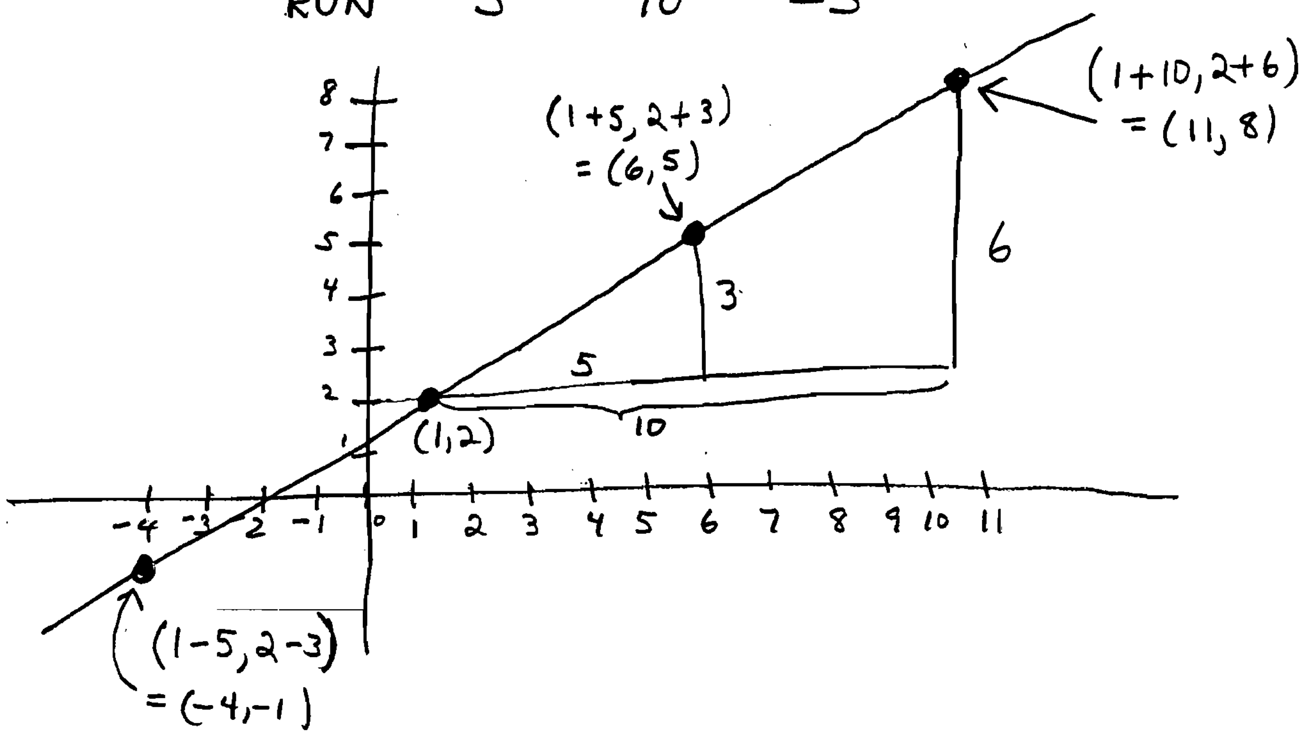


8. <u>NEGATIVE</u> <u>SLOPE</u>: AS YOU GO TO THE RIGHT, YOU GO DOWNHILL.



9. A LINE GOES THROUGH THE POINT (1,2) WITH SLOPE $\frac{3}{5}$. FIND THREE OTHER POINTS ON THE LINE.

$$m = \frac{\text{RISE}}{\text{RUN}} = \frac{3}{5} = \frac{6}{10} = \frac{-3}{-5}$$



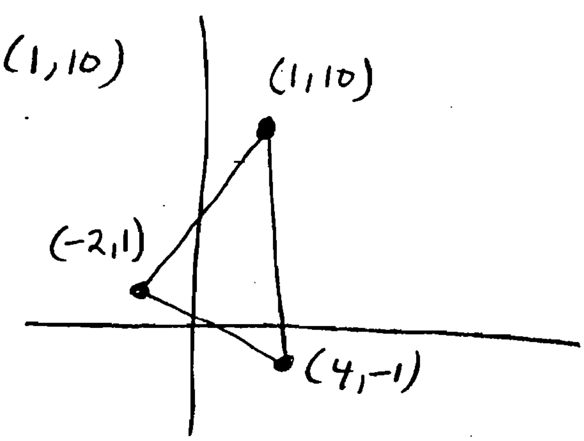
10. SHOW (-2,1), (1,10), AND (4,-1) FORM A RIGHT TRIANGLE

SLOPE BETWEEN (-2,1) AND (1,10)

$$m = \frac{10-1}{1-(-2)} = \frac{9}{3} = 3$$

SLOPE BETWEEN (-2,1) AND (4,-1)

$$m = \frac{-1-1}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3}$$



$-\frac{1}{3}$ AND 3, NEGATIVE RECIPROCALS, SO PERPENDICULAR

H. HOMEWORK (OIS)

1. IF POSSIBLE, FIND THE SLOPE OF THE LINE BETWEEN THE POINTS

   a. $(3,-10), (\frac{1}{2}, 4)$ b. $(-\frac{2}{3}, \frac{7}{5}), (-\frac{1}{2}, \frac{3}{7})$

   c. $(\frac{2}{3}, 7), (\frac{5}{6}, 7)$ d $(-\frac{2}{3}, 4), (-\frac{2}{3}, 5)$

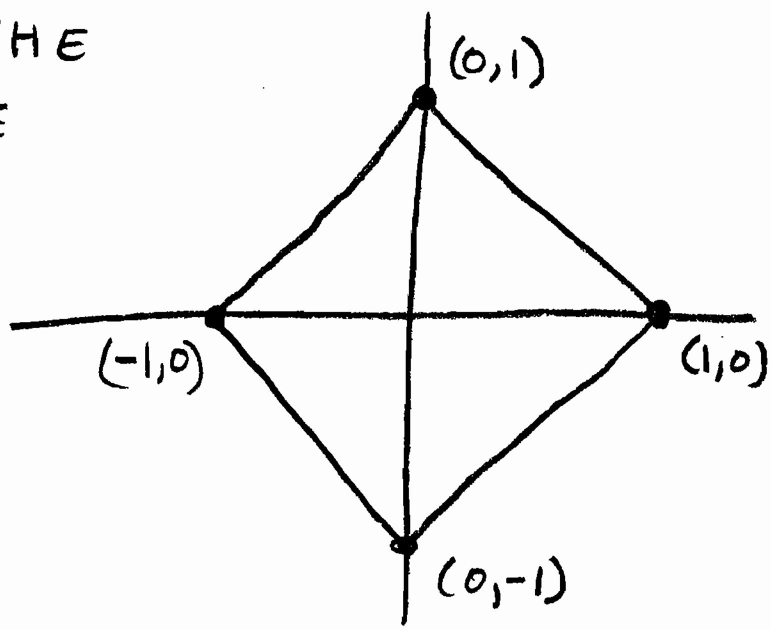
2. FIND THE SLOPE OF A LINE <u>PERPENDICULAR</u> TO THE LINE BETWEEN THE POINTS

   a. $(3,-7), (\frac{1}{4}, 5)$ b. $(-\frac{3}{8}, 2), (\frac{1}{4}, -\frac{2}{3})$

3. FIND 3 POINTS ON THE LINE WITH SLOPE $\frac{-3}{5}$ THROUGH THE POINT $(-3,2)$.

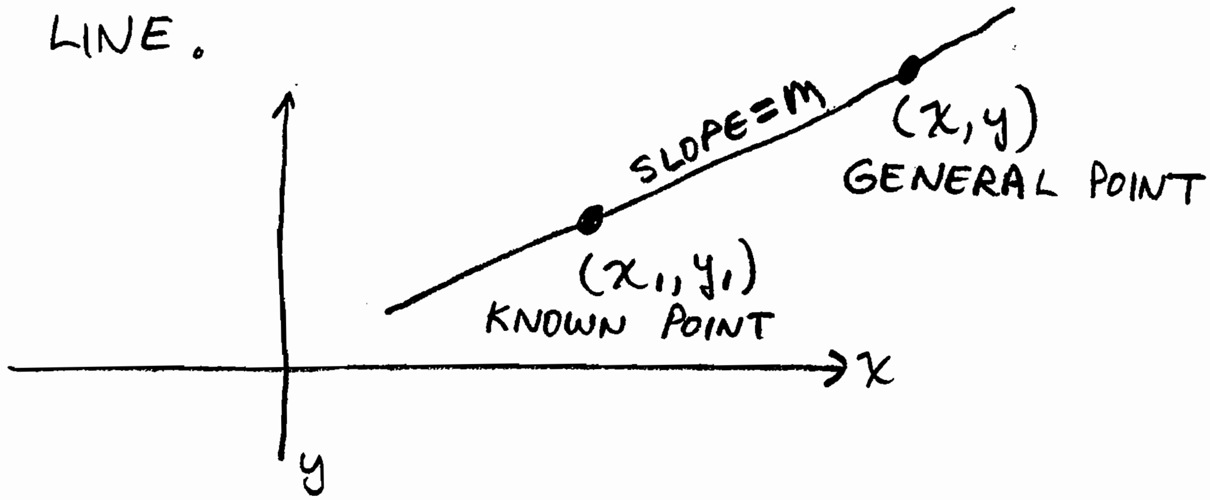
4. FIND THE SLOPE OF THE LINE THROUGH $(5,-2)$ AND THE MIDPOINT BETWEEN $(3,-6)$ AND $(-1,7)$.

5. PROVE THAT THE FIGURE AT THE RIGHT IS A SQUARE



I. EQUATIONS FOR A LINE

1. POINT-SLOPE EQUATION FOR A LINE.



$$m = \frac{y-y_1}{x-x_1} \qquad m(x-x_1) = y-y_1$$

POINT-SLOPE EQUATION

$$\boxed{y-y_1 = m(x-x_1)}$$ (DEF.)

SO TO GET AN EQUATION FOR A LINE, YOU NEED A <u>POINT</u> AND A <u>SLOPE</u>

2. FIND AN EQUATION FOR THE LINE THROUGH (2,-3) WITH SLOPE 5

$$y-(-3) = 5(x-2)$$

3. STANDARD FORM FOR AN EQUATION FOR A LINE

a. IN THE LAST EXAMPLE OUR ANSWER WAS $y-(-3)=5(x-2)$

$$y+3=5x-10$$

$$-5x+y=-13$$

$$Ax+By=C \quad \text{FORM}$$

b.

$$\boxed{\begin{array}{l}\text{STANDARD FORM}\\ Ax+By=C\\ \text{EITHER } A\neq 0 \text{ OR } B\neq 0\end{array}}$$

(DEF.)

4. THE GRAPH OF AN EQUATION IS THE SET OF ALL POINTS THAT SATISFY THE EQUATION.

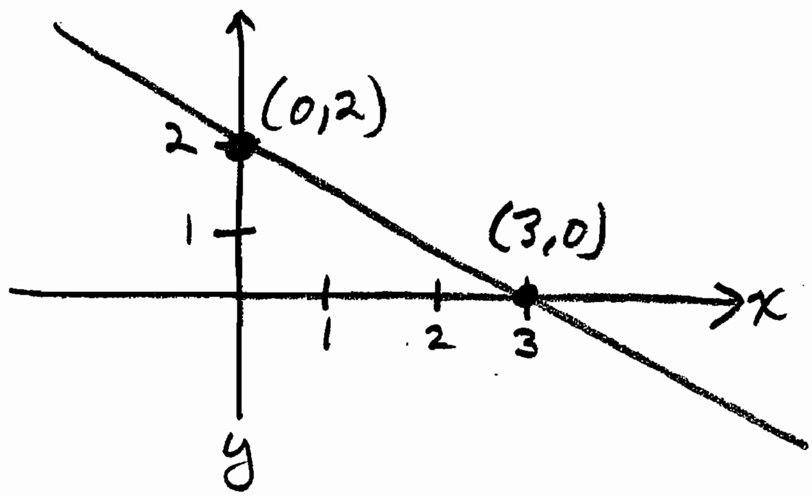
a. THE GRAPH OF AN EQUATION IS A STRAIGHT LINE IFF IT HAS AN EQUATION THAT CAN BE PUT IN THE FORM $Ax+By=C$ (EITHER $A\neq 0$ OR $B\neq 0$)

b. SKETCH THE GRAPH OF $2x+3y=6$.
THIS IS IN $Ax+By=C$ FORM SO ITS GRAPH IS A STRAIGHT LINE. 2 POINTS DETERMINE A LINE, SO DETERMINE 2 POINTS ON THE LINE AND DRAW THE LINE.

POINT 1: SET $x=0$. FIND y.
POINT 2: SET $y=0$. FIND x.

x	y
0	2
3	0

$2(0)+3y=6$
$2x+3(0)=6$



5. <u>2-POINT</u> <u>EQUATION</u> <u>FOR</u> <u>A</u> <u>LINE</u>

a. GIVEN 2 SPECIFIC POINTS $(x_1,y_1),(x_2,y_2)$

b. TO GET AN EQUATION YOU NEED A <u>POINT</u> AND A <u>SLOPE</u>, SO USE THE 2 POINTS TO GET THE SLOPE

$$m=\frac{y_2-y_1}{x_2-x_1}$$

SUBSTITUTE INTO $y-y_1=m(x-x_1)$

$$\boxed{\begin{array}{c}\text{2-POINT EQUATION}\\ y-y_1=\dfrac{y_2-y_1}{x_2-x_1}(x-x_1)\end{array}}\qquad (\underline{\text{DEF.}})$$

c. FIND AN EQUATION FOR THE LINE THROUGH THE 2 POINTS $(2,-3)$ AND $(5,7)$. PUT IN STANDARD FORM.

LET $(x_1,y_1)=(2,-3)$ $\quad (x_2,y_2)=(5,7)$

$$y-(-3)=\frac{7-(-3)}{5-2}(x-2)$$

$$y+3=\frac{10}{3}(x-2)$$

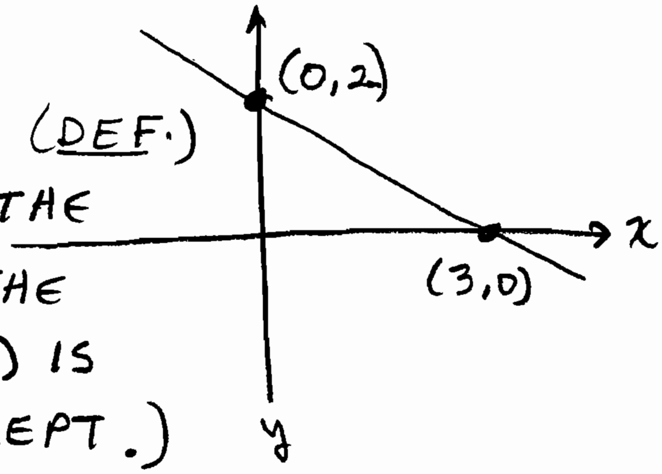
$$y+3=\frac{10}{3}x-\frac{20}{3}$$

$$-\frac{10}{3}x+y=-\frac{20}{3}-3=\frac{-20-9}{3}=-\frac{29}{3}$$

$$-\frac{10}{3}x+y=-\frac{29}{3}$$

6. INTERCEPTS

a. **X-INTERCEPT: (DEF.)**
A POINT WHERE THE GRAPH CROSSES THE x-AXIS. ($(3,0)$ IS AN x-INTERCEPT.)



b. **Y-INTERCEPT** (DEF.) : A POINT WHERE THE GRAPH CROSSES THE y-AXIS. ($(0,2)$ IS A y-INTERCEPT.)

c. FIND THE x AND y INTERCEPTS FOR $3x-4y=12$

i. TO GET THE x-INTERCEPT, SET $y=0$. SOLVE FOR x

$3x-4(0)=12$

$3x=12$

$x=4$

$(4,0)$ IS THE x-INTERCEPT

ii. TO GET THE y-INTERCEPT, SET $x=0$. SOLVE FOR y

$3(0)-4y=12 \qquad -4y=12$

$y=-3$

$(0,-3)$ IS THE y-INTERCEPT

7. SLOPE - INTERCEPT EQUATION

a. CONSIDER $y - y_1 = m(x - x_1)$. m IS THE SLOPE.

$y - y_1 = mx - mx_1$

$y = mx - mx_1 + y_1$

$y = mx + b$ WHERE $b = -mx_1 + y_1$

SET $x = 0$ TO FIND y-INTERCEPT

$y = m(0) + b = b$

$(0, b)$ IS THE y-INTERCEPT

DEFINITION

> SLOPE - INTERCEPT EQUATION
>
> $$y = mx + b$$
>
> WHERE SLOPE $= m$ AND
> y-INTERCEPT $= (0, b)$

b. FIND THE SLOPE AND y-INTERCEPT FOR THE LINE $2x + 3y = 6$.

SOLVE FOR y

$3y = -2x + 6$

$y = -\frac{2}{3}x + 2$

SLOPE $= -\frac{2}{3}$ $\quad$ y-INTERCEPT IS $(0, 2)$

c. FIND AN EQUATION FOR THE LINE PERPENDICULAR TO $4x+3y=12$ THAT PASSES THROUGH $(-2,5)$. PUT ANSWER IN STANDARD FORM.

NEED POINT AND SLOPE.

HAVE THE POINT $(-2,5)$. GET THE SLOPE.

SOLVE FOR y

$$3y = -4x + 12$$

$$y = -\frac{4}{3}x + 4$$

$-\frac{4}{3}$ = SLOPE OF $4x+3y=12$ LINE

$\frac{3}{4}$ = SLOPE OF DESIRED LINE PERPENDICULAR TO $4x+3y=12$ LINE.

POINT $(-2,5)$ SLOPE $\frac{3}{4}$

$$y-5 = \frac{3}{4}(x-(-2))$$

$$y-5 = \frac{3}{4}x + \frac{6}{4} = \frac{3}{4}x + \frac{3}{2}$$

$$-\frac{3}{4}x + y = 5 + \frac{3}{2} = \frac{10+3}{2} = \frac{13}{2}$$

$$-\frac{3}{4}x + y = \frac{13}{2}$$

8. HORIZONTAL LINE EQUATIONS

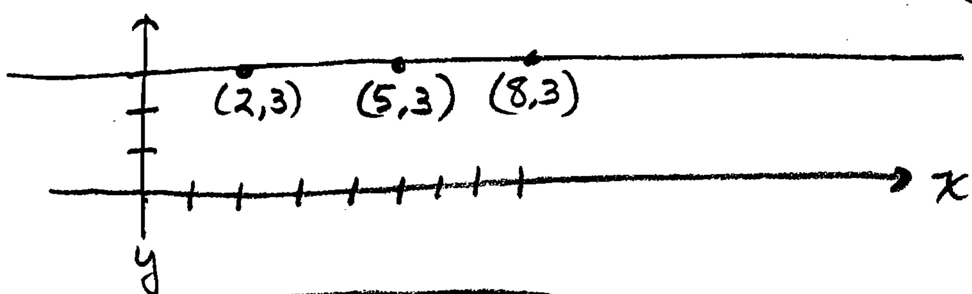
a. HORIZONTAL LINES HAVE SLOPE $m=0$

$$y = mx+b = 0x+b = b$$

HORIZONTAL LINE EQUATION

$$\boxed{y=b}$$

b. FIND AN EQUATION FOR THE HORIZONTAL LINE THROUGH (2,3)



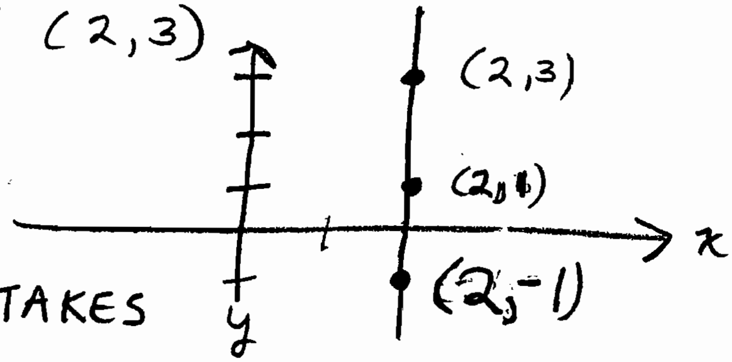
ANSWER $y=3$

NOTE ALL IT TAKES FOR A POINT ON THIS HORIZONTAL LINE TO SATISFY THE EQUATION IS FOR ITS y-COORDINATE TO EQUAL 3.

9. VERTICAL LINE EQUATION

$$\boxed{x=K}$$

a. FIND AN EQUATION FOR THE VERTICAL LINE THROUGH (2,3)



$$\boxed{x = 2}$$

NOTE: ALL IT TAKES FOR A POINT ON THIS VERTICAL LINE TO SATISFY THE EQUATION IS FOR ITS x-COORDINATE TO BE 2

10. EQUATIONS FOR VERTICAL LINE THAT WE STUDIED ARE IN STANDARD FORM (SAME FOR HORIZONTAL LINES)

HORIZONTAL LINE THROUGH (2,3)

$$y = 3$$

$$0x + 1y = 3$$

$$Ax + By = C$$

VERTICAL LINE THROUGH (2,3)

$$x = 2$$

$$1x + 0y = 2$$

$$Ax + By = C$$

J. HOMEWORK (OIS) FIND-AN-EQUATION-FOR-A-LINE TYPE OF PROBLEMS. PUT IN STANDARD FORM THE EQUATIONS FOR THE LINES THAT SATISFY THE FOLLOWING:

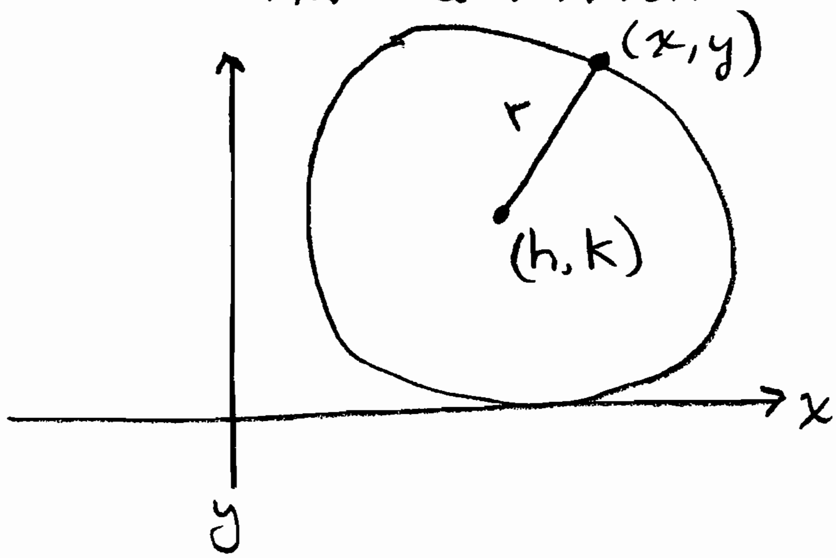
1. THROUGH $(0,1)$ AND $(1,0)$
2. THROUGH $(-5,-\frac{2}{3})$ WITH SLOPE $\frac{3}{5}$
3. HAS y-INTERCEPT $(0,4)$ WITH SLOPE -2
4. HAS x-INTERCEPT $(5,0)$ WITH SLOPE $6/7$
5. IS PARALLEL TO $3x-4y=6$ AND PASSES THROUGH $(-1,3)$.
6. IS PERPENDICULAR TO $2x+5y=10$ AND PASSES THROUGH $(-2,3)$.
7. IS THE PERPENDICULAR BISECTOR OF THE LINE SEQMENT BETWEEN $(-2,5)$ AND $(3,7)$

8. FIND AN EQUATION FOR THE LINE THAT PASSES THROUGH THE POINT $\frac{1}{4}$ OF THE DISTANCE BETWEEN $(2,3)$ AND $(4,7)$ ON THE LINE SEGMENT BETWEEN $(2,3)$ AND $(4,7)$. (THIS POINT IS CLOSEST TO THE POINT $(2,3)$). THE LINE HAS SLOPE 5.

[CHAPTER 11]

CIRCLES AND PARABOLAS

A. DERIVE AN EQUATION FOR A CIRCLE



$$\sqrt{(x-h)^2+(y-k)^2} = r$$

$$\boxed{(x-h)^2+(y-k)^2 = r^2}$$

EQUATION FOR A CIRCLE WITH CENTER (h, k) AND RADIUS r

STANDARD FORM (DEF.)

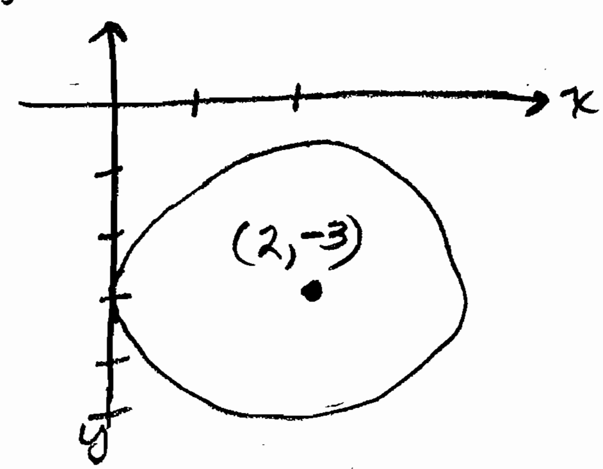
B. STATE CENTER, RADIUS, AND SKETCH

$$(x-2)^2+(y+3)^2 = 4$$

$$(x-2)^2+(y-[-3])^2 = 2^2$$

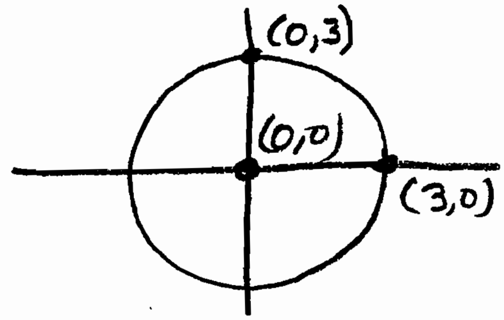
CENTER $(2,-3)$

RADIUS 2



C. $x^2+y^2=9$ IS AN EQUATION FOR THE CIRCLE WITH CENTER $(0,0)$ AND RADIUS 3

$$(x-0)^2+(y-0)^2=3^2$$



D. PUTTING AN EQUATION FOR A CIRCLE IN STANDARD FORM

GIVEN $2x^2+2y^2-12x+20y-30=0$

NOTE: BOTH THE SAME

GET x^2 AND y^2 COEFFICIENTS TO 1

$$x^2+y^2-6x+10y-15=0$$

GROUP x's AND y's, CONSTANT ON RIGHT

$$(x^2-6x\quad)+(y^2+10y\quad)=15$$

COMPLETE THE SQUARE ADDING THE SAME THING TO BOTH SIDES

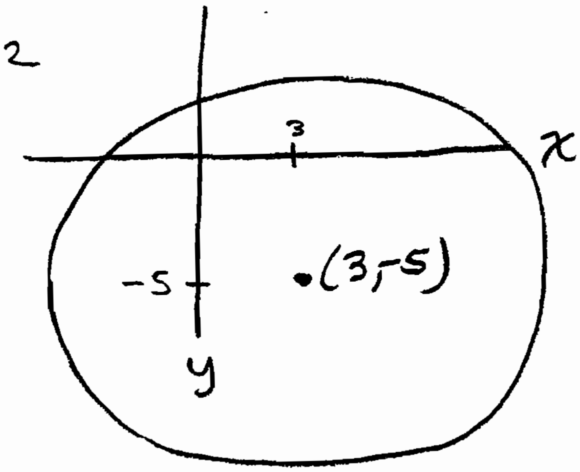
$$(x^2-6x+9)+(y^2+10y+25)=15+9+25$$

$$(x-3)^2+(y+5)^2=49$$

$$(x-3)^2+(y-[-5])^2=7^2$$

CENTER $(3,-5)$

RADIUS 7



E. MORE DIFFICULT ALGEBRA: PUT IN STANDARD FORM, NAME CENTER AND RADIUS, SKETCH

$$3x^2+3y^2+5x-7y-9=0$$

GET x^2 AND y^2 COEFFICIENTS TO 1

$$x^2+y^2+\frac{5}{3}x-\frac{7}{3}y-3=0$$

GROUP x's AND y's, CONSTANT OR RIGHT

$$\left(x^2+\frac{5}{3}x\qquad\right)+\left(y^2-\frac{7}{3}y\qquad\right)=3$$

$\left(\frac{1}{2}\left(\frac{5}{3}\right)\right)^2$ COMPLETE THE SQUARE ADDING THE SAME THING TO BOTH SIDES. $\left(\frac{1}{2}\left(\frac{-7}{3}\right)\right)^2$

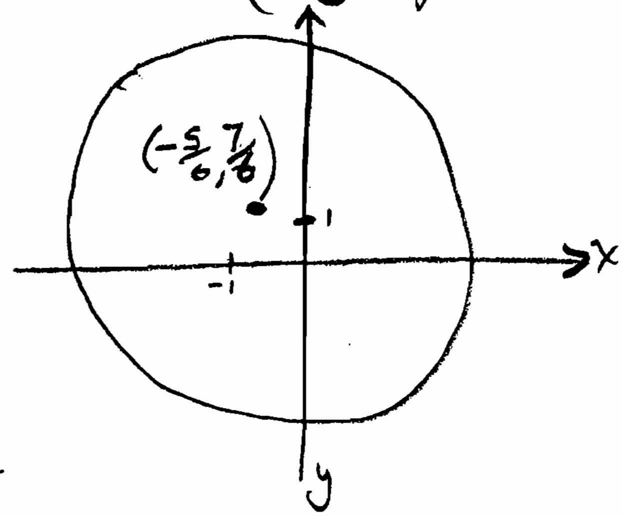
$$\left(x^2+\frac{5}{3}x+\frac{25}{36}\right)+\left(y^2-\frac{7}{3}y+\frac{49}{36}\right)=\frac{108}{36}+\frac{25}{36}+\frac{49}{36}$$

$$\left(x+\frac{5}{6}\right)^2+\left(y-\frac{7}{6}\right)^2=\frac{182}{36}$$

$$\left(x-\left[-\tfrac{5}{6}\right]\right)^2+\left(y-\tfrac{7}{6}\right)^2=\left(\frac{\sqrt{182}}{6}\right)^2$$

CENTER $\left(-\frac{5}{6},\frac{7}{6}\right)$

RADIUS $\frac{\sqrt{182}}{6}$



F. THE <u>GRAPH</u> OF

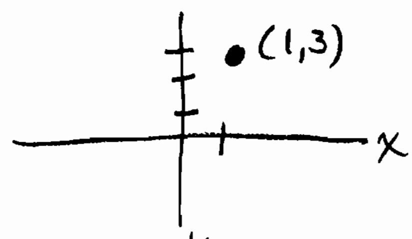
$x^2+y^2-2x-6y+10=0$ IS ONLY A <u>POINT</u>!

$$(x^2-2x \quad)+(y^2-6y \quad)=-10$$

$$(x^2-2x+1)+(y^2-6y+9)=-10+1+9$$

$$(x-1)^2+(y-3)^2=0$$

SOLUTION SET $\{(1,3)\}$



G. THE GRAPH OF $x^2+y^2-2x-6y+20=0$ DOES NOT EXIST. NO GRAPH.

$$(x^2-2x \quad)+(y^2-6y \quad)=-20$$

$$(x^2-2x+1)+(y^2-6y+9)=-20+1+9$$

$$(x-1)^2+(y-3)^2=-10$$

YOU CANNOT SQUARE 2 REALS, ADD TOGETHER AND GET A NEGATIVE NUMBER.

H. $ax^2+ay^2+bx+cy+d=0$ CAN EITHER HAVE NO GRAPH, A POINT GRAPH, OR A CIRCLE AS ITS GRAPH.

I. HOMEWORK (OIS)

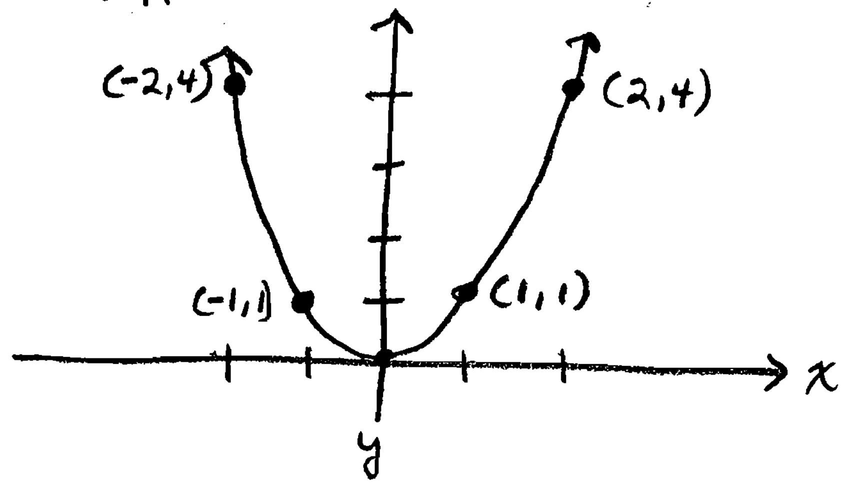
1. GIVE AN EQUATION IN STANDARD FORM FOR A CIRCLE WITH
   a. CENTER $(2,3)$ RADIUS 6
   b. CENTER $(-5,6)$ RADIUS 4
   c CENTER $\left(-\frac{2}{3}, -\frac{1}{5}\right)$ RADIUS $\frac{1}{7}$

2. FOR EACH OF THE FOLLOWING, PUT IN STANDARD FORM; STATE WHETHER IT IS NO GRAPH, POINT GRAPH, OR A CIRCLE GRAPH. SKETCH THOSE THAT HAVE A GRAPH
   a. $x^2+y^2-10x+6y+30=0$
   b. $x^2+y^2-4x-12y+44=0$
   c. $x^2+y^2+6x+8y=-25$
   d. $9x^2+9y^2-2x+4y-20=0$
   e. $15x^2+15y^2+6x+\frac{3}{5}=10y$

J. THE GRAPHS OF $y=x^2$, $y=-x^2$, $x=y^2$, AND $x=-y^2$ ARE PARABOLAS

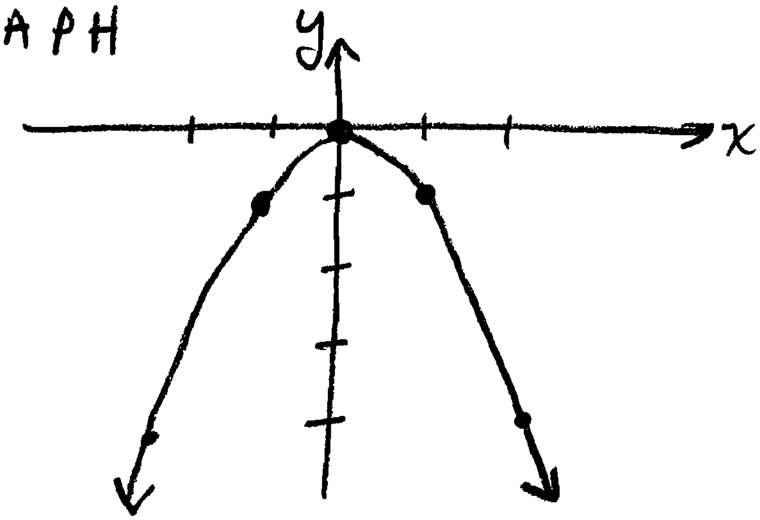
1. $y=x^2$ GRAPH

x	y
0	0
1	1
-1	1
2	4
-2	4

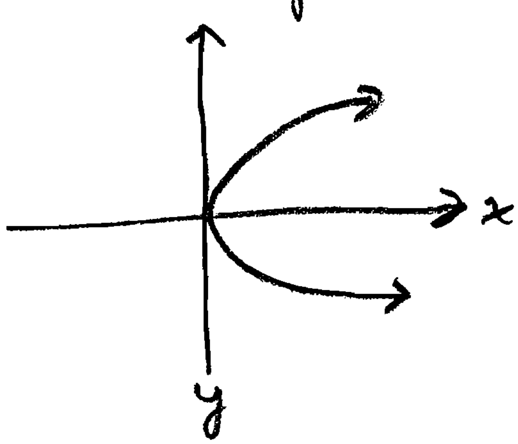


2. $y=-x^2$ GRAPH

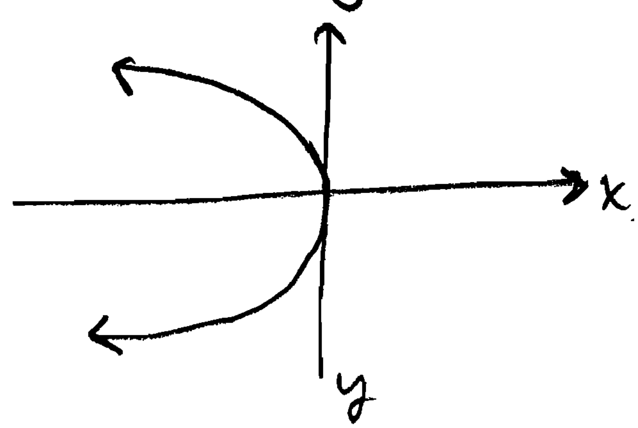
x	y
0	0
1	-1
-1	-1
2	-4
-2	-4



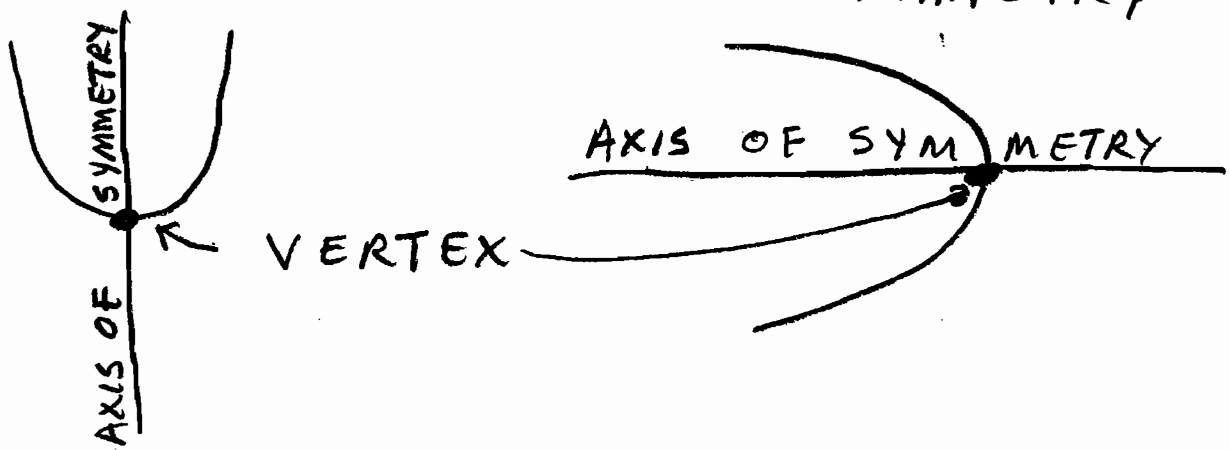
3. $x=y^2$ GRAPH



4. $x=-y^2$ GRAPH



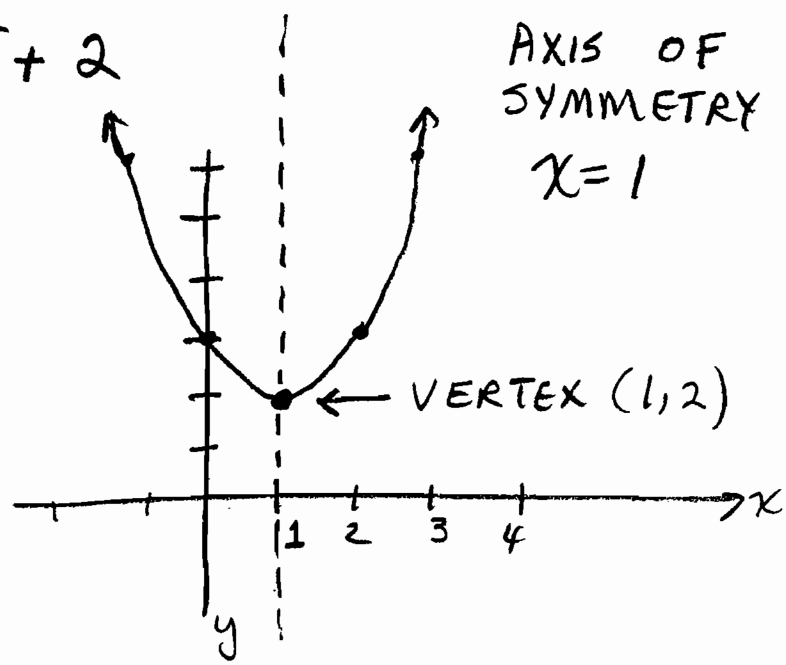
K. VERTEX AND AXIS OF SYMMETRY



L. PARABOLAS WHOSE VERTICES ARE <u>NOT</u> THE ORIGIN AND WHOSE AXES OF SYMMETRY ARE NOT COORDINATE AXES.

$$y = (x-1)^2 + 2$$

x	y
1	2
2	$(2-1)^2+2 = 3$
0	$(0-1)^2+2 = 3$
3	$(3-1)^2+2 = 6$
−1	$(-1-1)^2+2 = 6$



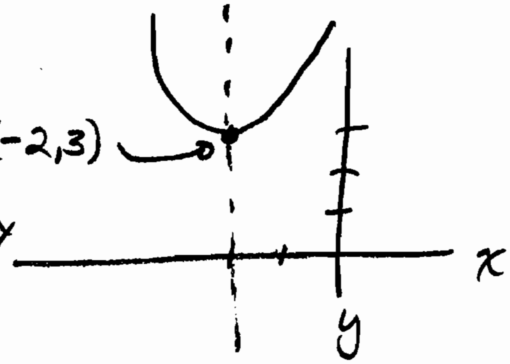
<u>NOTE</u>: THE GRAPH OF $y = x^2$ WAS TRANSLATED <u>RIGHT</u> 1 AND <u>UP</u> 2

M. SKETCH OF SEVERAL OTHER PARABOLAS BY JUST LOOKING AT THE FORM

1. $y = (x+2)^2 + 3$

LEFT 2
UP 3

VERTEX $(-2,3)$
AXIS OF SYMMETRY
$x = -2$

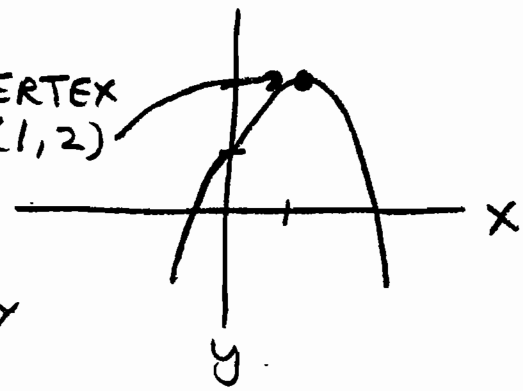


2. $y = -(x-1)^2 + 2$

$y = -x^2$ GRAPH
RIGHT 1
UP 2

VERTEX
$(1,2)$

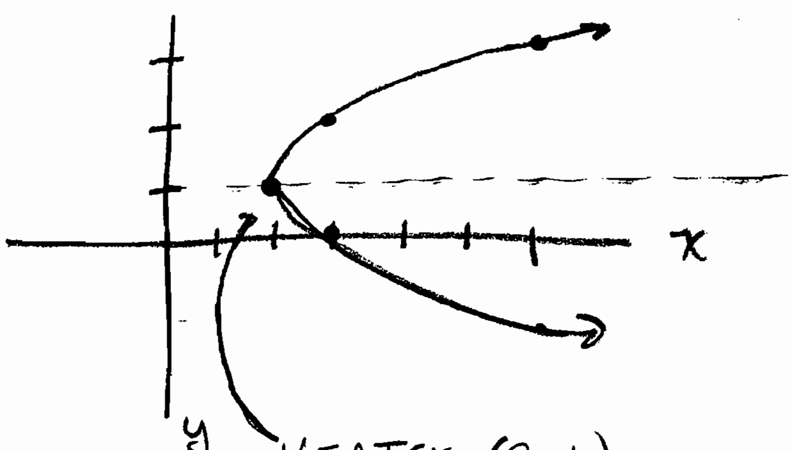
AXIS OF SYMMETRY
$x = 1$



3. $x = (y-1)^2 + 2$

x	y
$2 = (1-1)^2+2$	1
$3 = (2-1)^2+2$	2
$3 = (0-1)^2+2$	0
$6 = (3-1)^2+2$	3
$6 = (-1-1)^2+2$	-1

$x = y^2$ GRAPH
UP 1
RIGHT 2



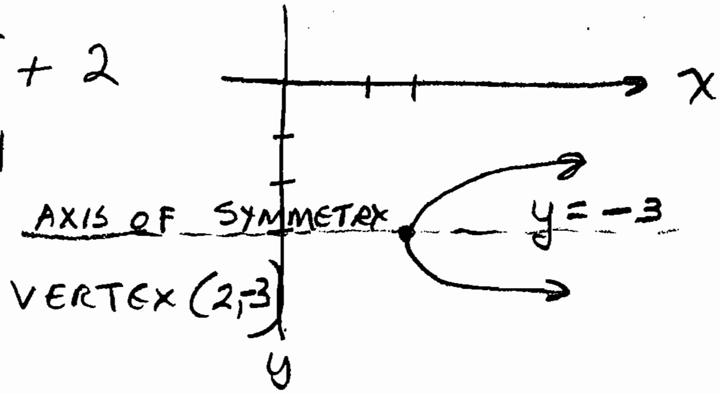
VERTEX $(2,1)$
AXIS OF SYMMETRY
$y = 1$

4. $x = (y+3)^2 + 2$

$x = y^2$ GRAPH

DOWN 3

RIGHT 2

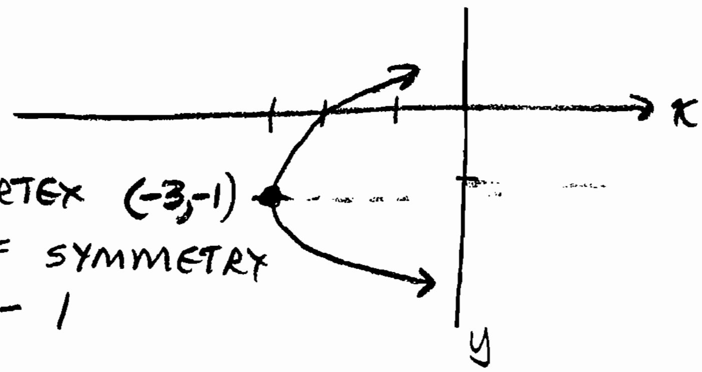


5. $x = (y+1)^2 - 3$

$x = y^2$ GRAPH

DOWN 1

LEFT 3



VERTEX (-3,-1)

AXIS OF SYMMETRY

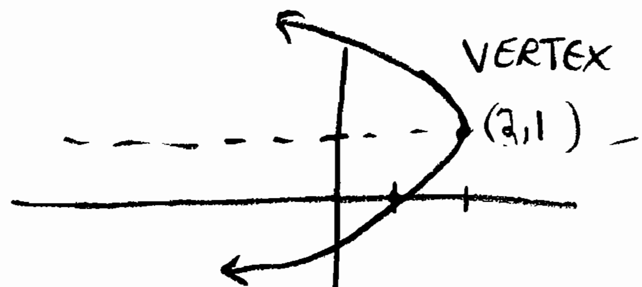
$y = -1$

6. $x = -(y-1)^2 + 2$

$x = -y^2$ GRAPH

UP 1

RIGHT 2



AXIS OF SYMMETRY

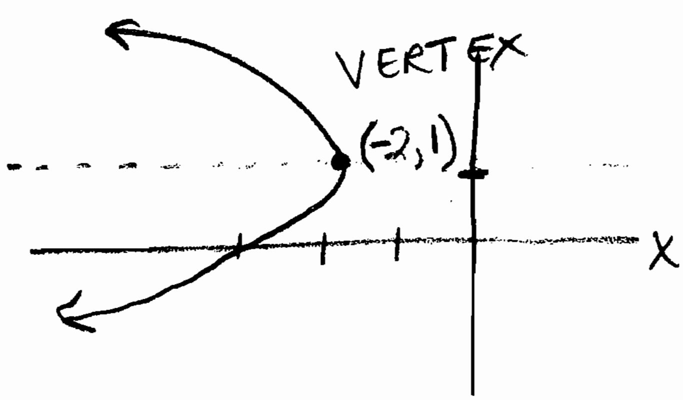
$y = 1$

7. $x = -(y-1)^2 - 2$

$x = -y^2$ GRAPH

UP 1

LEFT 2



AXIS OF SYMMETRY

$y = 1$

11-209 A

$\boxed{\text{HOMEWORK}}$

SKETCH, NAME VERTEX, AND AXIS OF SYMMETRY

A. $y = (x-3)^2 - 2$

B. $y = -(x+7)^2 + 12$

C. $x = (y-1)^2 - 7$

D. $x = -(y+3)^2 - 12$

E. $x = (y+3)^2 + 7$

N. THE EFFECT OF a IN $y = ax^2$

$y = 1x^2$

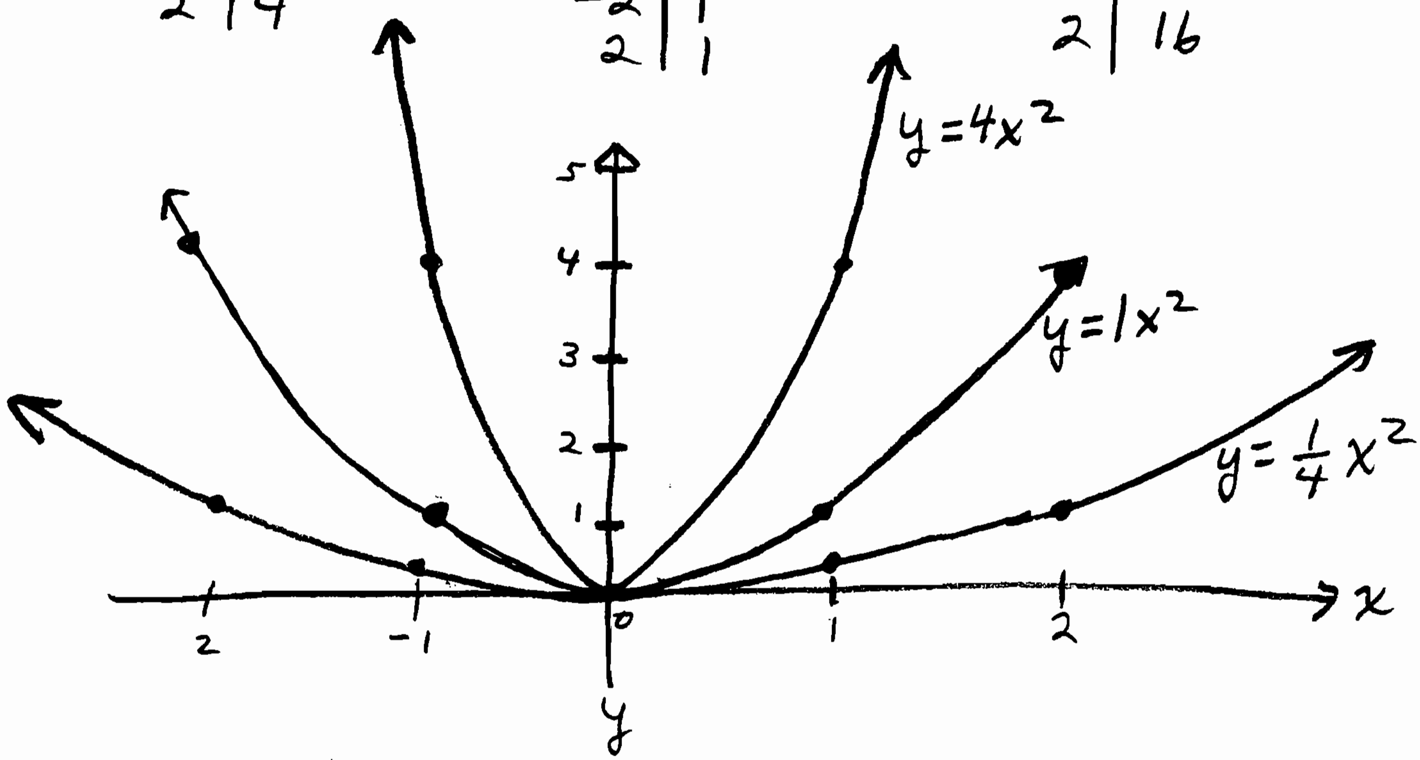
x	y
0	0
−1	1
1	1
−2	4
2	4

$y = \frac{1}{4}x^2$

x	y
0	0
−1	$\frac{1}{4}$
1	$\frac{1}{4}$
−2	1
2	1

$y = 4x^2$

x	y
0	0
−1	4
1	4
−2	16
2	16

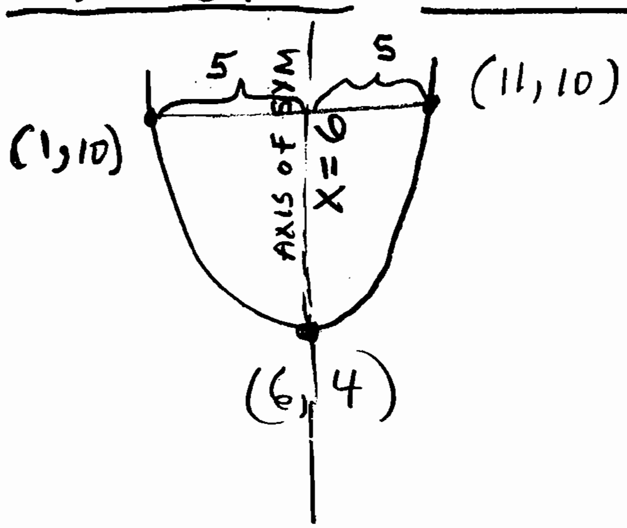


$y = x^2 = 1x^2$ "NORMAL" PARABOLA

$\begin{cases} y = ax^2 & 0<a<1 \\ y = (\text{SMALL POSITIVE})x^2 \end{cases}$ <u>COMPRESSED</u> PARABOLA

$\begin{cases} y = ax^2 & a>1 \\ y = (\text{BIG POSITIVE})x^2 \end{cases}$ <u>STRETCHED</u> PARABOLA

θ. SKETCHING WITH THE AID OF A SYMMETRIC PARTNER



(1,10) AND (11,10) ARE SYMMETRIC PARTNERS

SUPPOSE YOU ARE GIVEN $y = 4x^2 - 16x + 13$ HAS VERTEX (2,−3) FIND 2 PAIRS OF SYMMETRIC PARTNERS

$x = 0;\ y = 4(0^2) - 16(0) + 13$

$y = 13$

(0,13) IS ON GRAPH

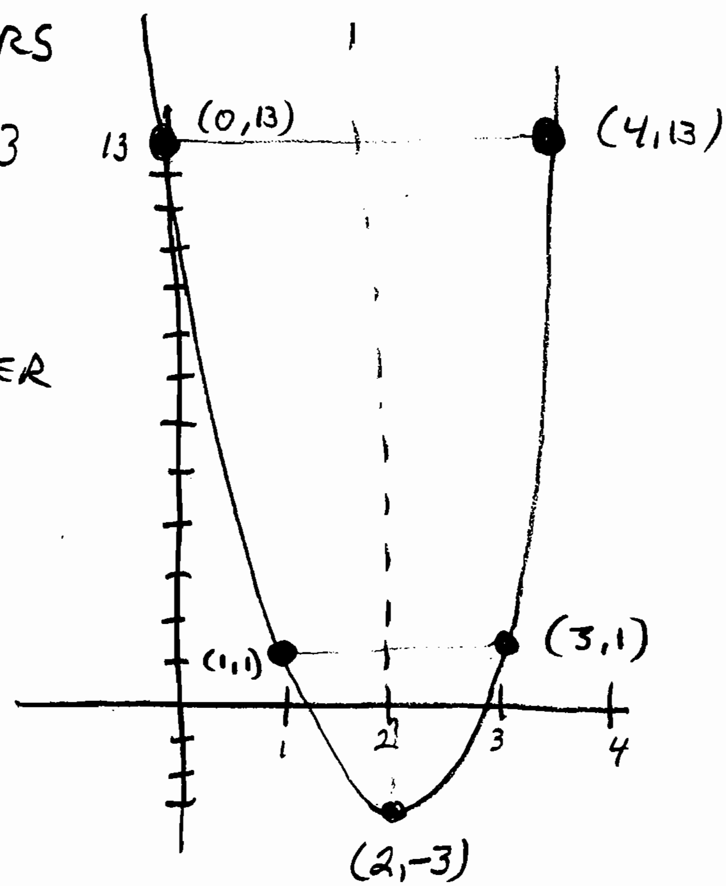
(4,13) SYMMETRIC PARTNER

$x = 1;\ y = 4(1^2) - 16(1) + 13$

$y = 1$

(1,1) IS ON GRAPH

(3,1) SYMMETRIC PARTNER



11-212

PUTTING THINGS TOGETHER

DEFINITION

STANDARD FORM FOR A PARABOLA

$$y = a(x-h)^2 + k \qquad a \neq 0$$

VERTEX (h, k) $\quad a > 0$ OPENS UP

$a < 0$ OPENS DOWN

$$x = a(y-k)^2 + h$$

VERTEX (h, k) $\quad a > 0$ OPENS RIGHT

$a < 0$ OPENS LEFT

1. FIND THE VERTEX, AXIS OF SYMMETRY, A PAIR OF SYMMETRIC PARTNERS, SKETCH

$$y = 4(x-2)^2 - 3$$

THINK

$y = x^2$ OPENS UP

$y = 4x^2$ STRETCH

$y = 4(x-2)^2$ RIGHT 2

$y = 4(x-2)^2 - 3$ DOWN 3

VERTEX $(2, -3)$

AXIS OF SYMMETRY $x = 2$

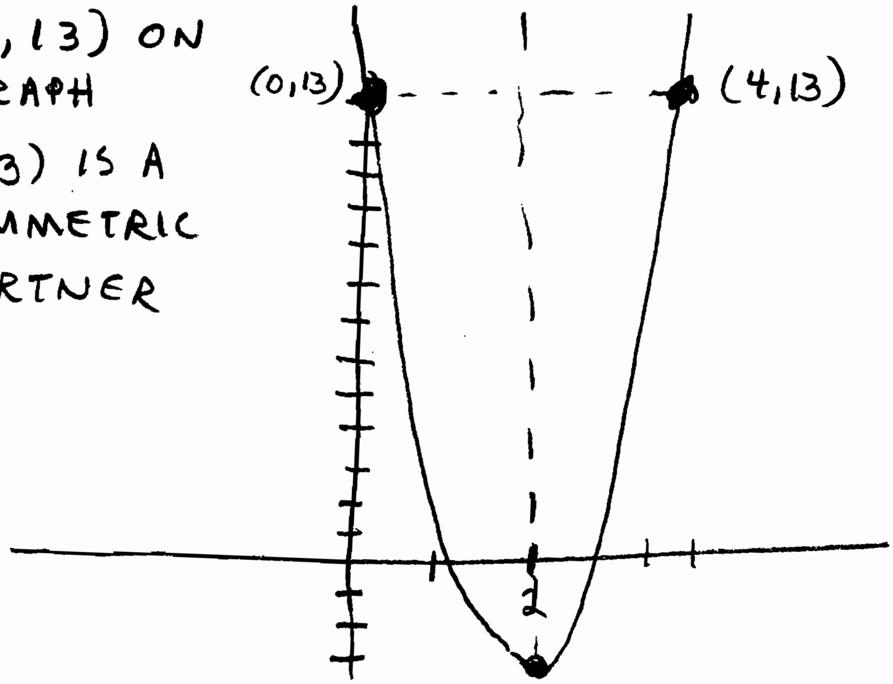
FIND A PAIR OF SYMMETRIC PARTNERS

$x = 0$ (GET AN EASY POINT)

$y = 4(0-2)^2 - 3 = 4(4) - 3 = 13$

(0,13) ON GRAPH

(4,13) IS A SYMMETRIC PARTNER



P. **JUGULAR #10:** COMPLETE THE SQUARE. PUT IN STANDARD FORM FOR A PARABOLA. FIND THE VERTEX, AXIS OF SYMMETRY, A PAIR OF SYMMETRIC PARTNERS, AND SKETCH

DO FOR $y = 3x^2 - 30x + 79$

11-214

$$y = 3x^2 - 30x + 79$$

GET x^2 COEFFICIENT EQUAL TO 1 INSIDE PARENTHESES THAT GROUP THE x-TERMS

LEAVE OUT

$$y = 3(x^2 - 10x \quad\quad) + 79$$

COMPLETE THE SQUARE INSIDE THE PARENTHESES. BE CAREFUL TO MAINTAIN THE EQUALITY

$$y = 3(x^2 - 10x + 25) - 75 + 79$$

NOTE: 25 WAS PUT <u>INSIDE</u> THE PARENTHESES, BUT THE 3 IN FRONT MEANS $3 \cdot 25$ WAS ADDED. SO TO MAINTAIN THE EQUALITY, $3 \cdot 25$, WHICH IS 75 IS TO BE SUBTRACTED.

$$y = 3(x-5)^2 + 4$$ <u>STANDARD</u> <u>FORM</u>

THINK:

$y = x^2$ OPENS UP

$y = 3x^2$ STRETCH

$y = 3(x-5)^2$ RIGHT 5

$y = 3(x-5)^2 + 4$ UP 4

<u>VERTEX</u> $(5, 4)$

AXIS OF SYMMETRY $x = 5$

FIND A PAIR OF SYMMETRIC PARTNERS

$y = 3x^2 - 30x + 79$

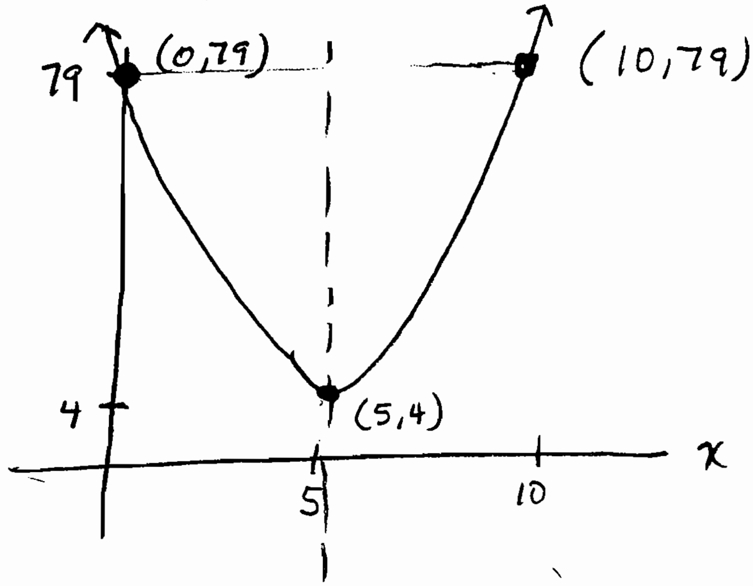
$x = 0$

$y = 3(0^2) - 30(0) + 79$

$y = 79$

$(0, 79)$ ON GRAPH

$(10, 79)$ SYMMETRIC PARTNER



Q. JUGULAR #10 AGAIN FOR $x = -2y^2 - 12y - 17$ COMPLETE THE SQUARE; PUT IN STANDARD FORM FOR A PARABOLA; FIND THE VERTEX, AXIS OF SYMMETRY, A PAIR OF SYMMETRIC PARTNERS, AND SKETCH

GET y^2 COEFFICIENT EQUAL TO 1 INSIDE PARENTHESES THAT GROUP THE y-TERMS

$$x = -2(y^2 + 6y \qquad) - 17$$

CAREFUL

LEAVE OUT OF THE PARENTHESES

COMPLETE THE SQUARE INSIDE THE PARENTHESES. BE CAREFUL TO MAINTAIN THE EQUALITY.

$$x = -2(y^2+6y+9)+18-17$$

$$x = -2(y+3)^2+1$$

THINK: $x=-y^2$ OPENS LEFT

$x=-2y^2$ STRETCH

$x=-2(y+3)^2$ DOWN 3

$x=-2(y+3)^2+1$ RIGHT 1

VERTEX $(1,-3)$ AXIS OF SYM: $y=-3$

SYMM. PARTNERS

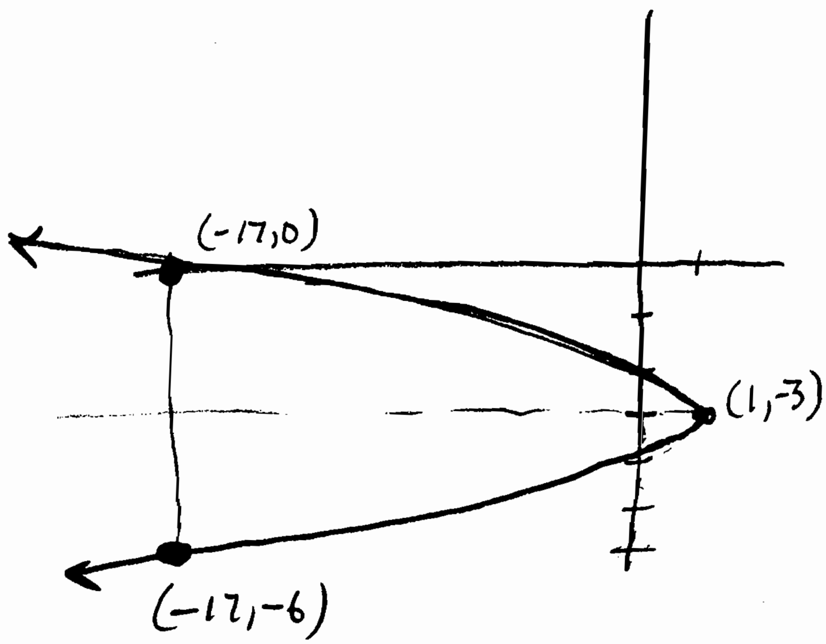
$y=0$

$x=-2(0^2)-12(0)-17$

$x=-17$

$(-17,0)$ ON GRAPH

$(-17,-6)$ SYMM. PARTNER



R. NOTE: $y = ax^2 + bx + c$

$a > 0$ PARABOLA OPENS UP

$a < 0$ PARABOLA OPENS DOWN

$x = ay^2 + by + c$

$a > 0$ PARABOLA OPENS RIGHT

$a < 0$ PARABOLA OPENS LEFT

S. HOMEWORK (OIS) PUT IN STANDARD FORM, FIND THE VERTEX, AXIS OF SYMMETRY, A PAIR OF SYMMETRIC PARTNERS, AND SKETCH

1. $3y - 4x^2 = 0$
2. $12x + 3y^2 = 0$
3. $y = -2(x-1)^2 + 3$
4. $x = -\frac{1}{2}(y+3)^2 + 4$
5. $y = -3x^2 + 12x - 11$
6. $y = \frac{1}{3}x^2 + 2x + 1$
7. $x = 2y^2 - 12y + 17$
8. $x = -\frac{1}{5}y^2 + 2y - 6$

[CHAPTER 12]

FUNCTION BASICS

$\boxed{\text{DEFINITION}}$

A. A <u>FUNCTION</u> IS A SET OF ORDERED PAIRS SUCH THAT NO 2 ORDERED PAIRS HAVE THE SAME FIRST TERM

1. $f = \{(1,3),(2,5),(4,7)\}$ FUNCTION
2. $g = \{(1,3),(2,3),(5,6)\}$ FUNCTION

IT IS OK TO HAVE 2 SECOND TERMS THE SAME

3. $h = \{(1,3),(2,5),(1,3)\} = \{(1,3),(2,5)\}$

YES A FUNCTION. THESE ARE NOT 2 DIFFERENT ORDERED PAIRS.

4. $n = \{(1,5),(2,7),(1,8)\}$ <u>NOT</u> A FUNCTION

5. THE TERMS OF THE ORDERED PAIRS OF A FUNCTION DO NOT HAVE TO BE NUMBERS.

$$s = \{(A,3),(B,\{1,2\}),((2,3),\{5\})\}$$

S <u>IS</u> A FUNCTION.

$\boxed{\text{DEFINITION}}$

B. THE <u>DOMAIN</u> OF A FUNCTION f, DENOTED $\mathrm{dom}(f)$, IS THE <u>SET</u> OF ALL FIRST TERMS OF f. THE <u>RANGE</u> OF f, DENOTED $\mathrm{ran}(f)$, IS THE SET OF ALL SECOND TERMS OF f.

1. $f = \{(1,3),(2,5),(4,7)\}$

   $\mathrm{dom}(f) = \{1,2,4\}$ $\mathrm{ran}(f) = \{3,5,7\}$

2. $g = \{(1,3),(2,3),(5,6)\}$

   $\mathrm{dom}(g) = \{1,2,5\}$ $\mathrm{ran}(g) = \{3,6\}$

3. $w = \{(1,2),(5,2)\}$

   $\mathrm{dom}(w) = \{1,5\}$ $\mathrm{ran}(w) = \{2\}$

   $\mathrm{ran}(w) \neq 2$

4. $s = \{(A,3),(B,\{1,2\}),((2,3),\{5\})\}$

   $\mathrm{dom}(s) = \{A, B, (2,3)\}$ $\mathrm{ran}(s) = \{3, \{1,2\}, \{5\}\}$

(NOT DEF. OF A FUNCTION) 12-220

C. A FUNCTION ASSOCIATES EACH ELEMENT IN THE DOMAIN WITH ONLY ONE ELEMENT IN THE RANGE. FOR FUNCTION f,

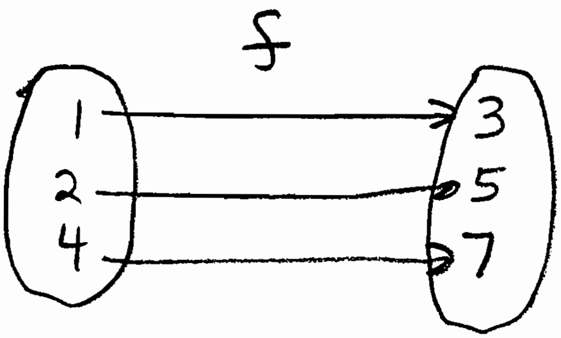
$f(x)$ DEF.

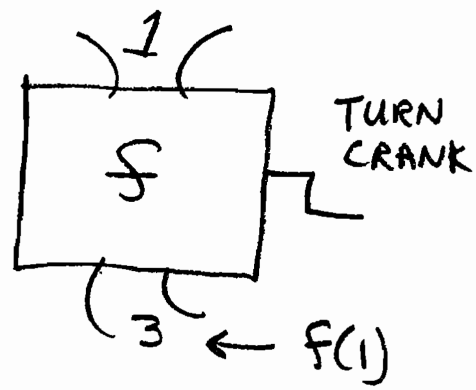
$f(x)$ IS THE SECOND TERM IN THE ORDERED PAIR OF f THAT HAS x AS ITS FIRST TERM. $f(x)$ IS READ "f OF x".

1. $f = \{(1,3), (2,5), (4,7)\}$ FUNCTION

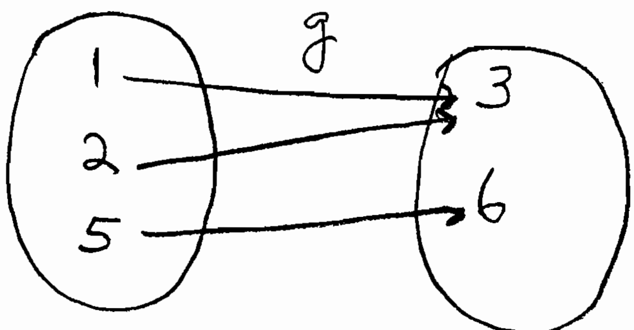
$f(1) = 3$ SAME AS $(1,3) \in f$

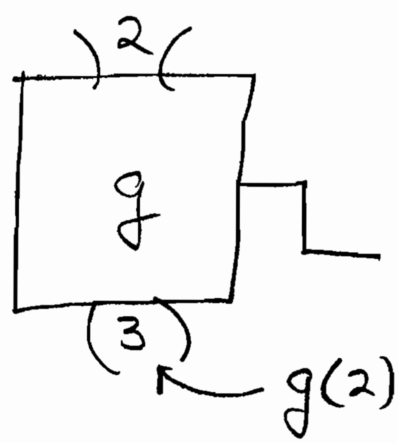
$f(2) = 5$ SAME AS $(2,5) \in f$





2. $g = \{(1,3), (2,3), (5,6)\}$

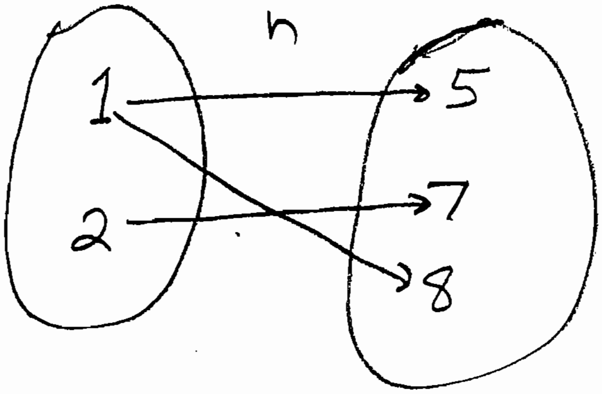


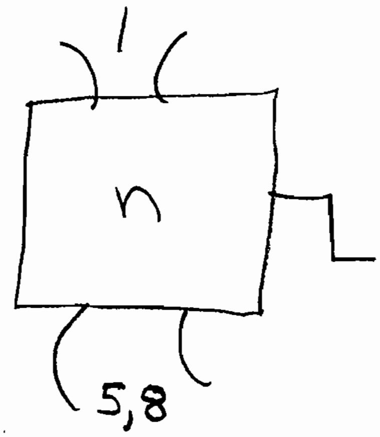


$g(1) = 3$ SAME AS $(1,3) \in g$

$g(2) = 3$ SAME AS $(2,3) \in g$

3. $n = \{(1,5), (2,7), (1,8)\}$ NOT A FUNCTION





1 IS NOT ASSOCIATED WITH ONLY ONE ELEMENT

BAD. TWO COME OUT!

D. EQUATIONS DEFINE FUNCTIONS

2nd Term

1. $y = x^2$ — 1st TERM DEFINES

$$m = \{(x,y) \mid y = x^2\}$$

$(2,4) \in m$ $\quad(-2,4) \in m$ $\quad(-3,9) \in m$ $\quad(3,9) \in m$

$m(2) = 4$ $\quad m(-2) = 4$ $\quad m(-3) = 9$ $\quad m(3) = 9$

$$\boxed{m(x) = x^2}$$

$x \leftarrow$ INDEPENDENT VARIABLE (1st TERMS)

$y \leftarrow$ DEPENDENT VARIABLE (2nd TERMS)

x IS CHOSEN INDEPENDENTLY. y DEPENDS ON WHAT x IS.

EVALUATING $m(x) = x^2$

SUBSTITUTE FOR x ON BOTH SIDES OF THE EQUATION.

$m(-5) = (-5)^2 = 25$

$m(a+b) = (a+b)^2 = a^2 + 2ab + b^2$

$m(-2x+1) = (-2x+1)^2 = 4x^2 - 4x + 1$

2. $y = x^2 + 2x + 1$

x INDEPENDENT VARIABLE (1st TERMS)

y DEPENDENT VARIABLE (2nd TERMS)

$x = 3 \quad y = 3^2 + 2(3) + 1 = 9 + 6 + 1 = 16$

$p = \{(x, y) \mid y = x^2 + 2x + 1\}$

$p(3) = 16$ so $(3, 16) \varepsilon p$

$\boxed{p(x) = x^2 + 2x + 1}$

$p(-2) = (-2)^2 + 2(-2) + 1 = 4 - 4 + 1 = 1$

$p(-2) = 1 \quad (-2, 1) \varepsilon p$

$p(a+b) = (a+b)^2 + 2(a+b) + 1 =$
$a^2 + 2ab + b^2 + 2a + 2b + 1$

$p(x+h) = (x+h)^2 + 2(x+h) + 1 =$
$x^2 + 2xh + h^2 + 2x + 2h + 1$

3. VARIABLES OTHER THAN x AND y IN AN EQUATION TO DEFINE A FUNCTION.

a. $3s + 2t = 7$

DEPENDENT VARIABLE (s) INDEPENDENT VARIABLE (t)

SOLVE FOR DEPENDENT VARIABLE

$$3s = -2t + 7$$

$$s = \frac{-2t+7}{3}$$

$$k = \left\{(t, s) \mid s = -\frac{2t+7}{3}\right\} = \left\{(t, s) \mid 3s + 2t = 7\right\}$$

$$s = k(t) = \frac{-2t+7}{3}$$

$$k(-4) = \frac{-2(-4)+7}{3} = \frac{8+7}{3} = \frac{15}{3} = 5 \quad (-4, 5) \in k$$

b. IN SOME EQUATIONS EITHER VARIABLE CAN BE THE INDEPENDENT VARIABLE.
FOR $3s + 2t = 7$, NOW LET s BE THE INDEPENDENT VARIABLE AND t THE DEPENDENT. SOLVE FOR t

$$2t = 7 - 3s. \qquad t = \frac{7-3s}{2}$$

$$l = \left\{(s, t) \mid t = \frac{7-3s}{2}\right\} = \left\{(s, t) \mid 3s + 2t = 7\right\}$$

$$t = l(s) = \frac{7-3s}{2}$$

$$l(5) = \frac{7-3(5)}{2} = \frac{7-15}{2} = \frac{-8}{2} = -4. \quad (5, -4) \in l$$

E. SOME EQUATIONS DO NOT DEFINE FUNCTIONS. CONSIDER $x^2+y^2=1$

1. TRY ASSOCIATING x WITH 1ST TERMS AND y WITH 2ND TERMS.

(0, 1) SATISFIES $x^2+y^2=1$ SINCE
$0^2+1^2=1$

(0, −1) SATISFIES $x^2+y^2=1$ SINCE
$0^2+(-1)^2=1$

→ 2 ORDERED PAIRS WITH THE SAME 1ST TERM. DOES NOT DEFINE A FUNCTION.

2. TRY ASSOCIATING y WITH 1ST TERMS AND x WITH 2ND TERMS.

$x=1$ AND $y=0$	$x=-1$ AND $y=0$
$1^2+0^2=1$	$(-1)^2+0^2=1$
$(y,x)=(0,1)$	$(y,x)=(0,-1)$
SATISFIES $x^2+y^2=1$	SATISFIES $x^2+y^2=1$

(0, 1) AND (0, −1) : 2 ORDERED PAIRS WITH THE SAME FIRST TERM. DOES NOT DEFINE A FUNCTION

F. FOR SOME EQUATIONS THAT DEFINE A FUNCTION, THE INDEPENDENT VARIABLE CAN ONLY BE CHOSEN ONE WAY

a. WE HAVE SEEN (SECTION D) THAT $y=x^2$ DEFINES A FUNCTION WHEN x IS THE INDEPENDENT VARIABLE (I.E. x IS ASSOCIATED WITH THE FIRST TERMS)

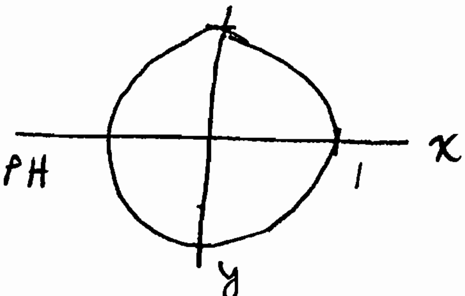
b. TRY ASSOCIATING y WITH 1^{ST} TERMS

$x=2$ AND $y=4$	$x=-2$ AND $y=4$
$4=2^2$ $(y,x)=(4,2)$ SATISFIES $y=x^2$	$4=(-2)^2$ $(y,x)=(4,-2)$ SATISFIES $y=x^2$

$(4,2)$ AND $(4,-2)$: 2 ORDERED PAIRS WITH THE SAME FIRST TERM. DOES NOT DEFINE A FUNCTION.

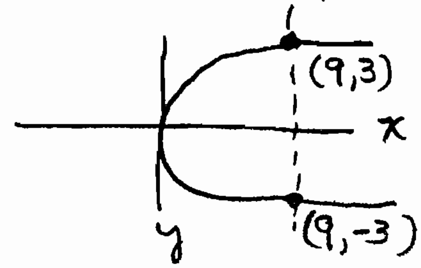
G. HOW TO TELL IF A GRAPH IS THE GRAPH OF A FUNCTION: NO VERTICAL LINE INTERSECTS THE GRAPH TWICE (ASSUMING HORIZONTAL AXIS ASSOCIATED WITH THE FIRST TERMS)

12-226

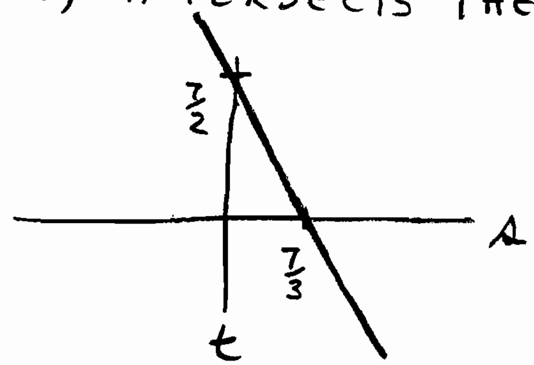
1. $x^2+y^2=1$ 

<u>NOT</u> A FUNCTION GRAPH

A VERTICAL LINE, THE y-AXIS, INTERSECTS THE GRAPH TWICE.

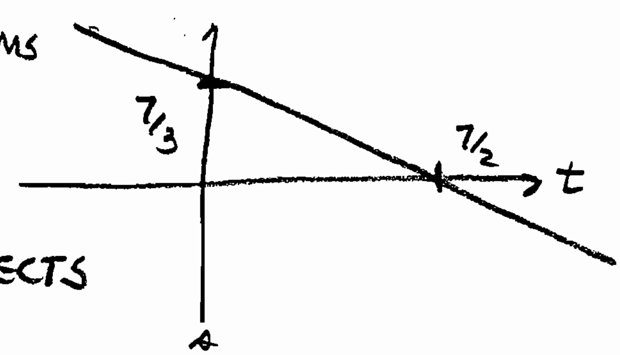
2. $x=y^2$. FIRST TERMS ASSOCIATED WITH x (HORIZONTAL AXIS) 

A VERTICAL LINE, $x=9$, INTERSECTS THE GRAPH TWICE.

3. $3s+2t=7$ 

s	t
0	$7/2$
$7/3$	0

s ASSOCIATED WITH 1ST TERMS (s-AXIS HORIZONTAL). <u>YES</u>, A FUNCTION GRAPH; NO VERTICAL LINE INTERSECTS THE GRAPH TWICE.

t ASSOCIATED WITH 1ST TERMS (t-AXIS HORIZONTAL). <u>YES</u>, A FUNCTION GRAPH; NO VERTICAL LINE INTERSECTS THE GRAPH TWICE. 

12-227

H. HOMEWORK (OIS)

1. DETERMINE WHETHER EACH IS A FUNCTION. FOR THE FUNCTIONS, NAME THE DOMAIN AND RANGE

   a. $f = \{(3,8), (2,8), (5,9)\}$

   b. $g = \{(3,8), (2,8)\}$

   c. $h = \{(8,3), (8,2)\}$

   d. $k = \{(A,B), (\{1\},(2,3)), (1,\{2,3\})\}$

2. LET $f(x) = 3x^2 - 2x + 4$. EVALUATE

   a. $f(5)$ b. $f(-5)$ c. $f(a+b)$

   d. $f(x+h)$ e. $\dfrac{f(x+h) - f(x)}{h}$

3. LET $g(x) = \dfrac{1}{\sqrt{2x+3}}$. EVALUATE

   a. $g(\frac{1}{2})$ b. $g(-1)$ c. $g(a+b)$

   d. $g(x+h)$ e. $\dfrac{g(x+h) - g(x)}{h}$

4. WHICH OF THESE EQUATIONS DEFINE FUNCTIONS. THOSE THAT DO, TELL WHAT CAN BE THE INDEPENDENT VARIABLE AND WHAT CAN BE THE DEPENDENT VARIABLE.

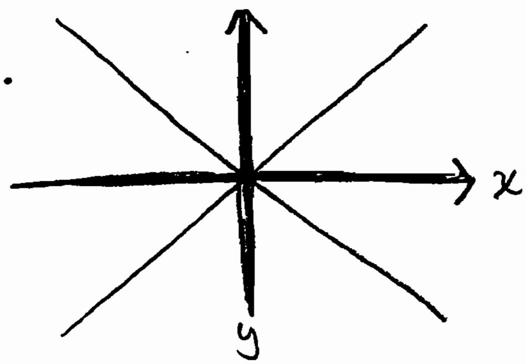
   a. $y = |x|$ b. $x = |y|$ c. $|x| = |y|$

   d. $4p + 3q = 1$ e. $y = x^2 - 6x + 2$

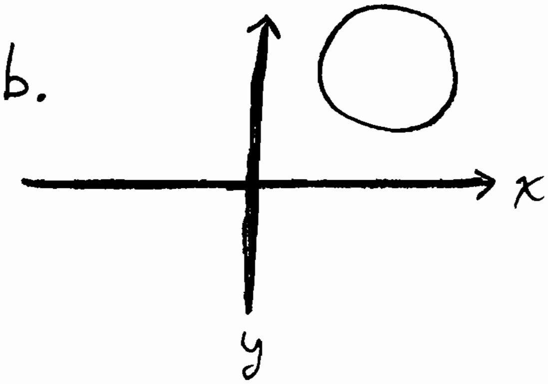
   f. $x^2 + y^2 - 6x + 12y + 45 = 0$

5. WHICH OF THE FOLLOWING ARE GRAPHS OF FUNCTIONS, ASSUMING THE HORIZONTAL AXIS IS ASSOCIATED WITH THE FIRST TERMS?

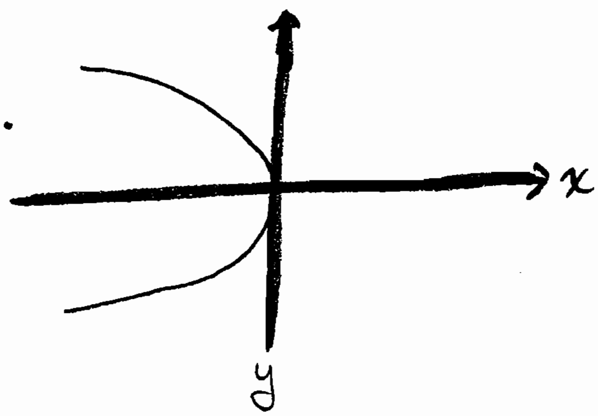
a.



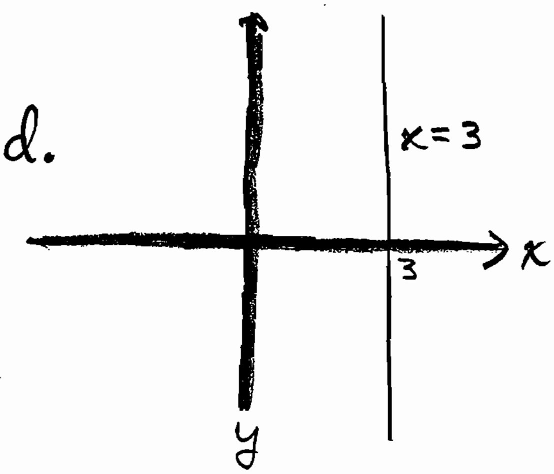
b.



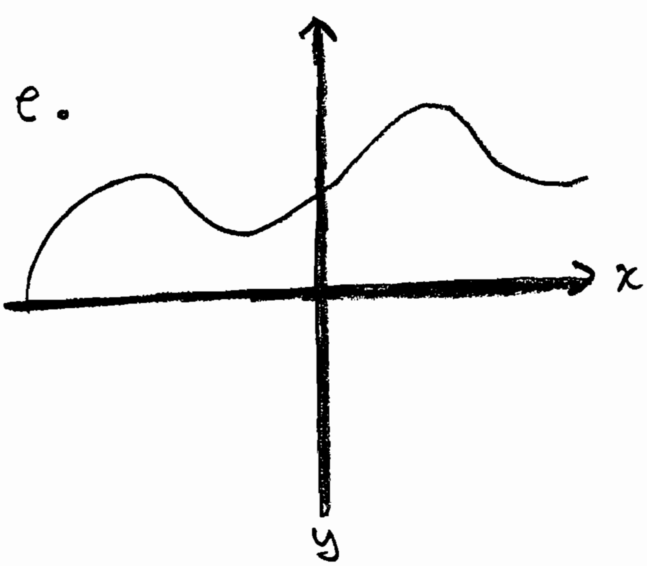
c.



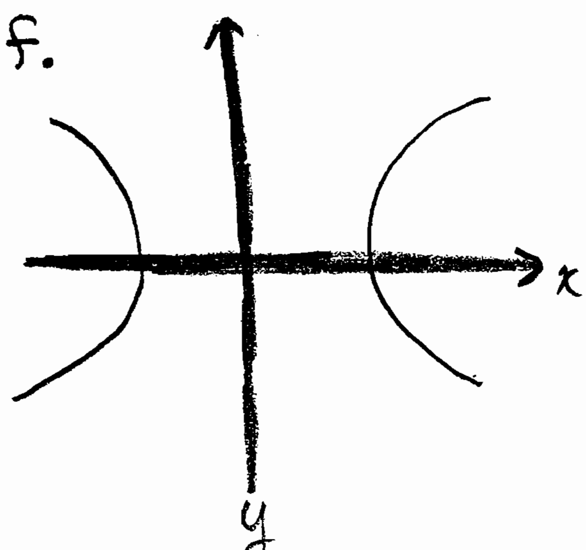
d.



e.



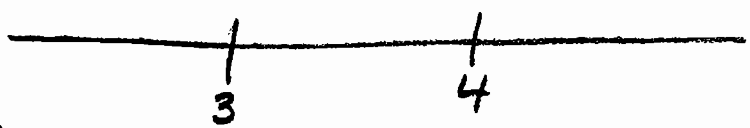
f.



I. THE UNDERSTOOD DOMAIN FOR A FUNCTION DEFINED BY A FORMULA: THE <u>SET</u> OF ALL VALUES FOR WHICH THE INDEPENDENT VARIABLE MAKES SENSE* IN THE EXPRESSION.

1. LET f BE THE FUNCTION DEFINED BY

$$f(x) = \frac{1}{x^2-7x+12} = \frac{1}{(x-3)(x-4)} \qquad \mathrm{dom}(f) = ?$$



$$\mathrm{dom}(f) = \{x \mid x \neq 3 \text{ AND } x \neq 4\}$$
$$= (-\infty, 3) \cup (3, 4) \cup (4, \infty)$$

2. LET g BE THE <u>REAL</u> VALUED FUNCTION DEFINED BY $g(x) = \sqrt{3-2x}$ $\quad \mathrm{dom}(g) = ?$

UNDER RADICAL ≥ 0

$$3 - 2x \geq 0$$
$$3 \geq 2x$$
$$\tfrac{3}{2} \geq x$$
$$x \leq \tfrac{3}{2}$$

$$\mathrm{dom}(g) = \{x \mid x \leq \tfrac{3}{2}\} = (-\infty, \tfrac{3}{2}]$$

* 3. RECALL, FOR THE REALS, DIVISION BY 0 AND $\sqrt{\text{NEGATIVE}}$ DO NOT MAKE SENSE, I.E. ARE UNDEFINED.

J. HOW TO GRAPH A FUNCTION f:
SET $y = f(x)$. GRAPH $y = f(x)$

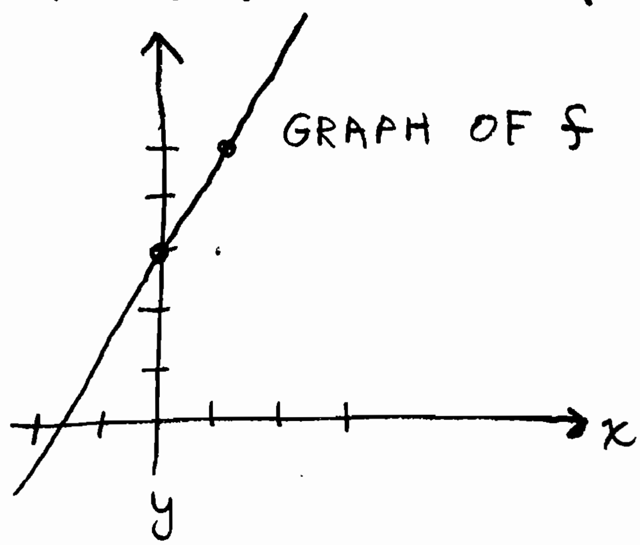
EXAMPLE: LET f BE THE FUNCTION DEFINED BY $f(x) = 2x + 3$. GRAPH IT.

LET $y = f(x)$

$y = 2x + 3$

LINEAR EQUATION

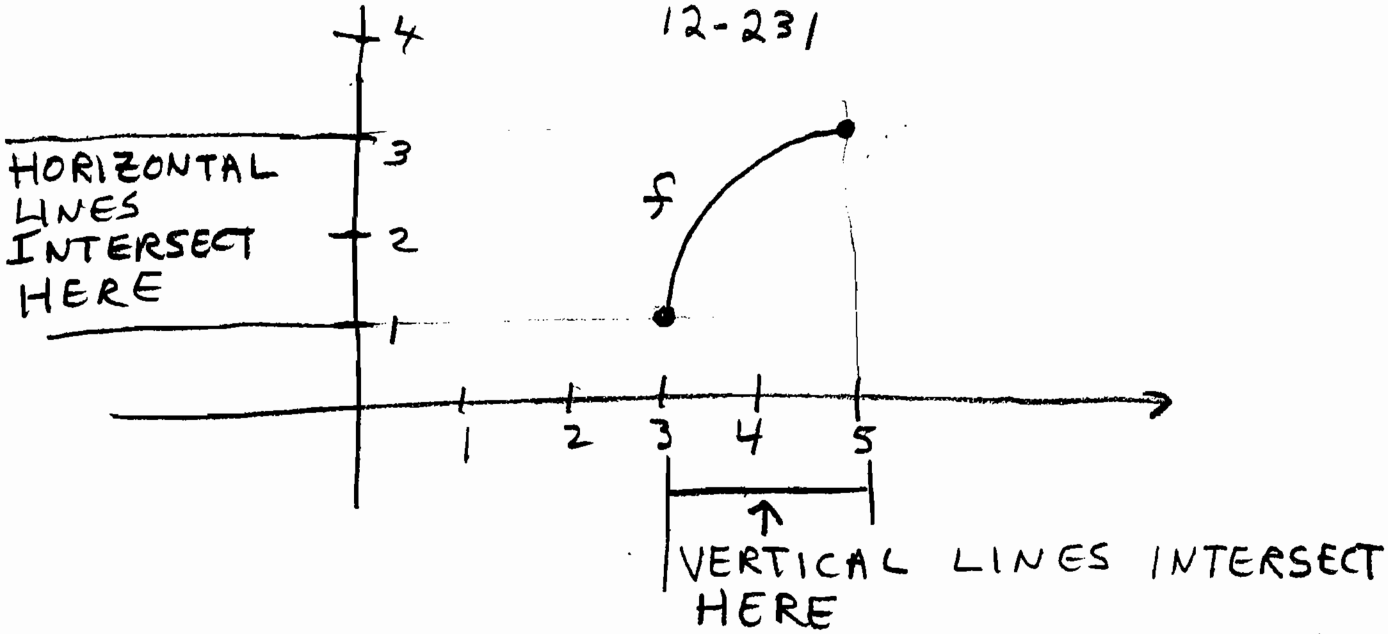
x	y
0	3
1	5



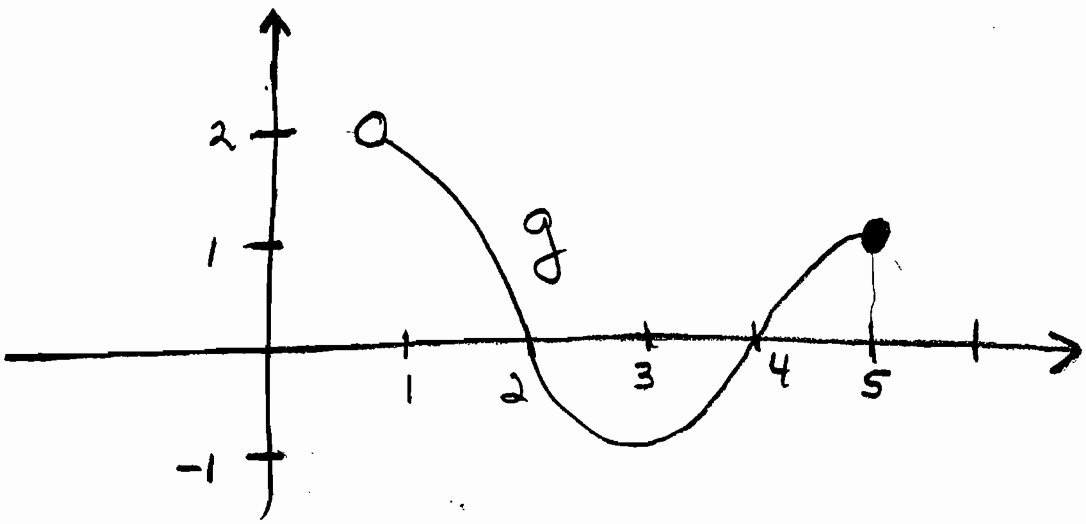
K. HOW TO LOOK AT THE GRAPH OF FUNCTION AND FIND ITS DOMAIN AND RANGE.

1. DOMAIN: THE SET OF ALL x-VALUES WHERE VERTICAL LINES THROUGH THOSE VALUES INTERSECT THE GRAPH

2. RANGE: THE SET OF ALL y-VALUES WHERE HORIZONTAL LINES THROUGH THOSE VALUES INTERSECT THE GRAPH



$\mathrm{dom}(f) = [3,5] \quad \mathrm{ran}(f) = [1,3]$



$\mathrm{dom}(g) = (1,5] \quad \mathrm{ran}(g) = [-1,2)$

L. METHODS FOR FINDING THE <u>RANGE</u> OF A FUNCTION (THIS CAN GET HARD!)

1. GRAPH THE FUNCTION. DETERMINE THE RANGE BY LOOKING AT THE GRAPH

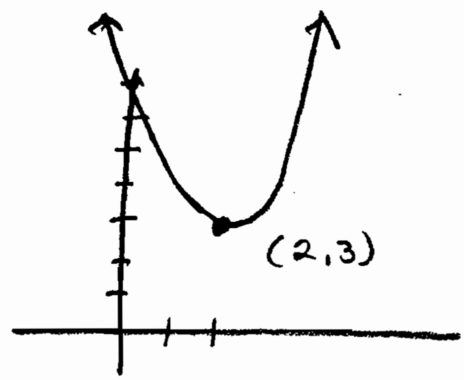
FIND THE RANGE FOR $f(x) = x^2 - 4x + 7$

GRAPH $y = x^2 - 4x + 7$

$$y = (x^2 - 4x \quad) + 7$$

$$y = (x^2 - 4x + 4) - 4 + 7$$

$$y = (x-2)^2 + 3$$



THINK a. $y = x^2$

b. $y = (x-2)^2$

RIGHT 2

c. $y = (x-2)^2 + 3$

UP 3

LOOK AT GRAPH. DETERMINE RANGE

$$\operatorname{ran}(f) = [3, \infty)$$

2. WORK WITH INEQUALITIES, BUILDING UP TO FUNCTION DEFINITION. (TO FIND $\operatorname{ran}(f)$)

FIND RANGE FOR $f(x) = x^2 - 4x + 7$

$$(x-2)^2 \geq 0$$

$$(x-2)^2 + 3 \geq 0 + 3$$

$$x^2 - 4x + 4 + 3 \geq 3$$

$$f(x) = x^2 - 4x + 7 \geq 3$$

$$\operatorname{ran}(f) = [3, \infty)$$

3. SET $y = f(x)$, SOLVE FOR x, FIND ANY RESTRICTIONS ON y. (TO FIND $\operatorname{ran}(f)$).

FIND RANGE FOR $f(x) = \dfrac{2x-1}{3x-2}$

$$y = \frac{2x-1}{3x-2}$$

$$y(3x-2) = 2x-1$$

$$3xy - 2y = 2x - 1$$

$$3xy - 2x = 2y - 1$$

$$x(3y-2) = 2y-1$$

$$x = \frac{2y-1}{3y-2} = \frac{2y-1}{3(y-\frac{2}{3})} \quad \text{SO } y \neq 2/3$$

$$\operatorname{ran}(f) = \{y \mid y \neq \tfrac{2}{3}\} = (-\infty, \tfrac{2}{3}) \cup (\tfrac{2}{3}, \infty)$$

4. USING ALL PREVIOUS MATH KNOWLEDGE YOU HAVE EVER LEARNED IN YOUR LIFE CAN ALSO BE ANOTHER WAY TO FIND $\operatorname{ran}(f)$. SO REALIZE FINDING THE RANGE CAN BE HARD. BE CREATIVE.

$$\operatorname{ran}(f) = \{f(x) \mid x \in \operatorname{dom}(f)\}$$

M. HOMEWORK (OIS)

1. FIND THE DOMAIN FOR EACH FUNCTION DEFINED BELOW.

a. $f(x) = \frac{1}{x-2}$ b. $f(x) = \frac{1}{\sqrt{x}}$

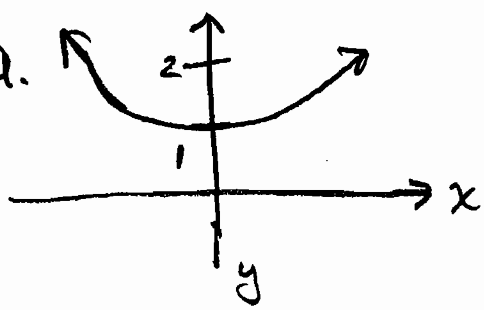
c. $f(x) = \sqrt{5x-1}$ d. $f(x) = \frac{1}{\sqrt{7-6x}}$

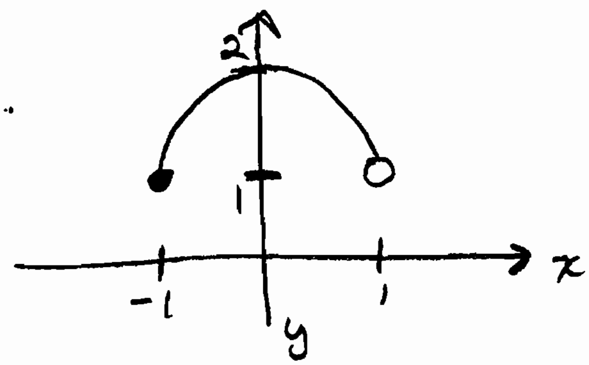
e. $f(x) = \frac{x}{|x|-2}$ f. $f(x) = \frac{3x}{x^2+x-6}$

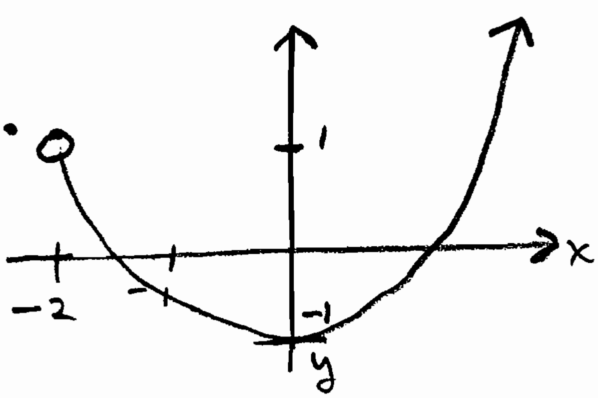
g. $f(x) = \frac{2x+1}{x^2+x-1}$ h. $f(x) = \frac{2x+1}{x^2+x+1}$

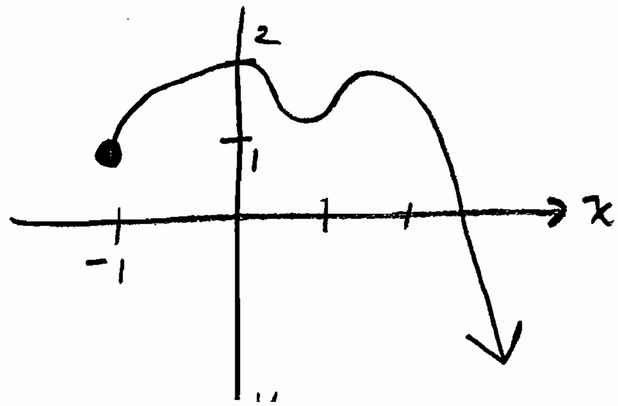
i. $f(x) = \frac{5x}{x^2+1}$

2. LOOK AT EACH GRAPH BELOW AND STATE ITS DOMAIN AND RANGE

a. 

b. 

c. 

d. 

3. FIND THE DOMAIN AND RANGE FOR THE FUNCTIONS BELOW BY FIRST GRAPHING THE FUNCTION AND THEN USING THE GRAPH TO FIND THE DOMAIN AND RANGE

a. $f(x) = 5x+2$ b. $f(x) = x^2+10x+23$

c. $f(x) = -2x^2+3x-7$

4. FIND THE RANGE FOR EACH OF THE FOLLOWING.

a. $f(x) = |x|$ b. $f(x) = -3|x|+2$

c. $f(x) = x^2+4$ d. $f(x) = \frac{3x+1}{2x-4}$

e. $f(x) = \frac{4x-3}{2x+1}$ f. $f(x) = \frac{1}{x^2+4}$

g. $f(x) = \frac{1}{x^2+3}$

N. PIECEWISE DEFINED FUNCTIONS

1. EXAMPLE:

$$f(x) = \begin{cases} 3x+1 & \text{IF } x \geq 2 \\ -2x & \text{IF } x < 2 \end{cases}$$

a. FIND $f(3)$. NOTE $3 \geq 2$, SO USE

$f(x) = 3x+1$

$f(3) = 3(3)+1 = 10$

b. FIND $f(-4)$. NOTE $-4 < 2$ SO USE

$f(x) = -2x$

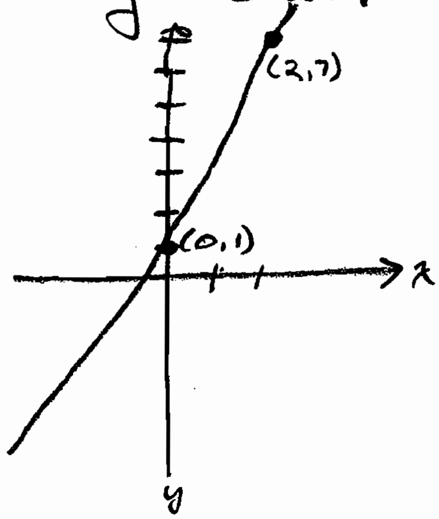
$f(-4) = -2(-4) = 8$

2. TO GRAPH A PIECEWISE DEFINED FUNCTION: (ILLUSTRATE FOR $f(x)$ ABOVE)

a. SET y EQUAL TO EACH PIECE AND GRAPH THESE IN THEIR ENTIRETY

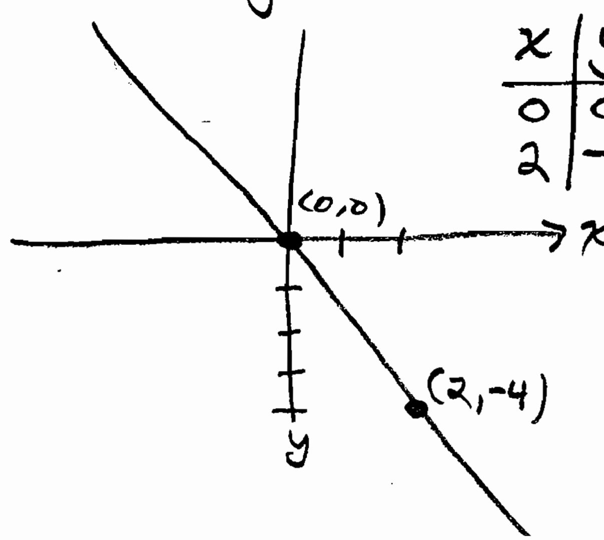
$y = 3x+1$

x	y
0	1
2	7



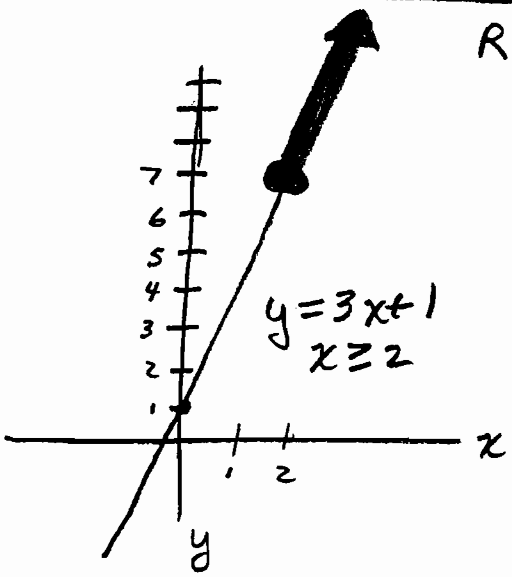
$y = -2x$

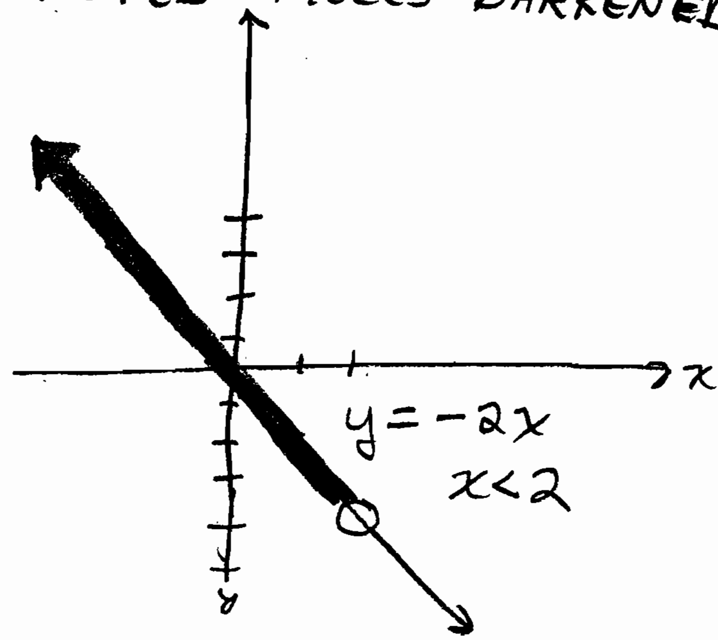
x	y
0	0
2	-4



b. RESTRICT THE GRAPHS ACCORDING TO f(x) DEFINITIONS

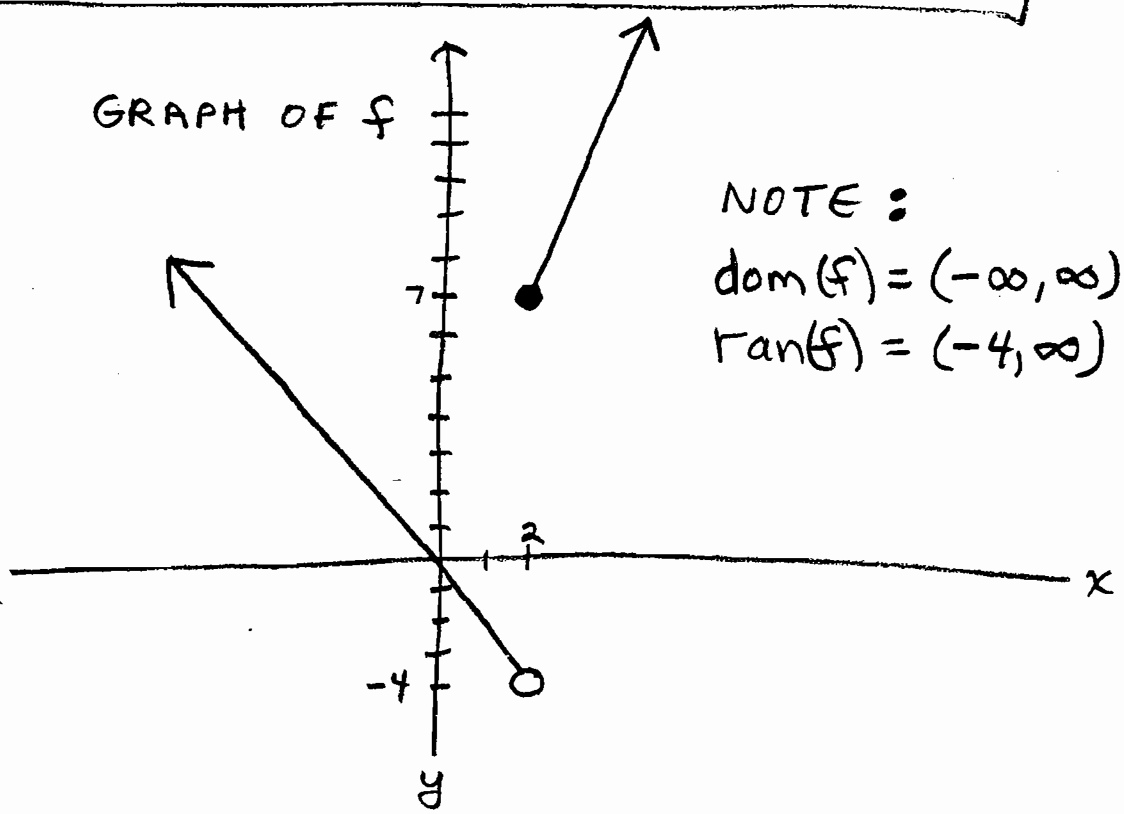
RESTRICTED PIECES DARKENED





c. PUT THE RESTRICTED PIECES ALL ON ONE GRAPH.

GRAPH OF f



NOTE:

$\text{dom}(f) = (-\infty, \infty)$

$\text{ran}(f) = (-4, \infty)$

3. FOR $f(x) = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases}$, FIND $f(3)$, $f(-2)$,

SKETCH, STATE DOMAIN AND RANGE

a. FIND $f(3)$. NOTE $3 \geq 0$, SO USE

$f(x) = x$

$f(3) = 3$

b. FIND $f(-2)$. NOTE $-2 < 0$, SO USE

$f(x) = -x$

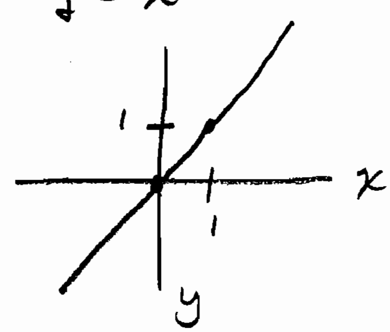
$f(-2) = -(-2) = 2$

c. SKETCH, FIND DOMAIN AND RANGE

SET y EQUAL TO EACH PIECE AND GRAPH THESE IN THEIR ENTIRETY

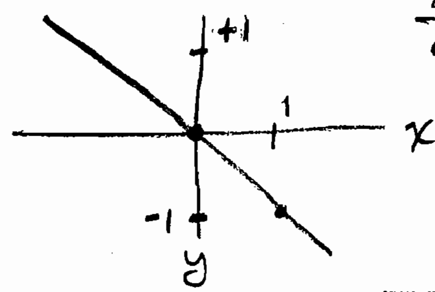
$y = x$

x	y
0	0
1	1



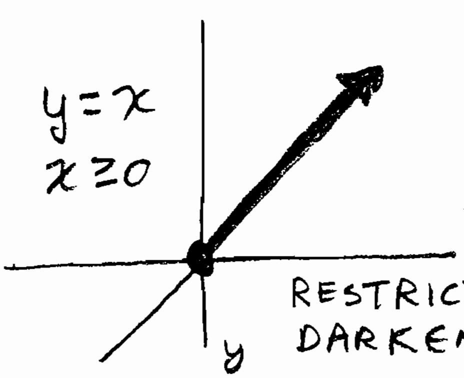
$y = -x$

x	y
0	0
1	-1



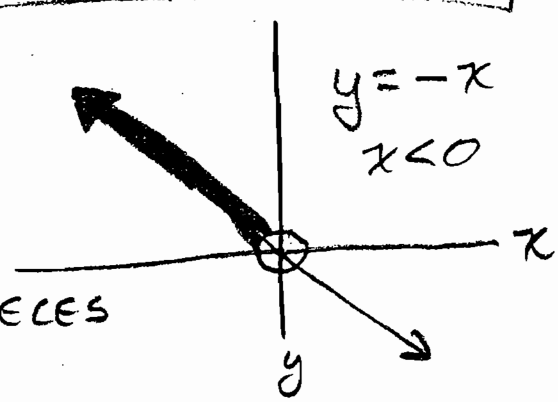
RESTRICT THE GRAPH ACCORDING TO $f(x)$ DEFINITIONS

$y = x$
$x \geq 0$

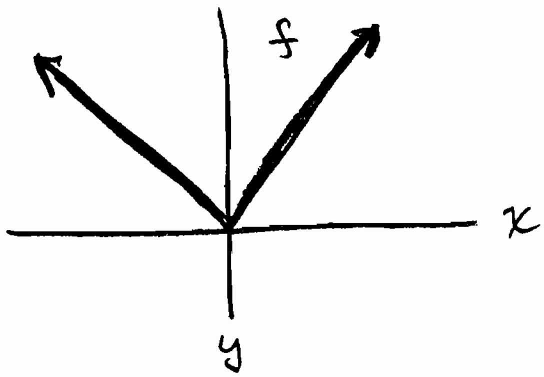


RESTRICTED PIECES DARKENED

$y = -x$
$x < 0$



PUT THE RESTRICTED PIECES ALL ON ONE GRAPH

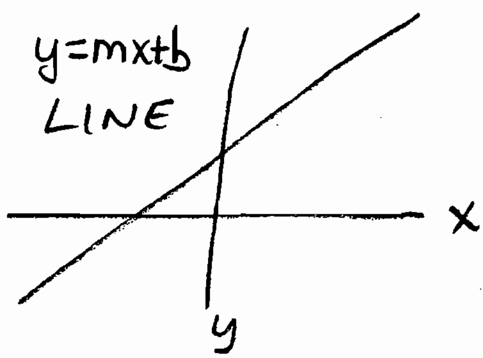


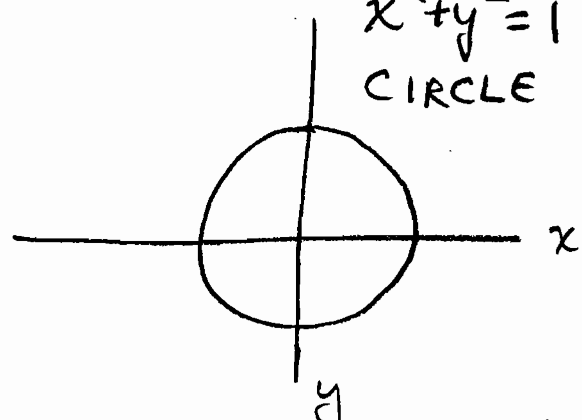
$\text{dom}(f) = (-\infty, \infty)$
$\text{ran}(f) = [0, \infty)$

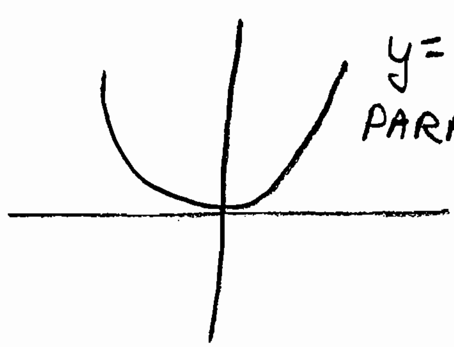
NOTE: $f(x) = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases}$ SHOULD LOOK FAMILIAR...

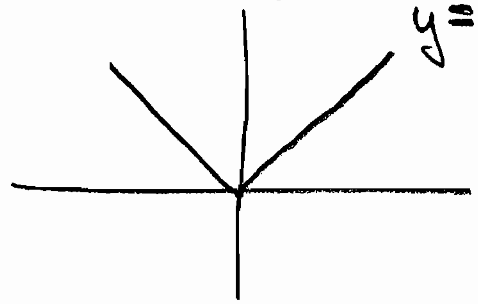
IT DOES: $|x| = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases}$ IT WAS THE ABSOLUTE VALUE FUNCTION IN DISGUISE!

QUICK REVIEW OF THE BASIC GRAPHS WE ARE TO KNOW









Ⓞ HOMEWORK (OIS) FOR EACH OF THE PIECEWISE DEFINED FUNCTIONS BELOW, EVALUATE AS INDICATED, SKETCH, GIVE DOMAIN AND RANGE

1. $f(x) = \begin{cases} 2x & \text{IF } x \geq 2 \\ x^2 & \text{IF } x < 2 \end{cases}$ $\quad f(-1), f(2), f(3)$

2. $f(x) = \begin{cases} x^2+1 & \text{IF } x > 1 \\ 1 & \text{IF } x < 1 \\ 3 & \text{IF } x = 1 \end{cases}$ $\quad f(1), f(2), f(0)$

3. $f(x) = \begin{cases} x^2 - 6x + 11 & \text{IF } x > 3 \\ x - 1 & \text{IF } x \leq 3 \end{cases}$ $\quad f(3), f(4), f(2)$

4. $f(x) = 2x+1 \quad \text{IF } x > 2$
   $= -3x \quad \text{IF } 0 \leq x \leq 2$
   $= -x+1 \quad \text{IF } -3 < x < 0$

   $f(2), f(0), f(-2), f(1), f(3)$

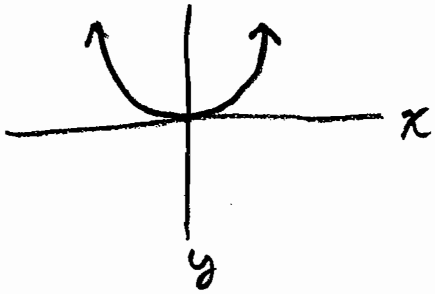
5. $f(x) = \begin{cases} 0 & \text{IF } 0 \leq x < 1 \\ 1 & \text{IF } 1 \leq x < 2 \\ 2 & \text{IF } 2 \leq x < 3 \\ 3 & \text{IF } 3 \leq x < 4 \end{cases}$ $\quad f(1/2), f(3/2), f(5/2), f(7/2)$

P. REFLECTIONS

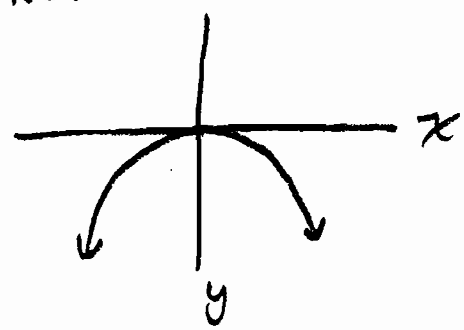
1. ABOUT THE x-AXIS : THE GRAPH OF $y = -f(x)$ IS THE REFLECTION ABOUT THE x-AXIS OF THE GRAPH OF $y = f(x)$

a. LET $f(x) = x^2$

$y = f(x) = x^2$
ORIGINAL



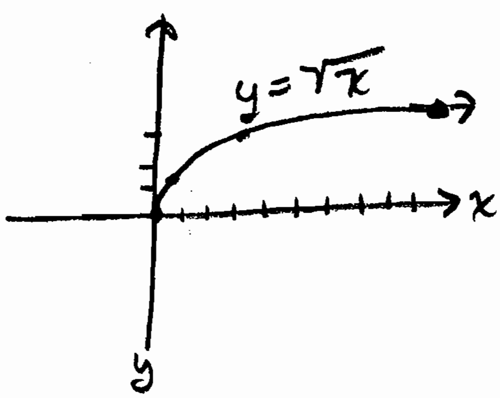
$y = -f(x) = -x^2$
REFLECT x-AXIS



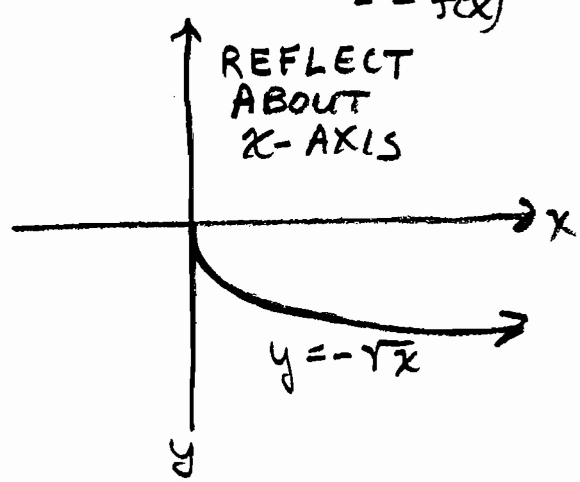
b. GRAPH $y = -\sqrt{x}$ USING REFLECTIONS

1st GRAPH $y = \sqrt{x} = f(x)$

x	y
0	0
1	1
4	2
9	3



2nd GRAPH $y = -\sqrt{x}$
$= -f(x)$

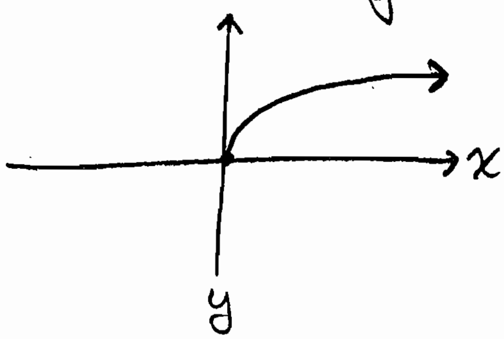


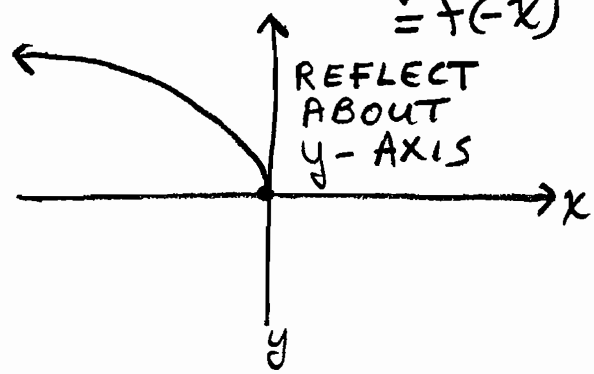
NOTE: ADD THE GRAPH OF $y = \sqrt{x}$ TO YOUR BASIC KNOWLEDGE.

2. ABOUT THE y-AXIS: THE GRAPH OF $y = f(-x)$ IS THE REFLECTION ABOUT THE y-AXIS OF THE GRAPH OF $y = f(x)$

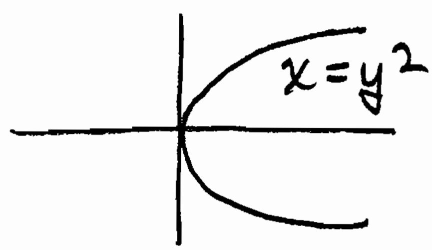
a. GRAPH $y = \sqrt{-x}$ USING REFLECTIONS

1^{st} GRAPH $y = \sqrt{x} = f(x)$ $\quad$ 2^{nd} GRAPH $y = \sqrt{-x} = f(-x)$





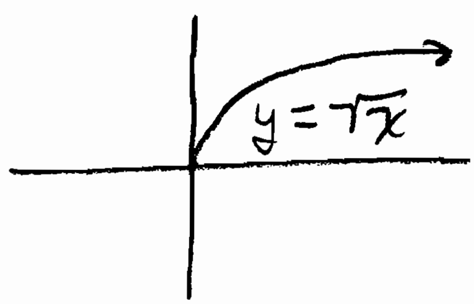
NOTE: THE GRAPH OF $y = \sqrt{x}$ IS JUST THE TOP HALF OF THE GRAPH OF $x = y^2$.



REASON: THE TOP HALF OF $x = y^2$ GRAPH HAS $y \geq 0$. SOLVE $x = y^2$ FOR y

$$\sqrt{x} = \sqrt{y^2} = |y| = y$$

$$y = \sqrt{x}$$

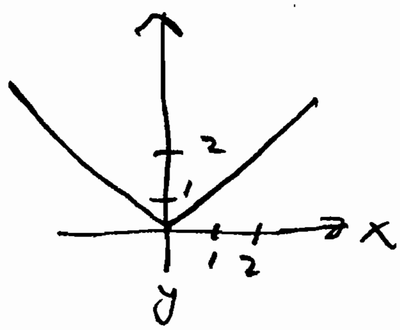


Q. TRANSLATIONS

1. TRANSLATE UP : $(K>0)$ THE GRAPH OF $y=f(x)+K$ IS A TRANSLATION UP K UNITS OF THE GRAPH OF $y=f(x)$

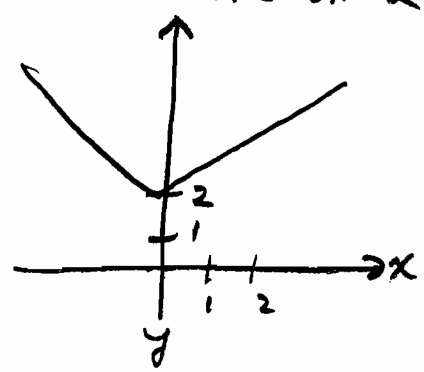
EXAMPLE : SKETCH THE GRAPH OF $y=|x|+2$

GRAPH OF $y=|x|=f(x)$



GRAPH OF $y=|x|+2=f(x)+2$

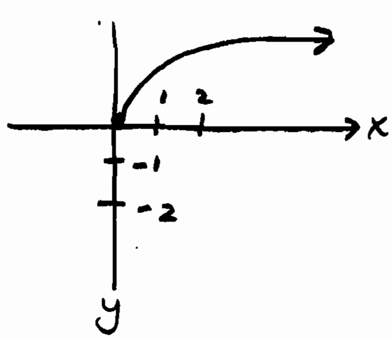
TRANSLATE UP 2



2. TRANSLATE DOWN : $(K>0)$ THE GRAPH OF $y=f(x)-K$ IS A TRANSLATION DOWN K UNITS OF THE GRAPH OF $y=f(x)$

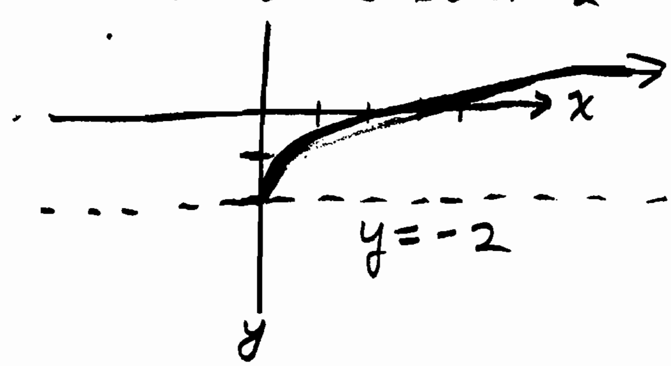
EXAMPLE : SKETCH GRAPH OF $y=\sqrt{x}-2$

GRAPH OF $y=\sqrt{x}=f(x)$



GRAPH OF $y=\sqrt{x}-2=f(x)-2$

TRANSLATE DOWN 2



3. TRANSLATE RIGHT: $(h>0)$ THE GRAPH OF $y=f(x-h)$ IS A TRANSLATION RIGHT h UNITS OF THE GRAPH OF $y=f(x)$.

a. FIRST, RECOGNIZING $f(x-h)$

i. LET $f(x)=2x^2-3x+4$

$$f(x-h)=2(x-h)^2-3(x-h)+4$$
$$f(x-5)=2(x-5)^2-3(x-5)+4$$

ii. LET $f(x)=|x|+3\sqrt{x}+7$

$$f(x-3)=|x-3|+3\sqrt{x-3}+7$$

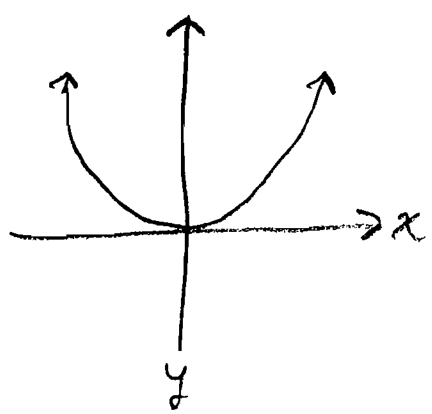
iii. LET $h(x)=5(x-1)^3-2\sqrt{x-1}+2(x-1)$

FIND $f(x)$ SO THAT $h(x)=f(x-1)$

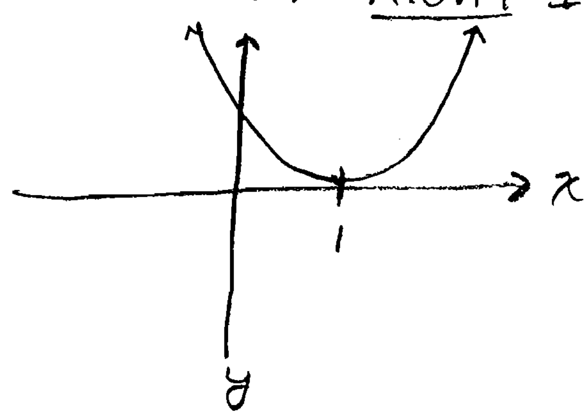
$$f(x)=5x^3-2\sqrt{x}+2x$$

b. EXAMPLE: SKETCH GRAPH OF $y=(x-1)^2$

GRAPH OF
$y=x^2=f(x)$



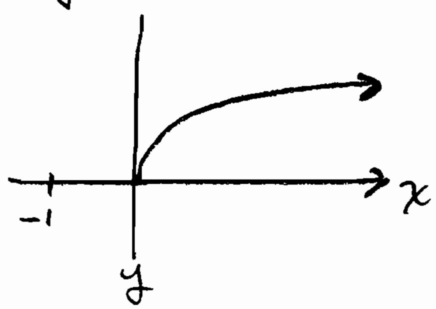
GRAPH OF
$y=(x-1)^2=f(x-1)$
TRANSLATE RIGHT 1



4. TRANSLATE LEFT: $(h>0)$ THE GRAPH OF $y=f(x+h)$ IS A TRANSLATION LEFT h UNITS OF THE GRAPH OF $y=f(x)$

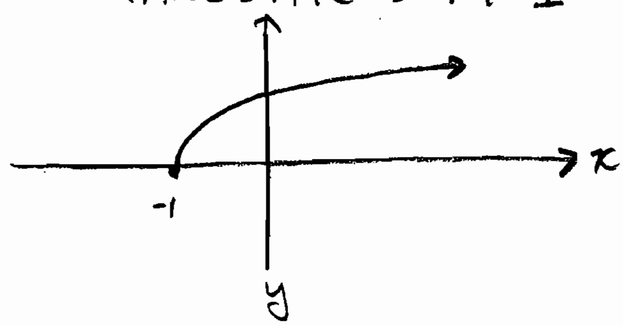
EXAMPLE: SKETCH GRAPH OF $y=\sqrt{x+1}$

GRAPH OF $y=\sqrt{x}=f(x)$



GRAPH OF $y=\sqrt{x+1}=f(x+1)$

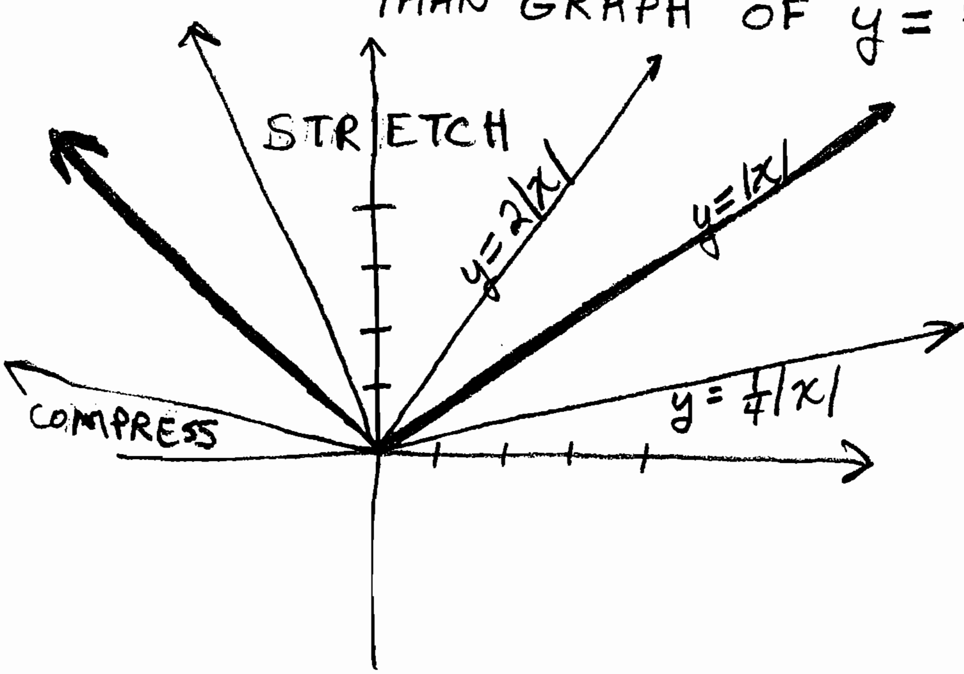
TRANSLATE LEFT 1



R. NARROW (STRETCH) / FLAT (COMPRESS)

$a>1$ GRAPH OF $y=af(x)$ STRETCH THAN GRAPH OF $y=f(x)$

$0<a<1$ GRAPH OF $y=af(x)$ COMPRESS THAN GRAPH OF $y=f(x)$

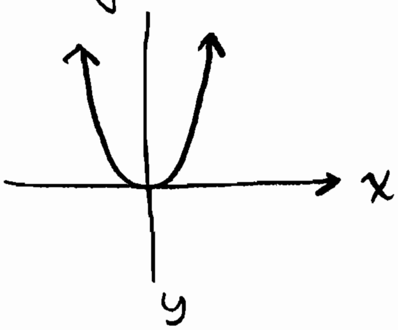


S. BY MEANS OF A SEQUENCE OF REFLECTIONS, TRANSLATIONS, STRETCH/COMPRESS, SKETCH THE GRAPHS.

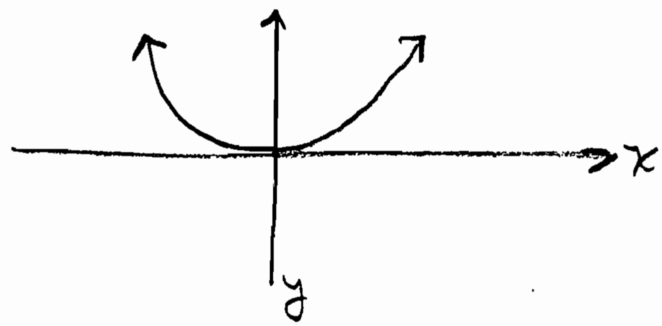
1. $y = \frac{1}{2}(x-1)^2 - 3$

THINK OF THE SEQUENCE: $y = x^2$, $y = \frac{1}{2}x^2$, $y = \frac{1}{2}(x-1)^2$, $y = \frac{1}{2}(x-1)^2 - 3$

$y = x^2$

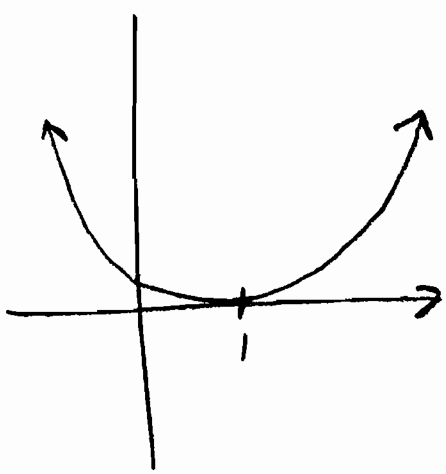


$y = \frac{1}{2}x^2$ COMPRESS



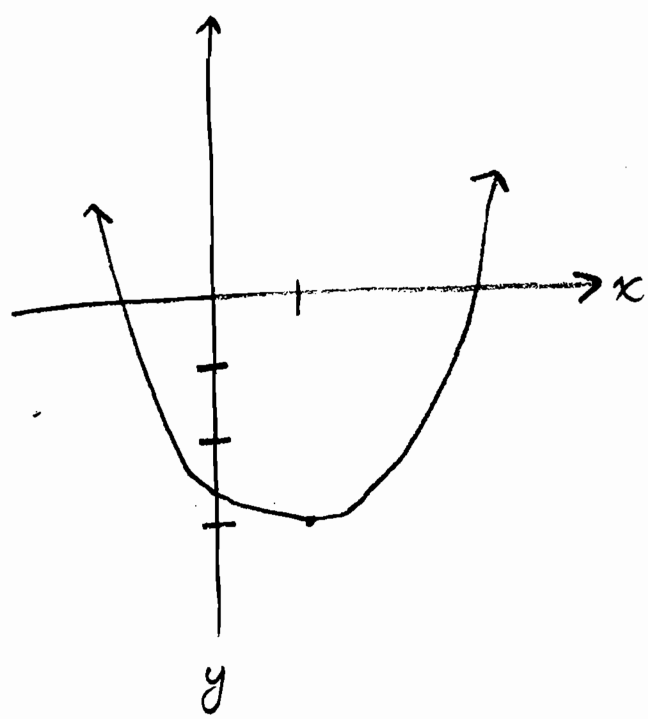
$y = \frac{1}{2}(x-1)^2$

RIGHT 1



$y = \frac{1}{2}(x-1)^2 - 3$

DOWN 3

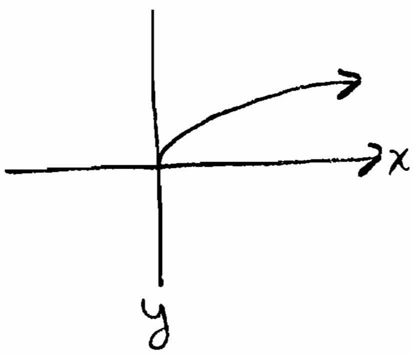


12-247

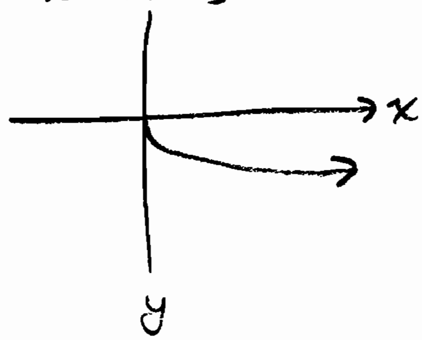
2. $y = -2\sqrt{x+1} + 3$

THINK OF THE SEQUENCE: $y = \sqrt{x}$, $y = -\sqrt{x}$, $y = -2\sqrt{x}$, $y = -2\sqrt{x+1}$, $y = -2\sqrt{x+1} + 3$.

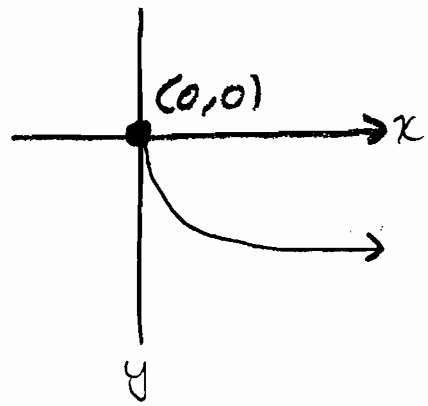
GRAPH $y = \sqrt{x}$



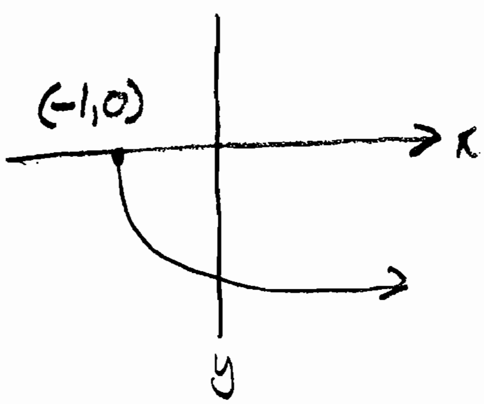
GRAPH $y = -\sqrt{x}$ REFLECT x- AXIS



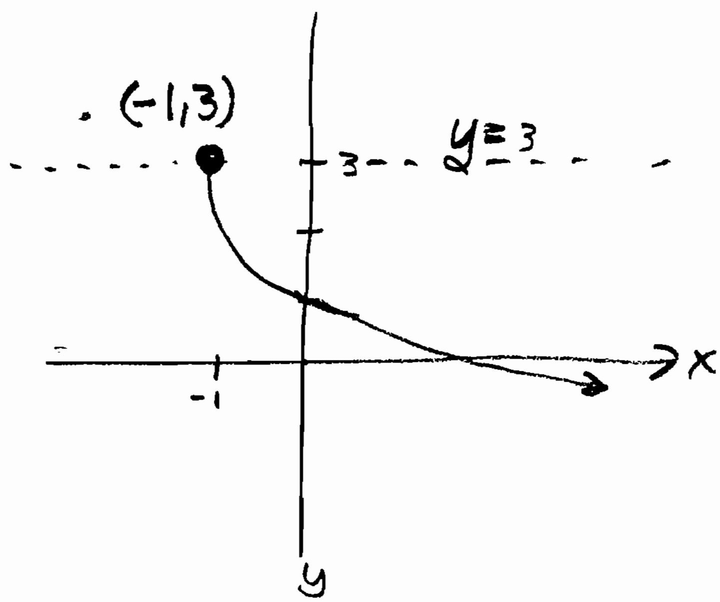
GRAPH $y = -2\sqrt{x}$ STRETCH



GRAPH $y = -2\sqrt{x+1}$ LEFT 1



GRAPH $y = -2\sqrt{x+1} + 3$ UP 3



T. HOMEWORK (OIS) FOR EACH OF THE FOLLOWING, BY MEANS OF A SEQUENCE OF REFLECTIONS, TRANSLATIONS, STRETCH / COMPRESS, SKETCH THE GRAPHS.

1. $y = -|x|$
2. $y = -3|x|$
3. $y = -3|x-2|$
4. $y = -3|x-2| + 1$
5. $y = -2(x+1)^2 + 3$
6. $y = \frac{1}{2}|x+3| - 4$
7. $y = \frac{1}{3}\sqrt{x-2} - 1$
8. $y = -2x^2 + 3x + 5$
9. $y = \frac{1}{2}x^2 - 2x + 3$

U. RATIONAL FUNCTION GRAPHS

1. RATIONAL FUNCTION $= \dfrac{\text{POLYNOMIAL}}{\text{POLYNOMIAL}}$

2. EXAMPLES OF RATIONAL FUNCTIONS

   a. $f(x) = \dfrac{6x-7}{3x-3}$ b. $f(x) = \dfrac{2x^2-3x+6}{5x^4-7x+1}$

   c. $f(x) = \dfrac{1}{x}$ d. $f(x) = 3x^2+7 = \dfrac{3x^2+7}{1}$

3. EXAMPLES OF <u>NOT</u> RATIONAL FUNCTIONS

   a. $n(x) = \dfrac{\sqrt{2x-1}}{3x+2}$ b. $n(x) = \dfrac{5x^2-7x^{1/3}+2}{3x^{1/5}-2}$

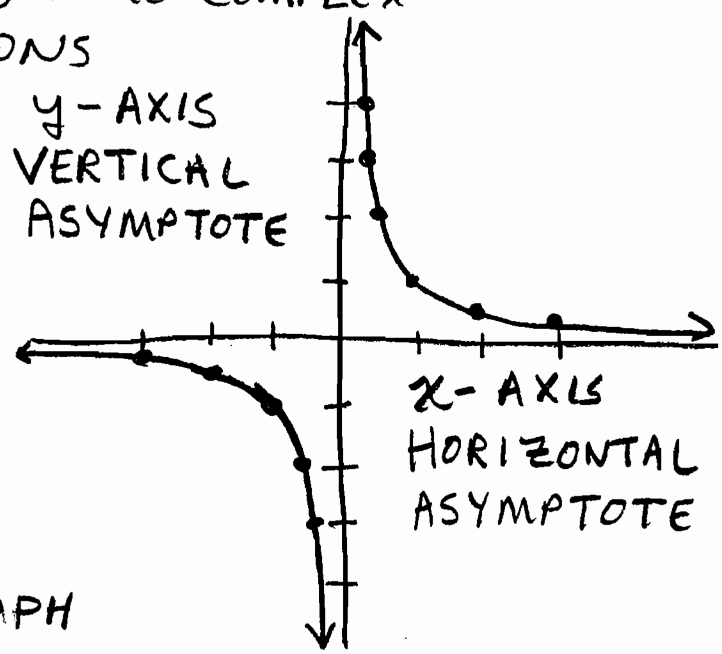
4. NOTE: POLYNOMIALS ARE RATIONAL FUNCTIONS

   POLYNOMIAL $= \dfrac{\text{POLYNOMIAL}}{1} = \dfrac{\text{POLYNOMIAL}}{\text{POLYNOMIAL}}$

5. THE GRAPH OF $y = \frac{1}{x}$ AS A BUILDING BLOCK TO GRAPHING MORE COMPLEX RATIONAL FUNCTIONS

x	y
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
$\frac{1}{2}$	$\frac{1}{1/2} = 2$
$\frac{1}{3}$	3
$\frac{1}{4}$	4

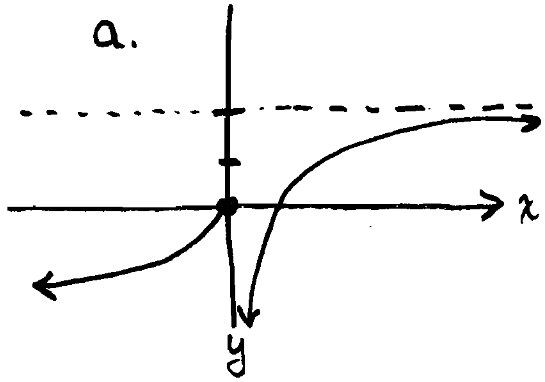
x	y
-1	-1
-2	$-\frac{1}{2}$
-3	$-\frac{1}{3}$
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3



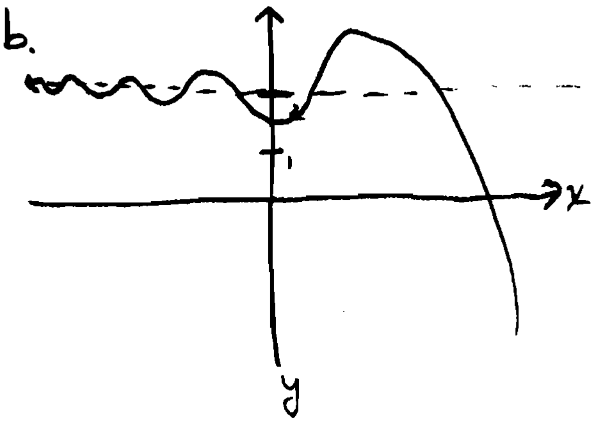
KNOW THIS GRAPH

6. $y=2$ IS A HORIZONTAL ASYMPTOTE IN EACH OF THE FOLLOWING GRAPHS

a.

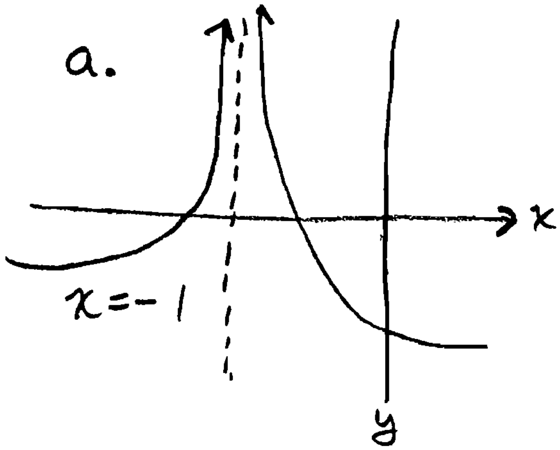


b.

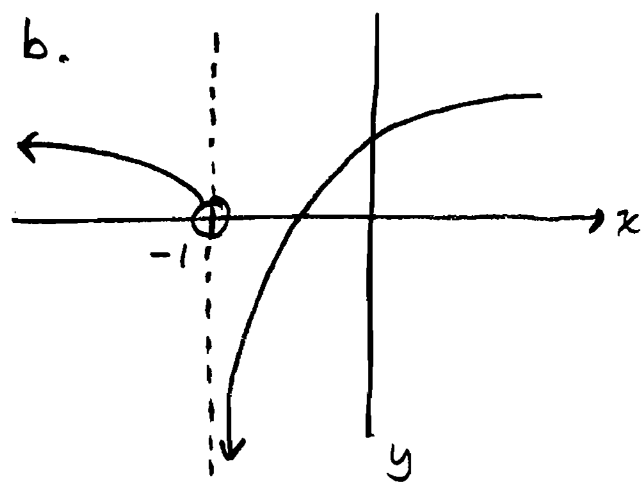


7. $x=-1$ IS A VERTICAL ASYMPTOTE IN EACH OF THE FOLLOWING GRAPHS

a.



b.



8. GRAPH $y = \dfrac{6x-7}{3x-3}$

AS A SEQUENCE OF REFLECTIONS, TRANSLATIONS, STRETCH/COMPRESS. NAME HORIZONTAL AND VERTICAL ASYMPTOTES. NAME THE x AND y INTERCEPTS.

12-251

a. LONG DIVISION

$$
\begin{array}{r}
2 \\
3x-3 \,\overline{\big)\, 6x-7} \\
-(6x-6) \\
\hline
-1
\end{array}
$$

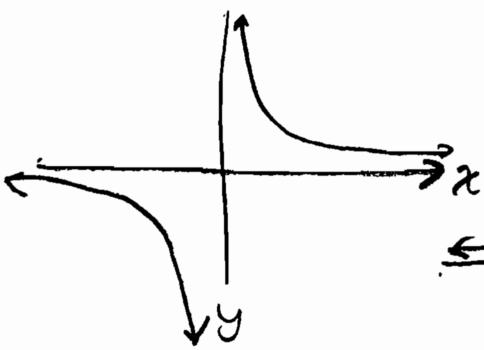
$$y = \frac{6x-7}{3x-3} = 2 + \frac{-1}{3x-3}$$

$$= \frac{-1}{3(x-1)} + 2$$

$$y = -\frac{1}{3}\left(\frac{1}{x-1}\right) + 2$$

b. SKETCHING SEQUENCE $y = \frac{1}{x}$, $y = -\frac{1}{x}$, $y = -\frac{1}{3}\cdot\frac{1}{x}$, $y = \left(-\frac{1}{3}\right)\frac{1}{(x-1)}$, $y = -\frac{1}{3}\left(\frac{1}{x-1}\right)+2$

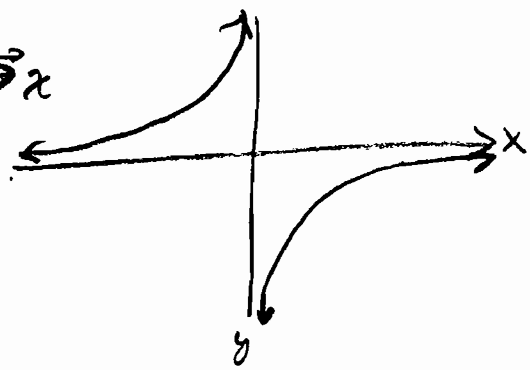
$y = \frac{1}{x}$



$y = -\frac{1}{x}$

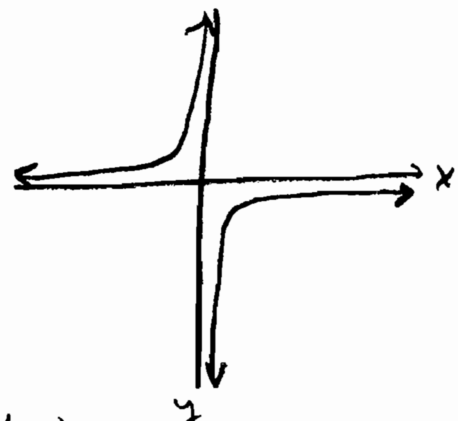
REFLECT

x- AXIS



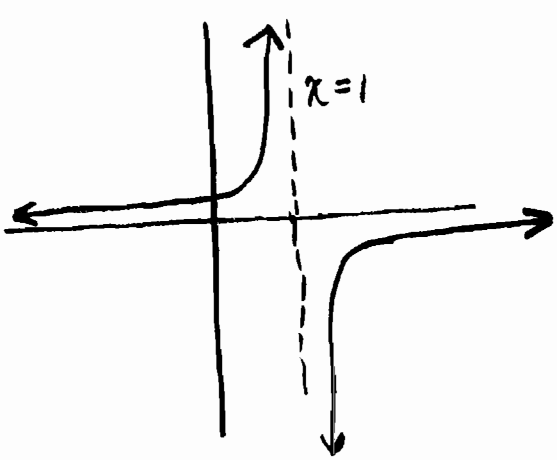
$y = -\frac{1}{3}\left(\frac{1}{x}\right)$

COMPRESS



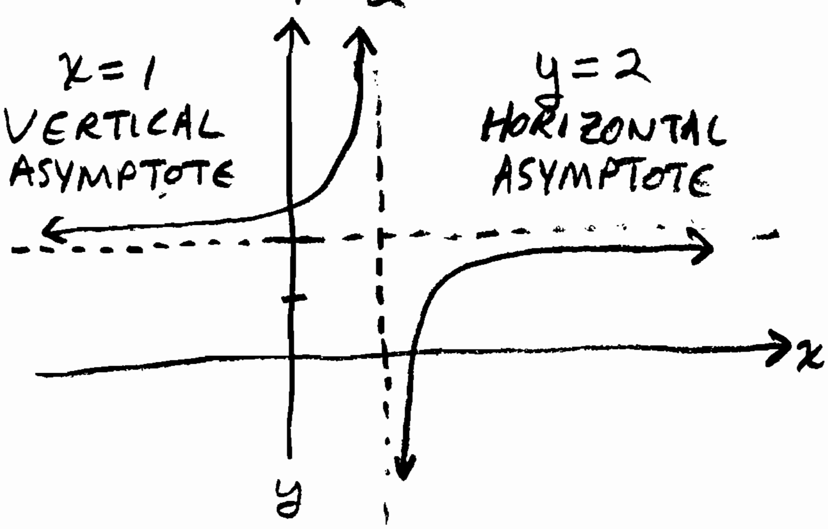
$y = \left(-\frac{1}{3}\right)\left(\frac{1}{x-1}\right)$

RIGHT 1



$y = -\frac{1}{3}\left(\frac{1}{x-1}\right)+2$

UP 2



c. NAME HORIZONTAL AND VERTICAL ASYMPTOTES

$y=2$ HORIZONTAL ASYMPTOTE

$x=1$ VERTICAL ASYMPTOTE

d. NAME x AND y INTERCEPTS.

FOR x-INTERCEPT SET $y=0$ IN $y=\frac{6x-7}{3x-3}$

$$0=\frac{6x-7}{3x-3}$$

$$6x-7=0$$
$$6x=7$$
$$x=7/6$$

$\left(\frac{7}{6},0\right)$ x-INTERCEPT

FOR y-INTERCEPT SET $x=0$ IN $y=\frac{6x-7}{3x-3}$

$$y=\frac{6(0)-7}{3(0)-3}=\frac{-7}{-3}=\frac{7}{3}$$

$\left(0,\frac{7}{3}\right)$ y-INTERCEPT

V. SYMMETRY

1. ABOUT THE y-AXIS

a. <u>FOR $f(x)$ NOTATION</u>

GRAPH OF f IS SYMMETRIC ABOUT THE y-AXIS IFF $f(x)=f(-x)$ (<u>DEF.</u>)

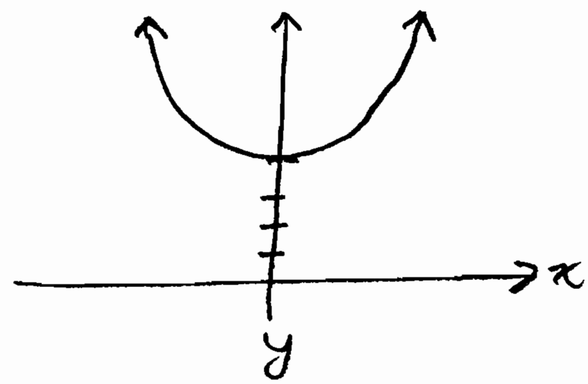
f IS CALLED AN <u>EVEN</u> FUNCTION

EXAMPLE: $f(x) = x^2 + 4$

$f(-x) = (-x)^2 + 4$

$= x^2 + 4$

$= f(x)$



SYMMETRIC ABOUT y-AXIS.

f IS AN <u>EVEN</u> FUNCTION

b. <u>SYMMETRY ABOUT y-AXIS</u> FOR <u>GRAPHS OF EQUATIONS</u>

IF YOU GET AN EQUIVALENT EQUATION BY SUBSTITUTING $-x$ FOR x, THE GRAPH IS SYMMETRIC ABOUT THE y-AXIS.

i. CONSIDER $y = x^2 + 4$

SUBSTITUTE $-x$ FOR x

$y = (-x)^2 + 4$

$y = x^2 + 4$ YES, SYMMETRIC ABOUT THE y-AXIS.

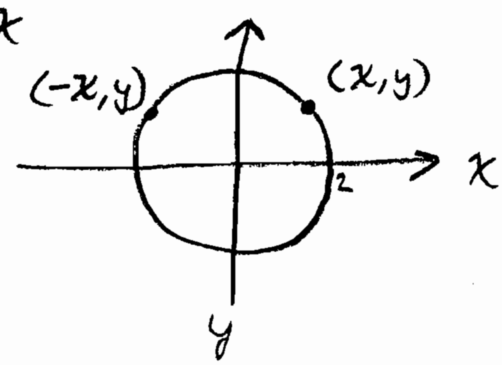
ii. CONSIDER $x^2 + y^2 = 4$ <u>NOT</u> A <u>FUNCTION</u>

SUBSTITUTE $-x$ FOR x

$(-x)^2 + y^2 = 4$

$x^2 + y^2 = 4$

YES, SYMMETRIC ABOUT y-AXIS



x	y
0	0
1	1
2	8
-1	-1
-2	-8

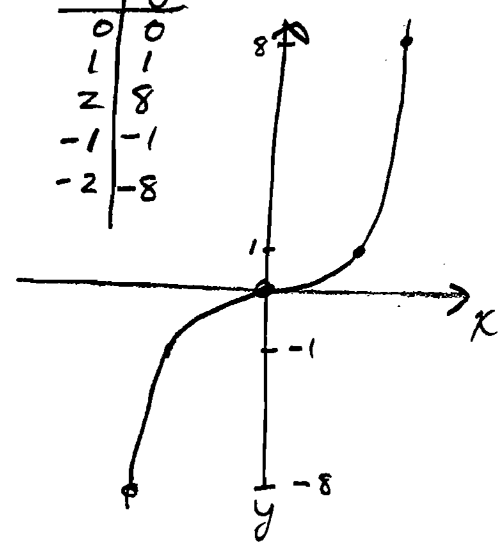
iii. CONSIDER $y = x^3$

SUBSTITUTE $-x$ FOR x

$y = (-x)^3$

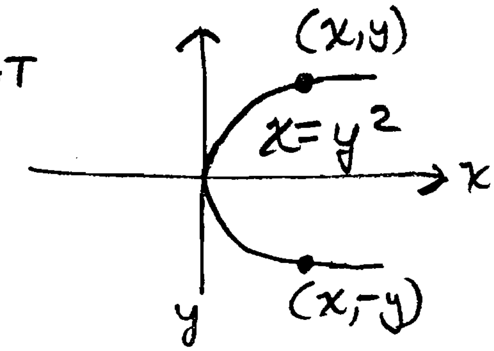
$y = -x^3$ NOT EQUIVALENT TO ORIGINAL $y = x^3$

NOT SYMMETRIC ABOUT y-AXIS



2. SYMMETRY ABOUT x-AXIS

a. $f(x)$ NOTATION NOT CONSIDERED SINCE IT IS NOT GENERALLY TRUE THAT GRAPHS SYMMETRIC ABOUT THE x-AXIS ARE FUNCTIONS

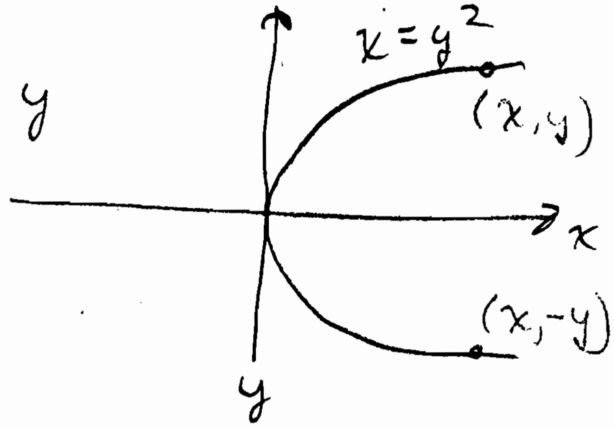


(GRAPH AT RIGHT IS SYMMETRIC ABOUT x-AXIS, BUT NOT A FUNCTION - VERTICAL LINE INTERSECTS TWICE)

b. SYMMETRIC ABOUT x-AXIS FOR GRAPHS OF EQUATIONS

IF YOU GET AN EQUIVALENT EQUATION BY SUBSTITUTING $-y$ FOR y, THE GRAPH IS SYMMETRIC ABOUT THE x-AXIS

i CONSIDER $x = y^2$
SUBSTITUTE $-y$ FOR y
$x = (-y)^2$
$x = y^2$ YES,
SYMMETRIC ABOUT
x-AXIS



ii CONSIDER $x = 2y + y^4$
SUBSTITUTE $-y$ FOR y
$x = 2(-y) + (-y)^4$
$x = -2y + y^4$ NO, NOT EQUIVALENT
TO ORIGINAL $x = 2y + y^4$
NOT SYMMETRIC ABOUT x-AXIS

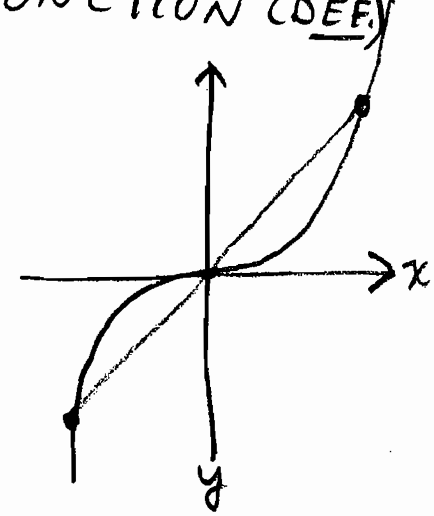
3. SYMMETRY ABOUT THE ORIGIN

a. FOR FUNCTIONAL $f(x)$ NOTATION
GRAPH OF f IS SYMMETRIC ABOUT
THE ORIGIN IFF $f(-x) = -f(x)$
f IS CALLED AN ODD FUNCTION (DEF.)

EXAMPLE: $f(x) = x^3$

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

YES,
SYMMETRIC ABOUT
ORIGIN. ODD FUNCTION



b. SYMMETRIC ABOUT THE ORIGIN FOR GRAPHS OF EQUATIONS (DEF.)

IF YOU GET AN EQUIVALENT EQUATION BY SUBSTITUTING $-x$ FOR x AND $-y$ FOR y, THE GRAPH IS SYMMETRIC ABOUT THE ORIGIN

i. CONSIDER $y = x^3$

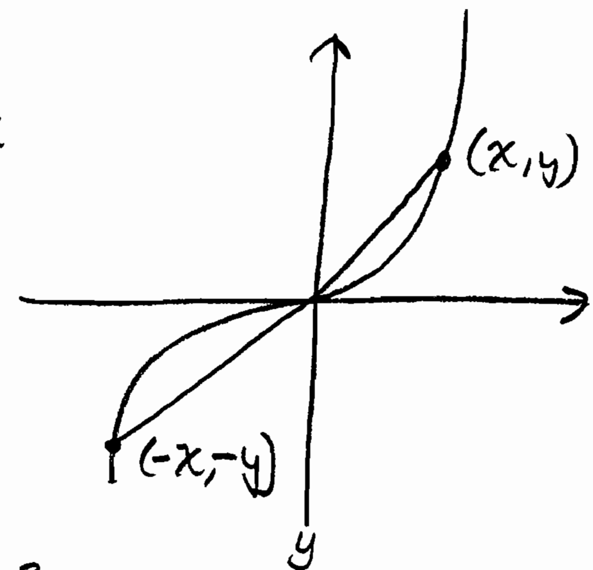
SUBSTITUTE $-x$ FOR x AND $-y$ FOR y

$-y = (-x)^3$

$-y = -x^3$

$y = x^3$ YES

SYMMETRIC ABOUT ORIGIN



ii. CONSIDER $2y^2 = x^3 - x^5$

SUBSTITUTE $-x$ FOR x AND $-y$ FOR y

$2(-y)^2 = (-x)^3 - (-x)^5$

$2y^2 = -x^3 - (-x^5)$

$2y^2 = -x^3 + x^5$. NO, NOT EQUIVALENT TO ORIGINAL $2y^2 = x^3 - x^5$

NOT SYMMETRIC ABOUT THE ORIGIN.

W. WEAPONS TO HELP GRAPH

1. FIND x AND y INTERCEPTS
2. FIND A KNOWN FUNCTION THAT YOU CAN REFLECT, TRANSLATE, NARROW/FLATTEN TO BECOME THE GRAPH
3. FIND OUT ABOUT SYMMETRY
   a. x-AXIS
   b. y-AXIS
   c. ORIGIN
4. PLOT ENOUGH POINTS

   YOUR INFORMATION ON SYMMETRY AND OTHER INFORMATION ABOVE MAY ALLOW YOU TO PLOT JUST A FEW POINTS, THEN DEDUCE WHAT THE REST OF THE GRAPH MUST LOOK LIKE.
5. YOU MAY BE ABLE TO USE ALGEBRA TO FIND RESTRICTIONS ON WHAT VALUES THE VARIABLES CAN TAKE ON.

X. USE GRAPHING WEAPONS TO HELP GRAPH

$4x^2+9y^2=36$

1. FIND x-INTERCEPTS. SET $y=0$

$4x^2+9(0^2)=36$

$4x^2=36$. $x^2=9$ $x=\pm 3$

$\boxed{(-3,0),(3,0) \text{ } x\text{-INTERCEPTS}}$

2. FIND y-INTERCEPTS. SET $x=0$

$4(0^2)+9y^2=36$

$9y^2=36$. $y^2=4$. $y=\pm 2$

$\boxed{(0,2),(0,-2) \text{ } y\text{-INTERCEPTS}}$

3. ALGEBRAIC ANALYSIS

$$\frac{4x^2}{36}+\frac{9y^2}{36}=\frac{36}{36}$$

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

$$\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$$

NOTE HOW THE INTERCEPTS LOOK VISIBLE IN THIS FORM

SOLVE FOR y

$9y^2=36-4x^2$

$$y^2=\frac{36-4x^2}{9}$$

$$|y|=\sqrt{\frac{36-4x^2}{9}}=\frac{\sqrt{4(9-x^2)}}{3}$$

$$|y| = \frac{2}{3}\sqrt{9-x^2}$$

TO BE DEFINED $9-x^2 \geq 0$

$9 \geq x^2$. $x^2 \leq 9$. $|x| \leq \sqrt{9}$

$|x| \leq 3$. $\boxed{-3 \leq x \leq 3 \text{ ONLY}}$

$\boxed{x \text{ VALUES FOR WHICH GRAPH EXISTS}}$

4. CHECK FOR y-AXIS SYMMETRY.
SUBSTITUTE $-x$ FOR x IN $4x^2+9y^2=36$

$4(-x)^2+9y^2=36$

$4x^2+9y^2=36$

$\boxed{\text{YES, SYMMETRIC ABOUT } y\text{-AXIS}}$

5. CHECK FOR x-AXIS SYMMETRY
SUBSTITUTE $-y$ FOR y IN $4x^2+9y^2=36$

$4x^2+9(-y)^2=36$

$4x^2+9y^2=36$

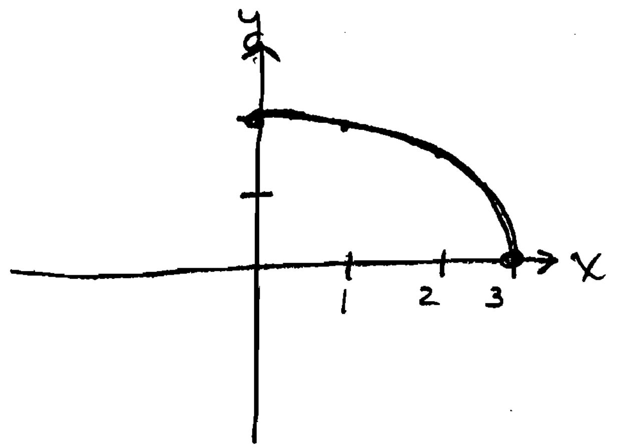
$\boxed{\text{YES, SYMMETRIC ABOUT } x\text{-AXIS}}$

6. DERIVATION OF ONLY POINTS NEEDED TO PLOT. DUE TO x AND y-AXIS SYMMETRY, ONLY NEED TO GET GRAPH IN QUADRANT I ($x \geq 0, y \geq 0$), THEN THE REST CAN BE DEDUCED.

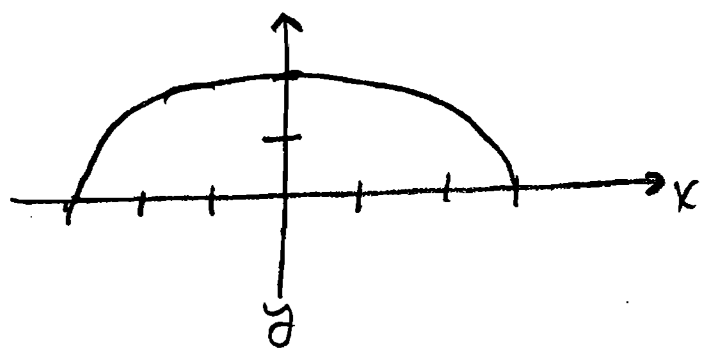
FOR $y \geq 0$ SINCE $|y| = \frac{2}{3}\sqrt{9-x^2}$

$y = \frac{2}{3}\sqrt{9-x^2}$

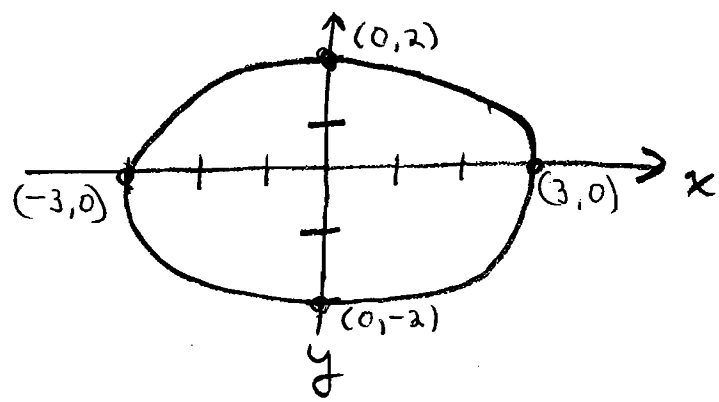
x	y
0	$\frac{2}{3}\sqrt{9-0^2} = 2$
1	$\frac{2}{3}\sqrt{9-1^2} \cong 1.9$
2	$\frac{2}{3}\sqrt{9-2^2} \cong 1.5$
2.5	$\frac{2}{3}\sqrt{9-(2.5)^2} \cong 1.1$
3	$\frac{2}{3}\sqrt{9-3^2} = 0$



DUE TO y-AXIS SYMMETRY, MORE OF THE GRAPH MUST LOOK LIKE



DUE TO x-AXIS SYMMETRY, THE WHOLE GRAPH MUST LOOK LIKE



THIS GRAPH IS CALLED AN ELLIPSE.

STANDARD FORM: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

"CENTER" (h,k)

WHAT WE GRAPHED HAD THE FORM $\frac{(x-0)^2}{3^2} + \frac{(y-0)^2}{2^2} = 1$

12-260.1

ELLIPSE CENTERED AT THE ORIGIN

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

NOTE: SET $y=0$

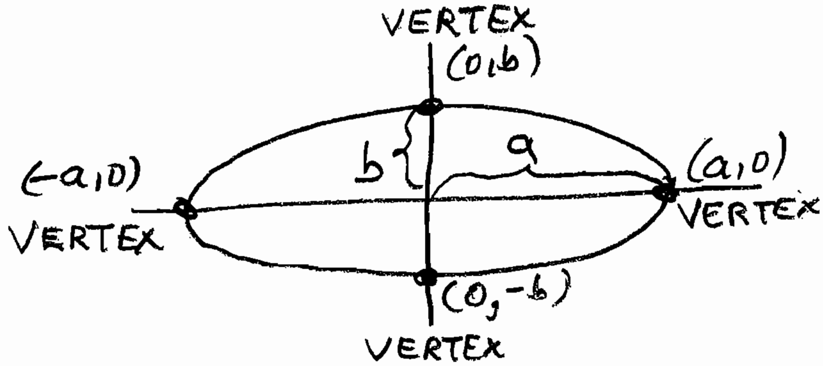
$$\frac{x^2}{a^2} = 1$$

$$x^2 = a^2$$

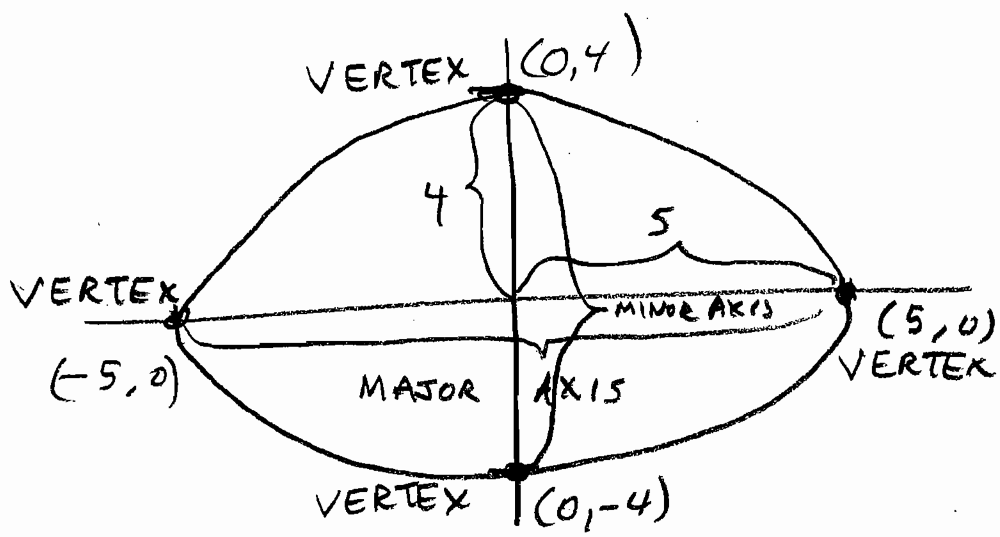
$$\sqrt{x^2} = \sqrt{a^2}$$

$$|x| = |a|$$

$$x = \pm a$$



$$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$



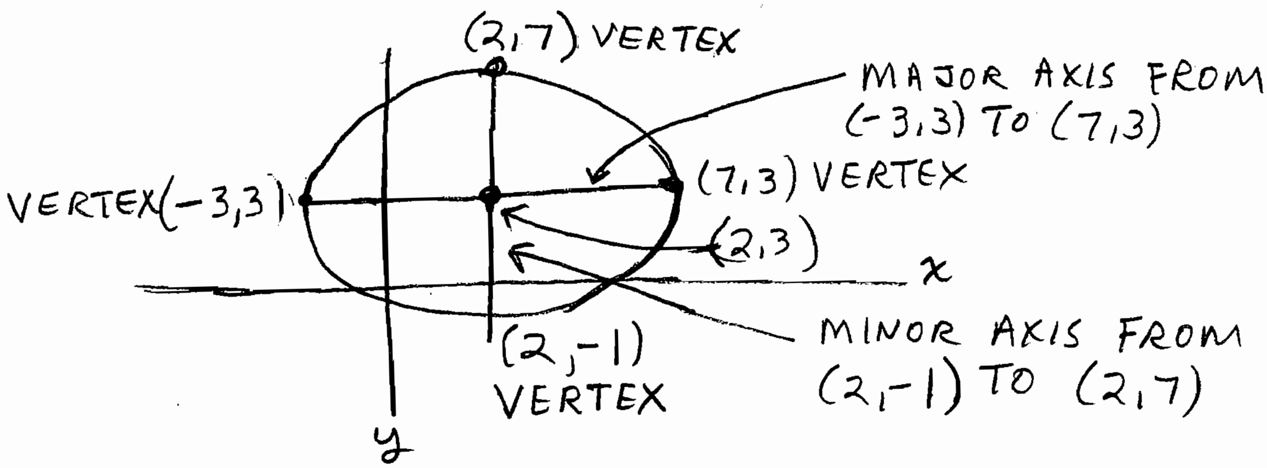
STANDARD FORM FOR AN ELLIPSE CENTERED AT (h,k)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

12-260A

ELLIPSES

SKETCH $\dfrac{(x-2)^2}{5^2} + \dfrac{(y-3)^2}{4^2} = 1$



PUT $4x^2+2y^2+24x-4y+30=0$
INTO STANDARD FORM FOR AN ELLIPSE, SKETCH, LABEL VERTICES, MAJOR AND MINOR AXES

X, Y GROUPED ON LEFT, CONSTANT ON RIGHT

$$4x^2+24x \quad +2y^2-4y \quad = -30$$

FACTOR OUT x^2, y^2 COEFFICIENTS

$$4(x^2+6x \quad) + 2(y^2-2y \quad) = -30$$

COMPLETE THE SQUARE

$$4(x^2+6x+9) + 2(y^2-2y+1) = -30+36+2$$

12-260 B

$$4(x+3)^2+2(y-1)^2=8$$

GET 1 ON RIGHT HAND SIDE

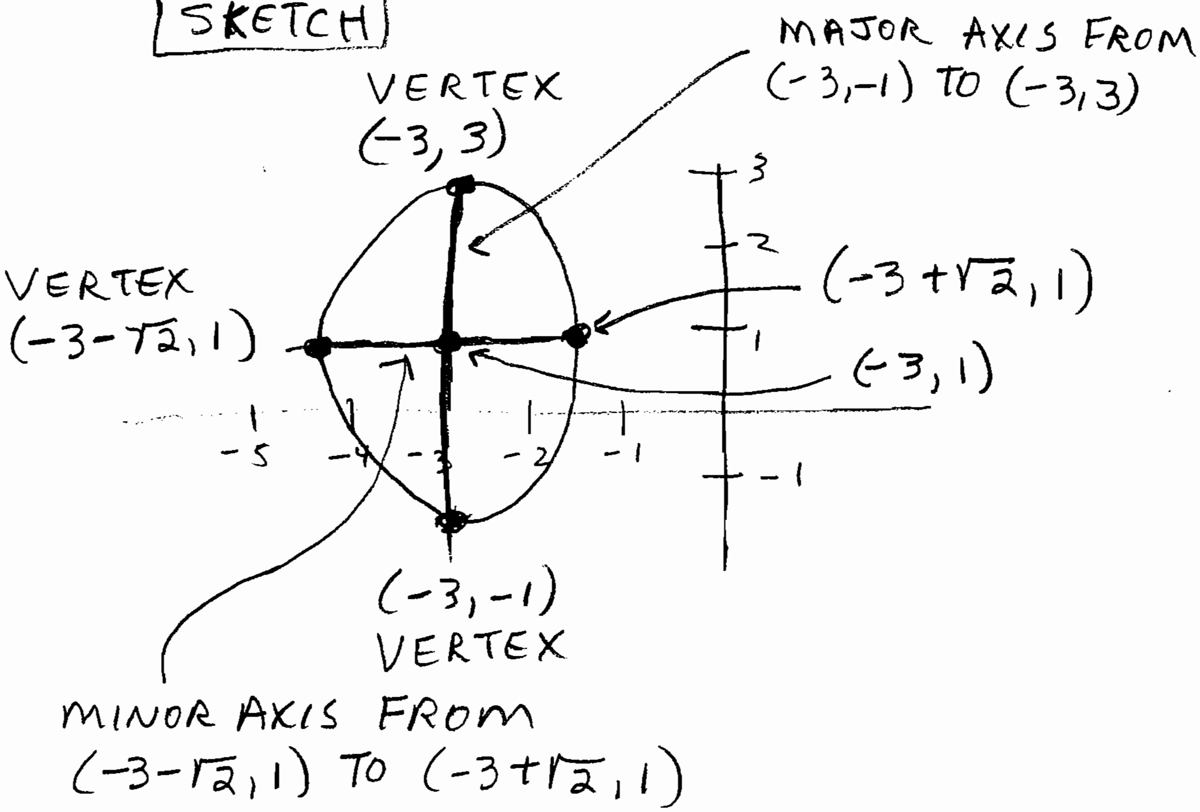
$$\frac{4(x+3)^2}{8}+\frac{2(y-1)^2}{8}=1$$

PUT IN STANDARD FORM

$$\frac{(x+3)^2}{2}+\frac{(y-1)^2}{4}=1$$

$$\frac{(x-[-3])^2}{(\sqrt{2})^2}+\frac{(y-1)^2}{2^2}=1$$

SKETCH

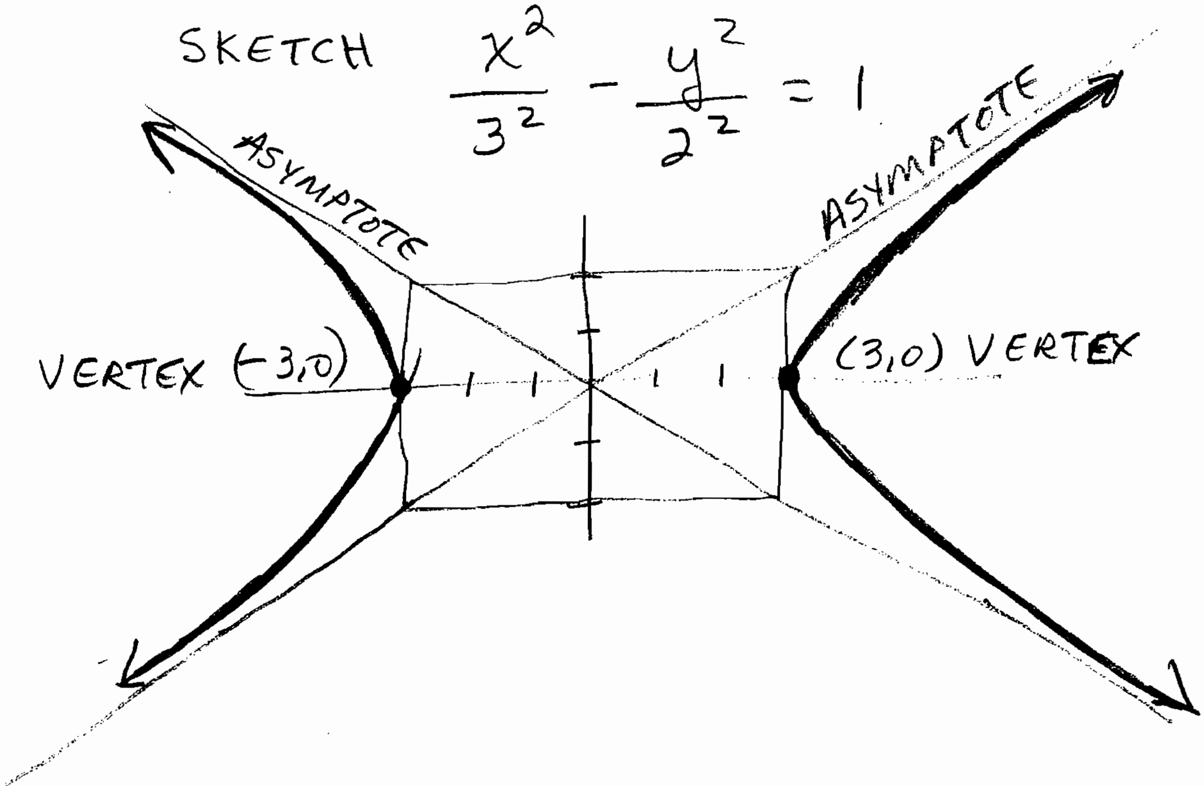


12 - 260 C

HYPERBOLAS

STANDARD FORM: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

SKETCH $\dfrac{x^2}{3^2} - \dfrac{y^2}{2^2} = 1$



PUT $9y^2 - 4x^2 - 16x - 18y - 43 = 0$ INTO STANDARD FORM FOR A HYPERBOLA, SKETCH, LABEL VERTICES.

$\boxed{\text{X, Y GROUPED ON LEFT, CONSTANT ON RIGHT}}$

$$9y^2 - 18y - 4x^2 - 16x = 43$$

12-260D

FACTOR OUT x^2, y^2 COEFFICIENTS

$$9(y^2-2y\quad)-4(x^2+4x\quad)=43$$

CAREFUL

COMPLETE THE SQUARE

$$9(y^2-2y+1)-4(x^2+4x+4)=43+9-16$$

$$9(y-1)^2-4(x+2)^2=36$$

GET 1 ON RIGHT HAND SIDE

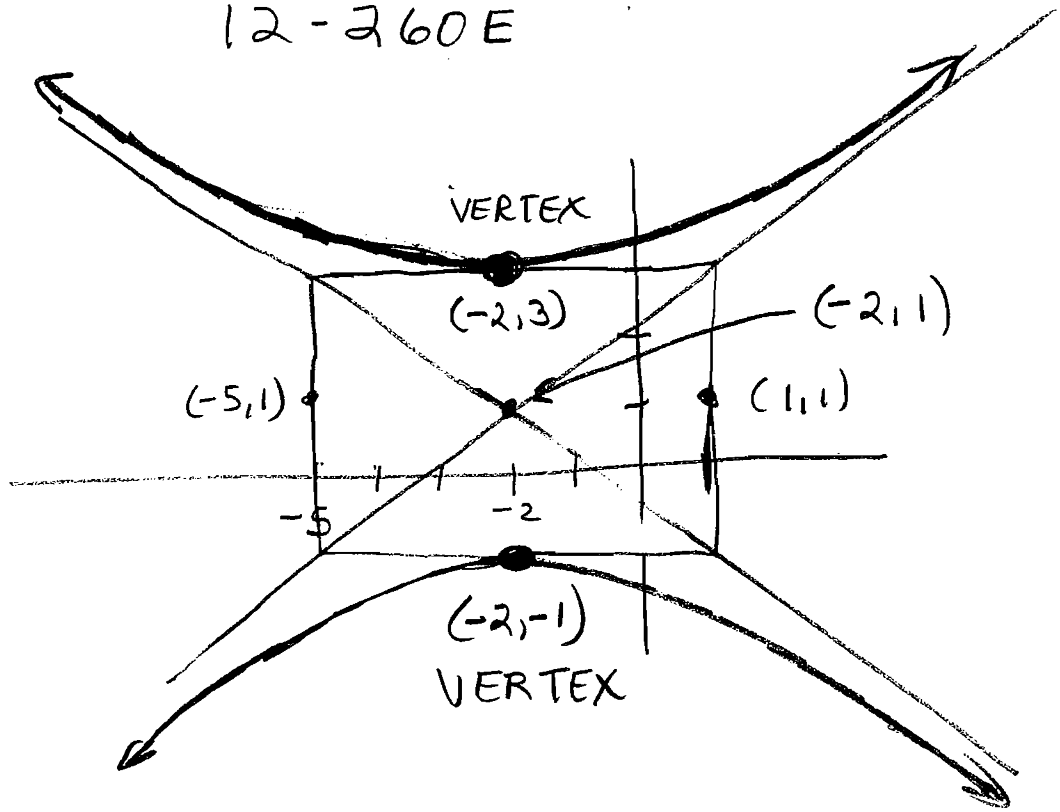
$$\frac{9(y-1)^2}{36}-\frac{4(x+2)^2}{36}=1$$

PUT IN STANDARD FORM

$$\frac{(y-1)^2}{4}-\frac{(x+2)^2}{9}=1$$

$$\frac{(y-1)^2}{2^2}-\frac{(x-[-2])^2}{3^2}=1$$

12-260E



CONIC SECTIONS

ELLIPSES, HYPERBOLAS, AND PARABOLAS ARE CONIC SECTIONS

Y. HOMEWORK (OTS)

1. GRAPH EACH OF THE FOLLOWING AS A SEQUENCE OF REFLECTIONS, TRANSLATIONS, STRETCH/COMPRESS, NAME THE HORIZONTAL AND VERTICAL ASYMPTOTES. NAME THE x AND y-INTERCEPTS

   a. $y = \dfrac{-x-1}{x+3}$ b. $f(x) = \dfrac{6x-17}{2x-4}$

2. TELL WHETHER THESE FUNCTIONS ARE EVEN OR ODD, WHETHER THEIR GRAPHS ARE SYMMETRIC ABOUT y-AXIS OR ORIGIN

   a. $f(x) = -7x^5 + 2x^3$ b. $f(x) = 3x^4 - 2x^2 + 7$

   c. $f(x) = -2x^2 + x^3$ d. $f(x) = 7$

3. TELL WHETHER THESE GRAPHS ARE SYMMETRIC ABOUT x-AXIS, y-AXIS, ORIGIN

   a. $y^3 = 3x^5 - x^3$ b. $-y^2 = x^4y^6 - y$

   c. $y^2 = x^2y^4 + x^4$ d. $y = \dfrac{1}{x}$

12-261 A

4. USE GRAPHING WEAPONS TO HELP GRAPH

a. $3x^2+4y^2=1$ b. $4x^2-9y^2=36$

5. PUT INTO STANDARD FORM FOR AN ELLIPSE, SKETCH, LABEL VERTICES, MAJOR AXIS, MINOR AXIS.

a. $\dfrac{(x-5)^2}{9}+\dfrac{(y+2)^2}{4}=1$

b. $16x^2+9y^2+64x+54y+1=0$

c $4x^2+3y^2-8x-30y+67=0$

6. PUT INTO STANDARD FORM FOR A HYPERBOLA, SKETCH, LABEL VERTICES

a. $\dfrac{(x-2)^2}{9}-\dfrac{(y+3)^2}{16}=1$

b. $16x^2-9y^2+64x-54y-161=0$

c. $2y^2-4x^2-4y-24x-35=0$

[CHAPTER 13]

FUNCTION OPERATIONS

A. +, −, ×, ÷ OF FUNCTIONS

$$\left.\begin{aligned}(f+g)(x) &= f(x)+g(x)\\ (f-g)(x) &= f(x)-g(x)\\ (fg)(x) &= f(x)g(x)\end{aligned}\right\}\ \underline{\text{DEF.}}$$

NOTE: FOR RIGHT HAND SIDE OF THE EQUATION TO MAKE SENSE $x \in dom(f)$ <u>AND</u> $x \in dom(g)$ (I.E. $x \in dom(f) \cap dom(g)$)

$$dom(f+g) = dom(f-g) = dom(fg)$$
$$= dom(f) \cap dom(g) \quad (\underline{\text{DEF.}})$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$ NOTE: FOR RIGHT HAND SIDE TO MAKE SENSE, $x \in dom(f)$ AND $x \in dom(g)$ AND $g(x) \neq 0$

(<u>DEF.</u>)

$$dom\left(\frac{f}{g}\right) = \{x \mid x \in dom(f) \text{ AND } x \in dom(g) \text{ AND } g(x) \neq 0\}$$
$$= \{x \mid x \in dom(f) \cap dom(g) \text{ AND } g(x) \neq 0\}$$

B. EXAMPLE FOR FUNCTIONS DEFINED BY THE LISTING METHOD. LET f AND g BE:

$$f=\{(1,5),(2,6),(3,7)\} \quad g=\{(2,8),(3,0),(4,9)\}$$

$$\mathrm{dom}(f)=\{1,2,3\} \quad \mathrm{dom}(g)=\{2,3,4\}$$

$$\mathrm{dom}(f\pm g)=\mathrm{dom}(fg)=\mathrm{dom}(f)\cap \mathrm{dom}(g)$$

$$=\{2,3\}$$

$$f+g=\{(2,6+8),(3,7+0)\}=\{(2,14),(3,7)\}$$

$$f-g=\{(2,6-8),(3,7-0)\}=\{(2,-2),(3,7)\}$$

$$fg=\{(2,6\cdot 8),(3,7\cdot 0)\}=\{(2,48),(3,0)\}$$

$$\mathrm{dom}\left(\frac{f}{g}\right)=\{x \,/\, x\in \mathrm{dom}(f)\cap \mathrm{dom}(g) \text{ AND } g(x)\neq 0\}$$

$$=\{2\}$$

$$\frac{f}{g}=\{(2,\tfrac{6}{8})\}=\{(2,\tfrac{3}{4})\}$$

NEXT WE LOOK AT $f\pm g$, fg, $\frac{f}{g}$ WHEN DEFINED BY FORMULAS FOR $f(x)$ AND $g(x)$

C. EXAMPLES OF $f \pm g$, fg, $\frac{f}{g}$ FOR FUNCTIONS DEFINED BY FORMULAS.

LET $f(x) = (x-2)^{3/2}$ $\quad g(x) = \dfrac{x-3}{x-2}$

TO FIND dom(f), $x-2 \geq 0$. $x \geq 2$

$\text{dom}(f) = [2, \infty)$

$\text{dom}(g) = \{x \mid x \neq 2\} = (-\infty, 2) \cup (2, \infty)$

$\text{dom}(f \pm g) = \text{dom}(fg) = \text{dom}(f) \cap \text{dom}(g)$

$= [2, \infty) \cap \{x \mid x \neq 2\} = (2, \infty)$

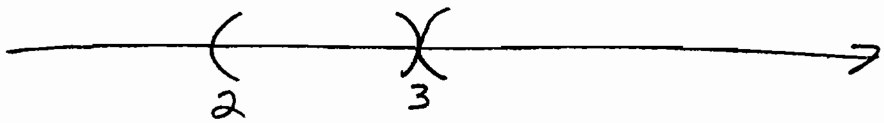
$$(f \pm g)(x) = f(x) \pm g(x) = (x-2)^{3/2} \pm \frac{x-3}{x-2}$$

$$(fg)(x) = f(x)\,g(x) = \frac{(x-2)^{3/2}(x-3)}{x-2} = (x-2)^{1/2}(x-3)$$

NOTE: TO LOOK AT THE SIMPLIFIED FORMULA FOR $f(x)g(x)$ YOU MIGHT THINK $\text{dom}(fg) = [2, \infty)$, BUT NOT WE HAVE PREVIOUSLY FOUND IT TO BE $(2, \infty)$. MORAL: FIND THE DOMAIN FOR $f \pm g$, fg, $\frac{f}{g}$ BEFORE YOU FIND THE FORMULA.

FIND dom $\frac{f}{g} = \{x \mid x \in \mathrm{dom}(f) \cap \mathrm{dom}(g) \text{ AND } g(x) \neq 0\}$

$$\boxed{\begin{array}{ccc} x \in \mathrm{dom}(f) \cap \mathrm{dom}(g) & \text{AND} & g(x) \neq 0 \\ \downarrow & & \swarrow \\ x \in (2, \infty) & \text{AND} & x \neq 3 \end{array}}$$



$$\mathrm{dom}\left(\frac{f}{g}\right) = (2,3) \cup (3, \infty)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{(x-2)^{3/2}}{\frac{(x-3)}{(x-2)}} = \frac{(x-2)^{3/2} \cdot (x-2)}{x-3}$$

$$\left(\frac{f}{g}\right)(x) = \frac{(x-2)^{5/2}}{x-3}$$

D. COMPOSITION OF FUNCTIONS

1. <u>NOTATION</u> $f \circ g$ IS READ "f COMPOSITION g"

"OF" NOT "TIMES"

2. <u>DEFINITION</u> $(f \circ g)(x) = f(g(x))$

(<u>DEF.</u>) 3. $\mathrm{dom}(f \circ g) = \{x \mid x \in \mathrm{dom}(g) \text{ AND } g(x) \in \mathrm{dom}(f)\}$

4. FOR $f(x)=\sqrt{x}+2x^2-7$ AND $g(x)=2x-1$

$$\left.\begin{aligned}&(f\circ g)(x)=f(g(x))=f(2x-1)=\\&\sqrt{2x-1}+2(2x-1)^2-7\end{aligned}\right\}\text{WAY 1}$$

$$\left.\begin{aligned}&(f\circ g)(x)=f(g(x))=\sqrt{g(x)}+2\,g(x)^2-7\\&=\sqrt{2x-1}+2(2x-1)^2-7\end{aligned}\right\}\text{WAY 2}$$

FIND $\mathrm{dom}(f\circ g)=\{x \mid x\in\mathrm{dom}(g) \text{ AND } g(x)\in\mathrm{dom}(f)\}$

$$\boxed{\begin{aligned}&\mathrm{dom}(g)=(-\infty,\infty) \quad \mathrm{dom}(f)=\{x \mid x\ge 0\}\\&x\in\mathrm{dom}(g) \text{ AND } g(x)\in\mathrm{dom}(f)\\&x\in(-\infty,\infty) \text{ AND } 2x-1\ge 0\\&x\in(-\infty,\infty) \text{ AND } 2x\ge 1\\&x \text{ IS REAL AND } x\ge \tfrac{1}{2}\end{aligned}}$$

$\mathrm{dom}(f\circ g)=[\tfrac{1}{2},\infty)$

$(g\circ f)(x)=g(f(x))=g(\sqrt{x}+2x^2-7)=$

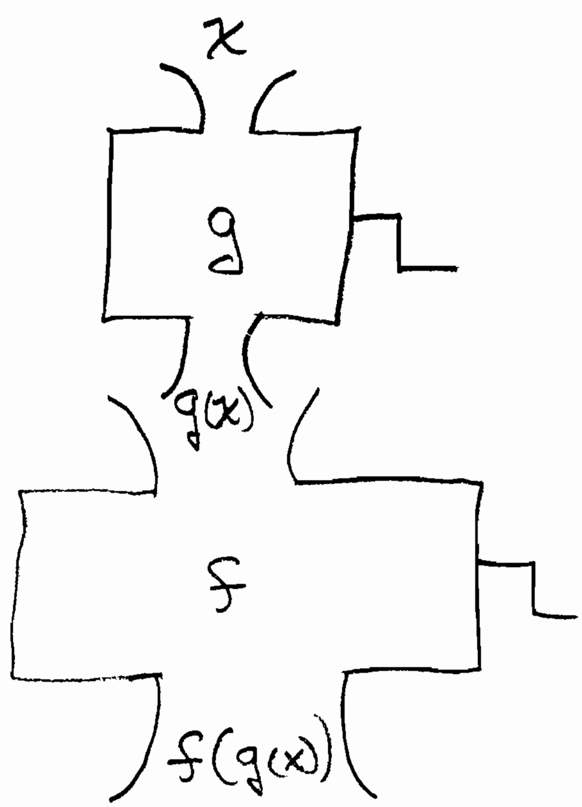
$2(\sqrt{x}+2x^2-7)-1=2\sqrt{x}+4x^2-14-1$

$=2\sqrt{x}+4x^2-15$

5. DRILL ON COMPOSITION FORMULAS

$$(h \circ g)(x) = h(g(x))$$
$$(g \circ h)(x) = g(h(x))$$
$$(w \circ f)(x) = w(f(x))$$
$$(f \circ w)(7) = f(w(7))$$
$$(h \circ w)(2) = h(w(2))$$

6. PICTURE OF $(f \circ g)(x) = f(g(x))$



7. DRILL ON EVALUATING $h \circ w$

$h(x) = 2\sqrt{x} - 4x^3 + 21 \quad w(x) = 3x+2$

$(h \circ w)(x) = h(w(x)) = h(3x+2) =$

$2\sqrt{3x+2} - 4(3x+2)^3 + 21$

$(h \circ w)(x) = h(w(x)) = 2\sqrt{w(x)} - 4w(x)^3 + 21$

$= 2\sqrt{3x+2} - 4(3x+2)^3 + 21$

$(h \circ w)(1) = 2\sqrt{3(1)+2} - 4(3(1)+2)^3 + 21$

$= 2\sqrt{5} - 4(5^3) + 21 = 2\sqrt{5} - 4(125) + 21$

$= 2\sqrt{5} - 500 + 21 = 2\sqrt{5} - 479$

$(h \circ w)(1) = h(w(1)) = h(3(1)+2) = h(5)$

$= 2\sqrt{5} - 4(5^3) + 21 = 2\sqrt{5} - 479$

8. WRITING A FUNCTION AS A COMPOSITION OF TWO FUNCTIONS

a. LET $H(x) = \sqrt{x-3}$. FIND f, g SUCH THAT $H(x) = (f \circ g)(x)$. LET $f(x) = \sqrt{x}$ $g(x) = x-3$.

$(f \circ g)(x) = f(g(x)) = f(x-3) = \sqrt{x-3} = H(x)$

b. LET $H(x) = 5(x-3)^2 + (x-3)^{1/3} + 2$. FIND f, g SUCH THAT $H(x) = (f \circ g)(x)$

LET $f(x) = 5x^2 + x^{1/3} + 2 \quad g(x) = x - 3$

$(f \circ g)(x) = f(g(x)) = f(x-3) =$

$5(x-3)^2 + (x-3)^{1/3} + 2 = H(x)$

c. LET $H(x) = -4(x-3)^2 + 2(x-3)^{1/5} - x + 2$.

FIND f, g SUCH THAT $H(x) = (f \circ g)(x)$

$H(x) = -4(x-3)^2 + 2(x-3)^{1/5} - (x-3) - 1$.

LET $f(x) = -4x^2 + 2x^{1/5} - x - 1 \quad g(x) = x - 3$

$(f \circ g)(x) = f(g(x)) = f(x-3) =$

$-4(x-3)^2 + 2(x-3)^{1/5} - (x-3) - 1 =$

$-4(x-3)^2 + 2(x-3)^{1/5} - x + 2 = H(x)$

9. COMPOSITION OF FUNCTIONS DEFINED BY THE LISTING METHOD

$f = \{(1,7), (2,6), (3,8)\} \quad g = \{(5,2), (9,3), (4,10)\}$

$f \circ g = \{(5,6), (9,8)\}$ NOTE $(f \circ g)(5) =$

$f(g(5)) = f(2) = 6$, SO $(5,6) \in f \circ g$

13-270

E. HOMEWORK (OIS)

1. LET $f = \{(7,2), (8,1), (3,5)\}$ $g = \{(3,4), (7,0)\}$
   BY THE LISTING METHOD, NAME $f \pm g$, fg, AN $\frac{f}{g}$.

2. LET $f(x) = \sqrt{4-x}$ $g(x) = \frac{x}{(4-x)^{1/3}}$ $h(x) = \frac{1}{2x^3}$
   a. FIND FORMULAS FOR $(f \pm g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$
   b. FIND DOMAINS FOR $(f \pm g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$
   c. FIND FORMULAS FOR $(g \pm h)(x)$, $(gh)(x)$, $\left(\frac{g}{h}\right)(x)$
   d. FIND DOMAINS FOR $(g \pm h)(x)$, $(gh)(x)$, $\left(\frac{g}{h}\right)(x)$

3. LET $f(x) = \sqrt{4-3x}$ $g(x) = 2x+1$ $h(x) = \sqrt{2-x}$
   a. FIND FORMULAS FOR $(f \circ g)(x)$, $(f \circ h)(x)$
   b. FIND DOMAINS FOR $f \circ g$ AND $f \circ h$
   c. FIND A FORMULA FOR $(g \circ f)(x)$

4. FIND FUNCTIONS f, g SUCH THAT
   $H(x) = (f \circ g)(x)$
   a. $H(x) = \sqrt{3-7x}$
   b. $H(x) = (2x-3)^3 + 4(2x-3)^2 - \pi$
   c. $H(x) = \sqrt{2+7x} + 5(2+7x)^{1/3} + 4x$

F. INVERSE FUNCTIONS

1. NOTATION: f^{-1} IS READ "f INVERSE"
2. $f^{-1} = \{(y, x) \mid (x, y) \in f\}$ (DEF.)
3. IF $f = \{(1,3), (2,6)\}$, THEN
   $f^{-1} = \{(3,1), (6,2)\}$
   a. $f(1) = 3 \qquad f^{-1}(3) = 1$
   b. $f(2) = 6 \qquad f^{-1}(6) = 2$
   c. $f(f^{-1}(3)) = f(1) = 3$
   d. $f^{-1}(f(1)) = f^{-1}(3) = 1$
4. IN GENERAL

$$\boxed{\begin{array}{ll} f(x) = y \quad \text{MEANS} & f^{-1}(y) = x \\ f(f^{-1}(y)) = y & f^{-1}(f(x)) = x \end{array}}$$

IF ALL VALUES DEFINED, WE CAN SAY:

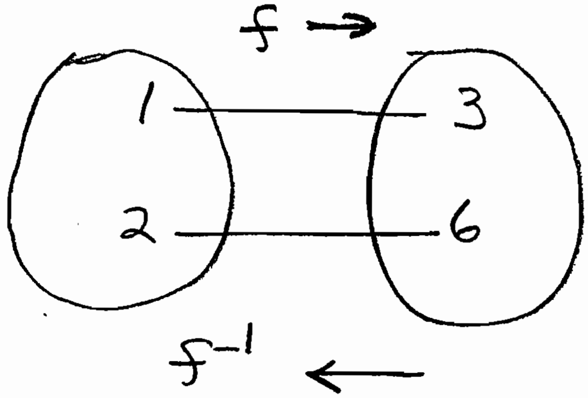
IF $h(2) = 3$, $h^{-1}(3) = 2$, $(2,3) \in h$

IF $h^{-1}(7) = 5$, $h(5) = 7$, $(7,5) \in h^{-1}$

$h(h^{-1}(6)) = 6$, $h^{-1}(h(20)) = 20$

5. TAKING THE INVERSE OF A FUNCTION REVERSES THE ASSOCIATION

$f = \{(1,3),(2,6)\}$ $\quad f^{-1} = \{(3,1),(6,2)\}$



6. IT IS NOT NECESSARILY TRUE THAT f^{-1} IS A FUNCTION

$f = \{(7,5),(8,5)\}$ $\quad f^{-1} = \{(5,7),(5,8)\}$

f^{-1} IS NOT A FUNCTION

7. (DEF.) ONE-TO-ONE FUNCTIONS (1-1 FUNCTIONS)

GIVEN f IS A FUNCTION. f IS <u>ONE-TO-ONE</u> IF AND ONLY IF NO TWO ORDERED PAIRS OF f HAVE THE SAME <u>SECOND</u> TERMS

$f = \{(1,3),(2,6)\}$ $\quad$ 1-1

$f = \{(7,5),(8,5)\}$ $\quad$ NOT 1-1

8. FOR ONE-TO-ONE FUNCTIONS f, f^{-1} IS A FUNCTION

TERMINOLOGY THAT MEANS THE SAME

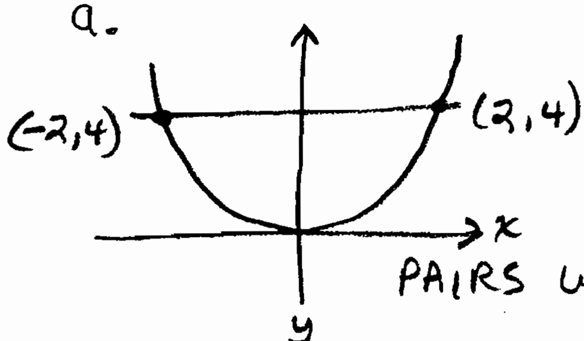
f IS 1-1

f IS INVERTIBLE

f^{-1} IS A FUNCTION

9. HOW TO LOOK AT THE GRAPH OF A FUNCTION AND TELL IF IT IS 1-1:
f IS A 1-1 FUNCTION IF AND ONLY IF NO HORIZONTAL LINE INTERSECTS THE GRAPH OF f TWICE

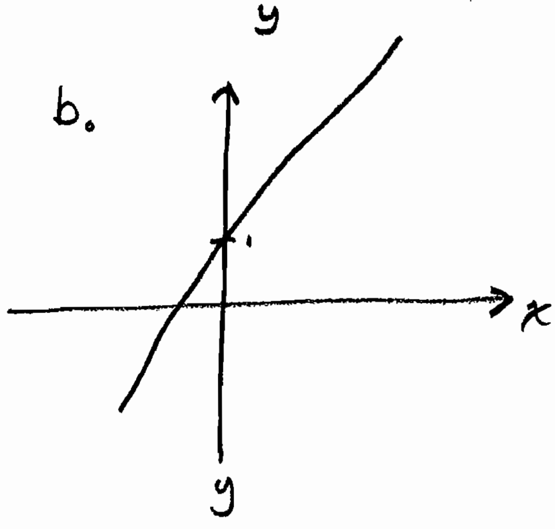
a.



$f(x) = x^2$

NOT 1-1. A HORIZONTAL LINE INTERSECTS THE GRAPH TWICE; 2 ORDERED PAIRS WITH THE SAME 2ND TERM

b.



$f(x) = 2x+1$ 1-1.

NO HORIZONTAL LINE INTERSECTS THE GRAPH TWICE.

10. THE NEED FOR A FORMULA FOR f^{-1}

$f(x) = 2x + 1$ IS 1-1

$f(2) = 2(2)+1 = 5$ SO $f^{-1}(5) = 2$

$f(3) = 2(3)+1 = 7$ SO $f^{-1}(7) = 3$

BUT WHAT IS $f^{-1}\left(\frac{5}{2}\right)$? HARD TO FIND

11. FINDING A FORMULA FOR f^{-1} IN TERMS OF y

GIVEN $f(x) = 2x+1$

$\boxed{\text{SET } y = f(x)}$

$y = 2x+1$

$\boxed{\text{SOLVE FOR } x}$

$y - 1 = 2x$

$\frac{y-1}{2} = x$

$\boxed{x \text{ IS } f^{-1}(y)}$

$f^{-1}(y) = \frac{y-1}{2}$

NOTE: $f^{-1}(5) = \frac{5-1}{2} = \frac{4}{2} = 2$

$f^{-1}\left(\frac{5}{2}\right) = \frac{\frac{5}{2}-1}{2} = \frac{\frac{3}{2}}{\frac{2}{1}} = \frac{3}{2} \cdot \frac{1}{2} = 3/4$

12. ONCE A FORMULA IS FOUND FOR f^{-1} IS FOUND IN TERMS OF y, YOU CAN PUT IT IN TERMS OF OTHER VARIABLES.

$$f^{-1}(y) = \frac{y-1}{2}, \quad f^{-1}(w) = \frac{w-1}{2}, \quad f^{-1}(x) = \frac{x-1}{2}$$

13. VERIFICATION OF EARLIER FORMULAS

$$f(x) = 2x+1 \qquad f^{-1}(x) = \frac{x-1}{2}$$

$$f(f^{-1}(x)) = f\left(\frac{x-1}{2}\right) = 2\left(\frac{x-1}{2}\right)+1 = x-1+1 = x$$

$$f^{-1}(f(x)) = f^{-1}(2x+1) = \frac{2x+1-1}{2} = \frac{2x}{2} = x$$

14. LET $f(x) = x^2-6x+11 \quad x<3$

f IS 1-1. FIND A FORMULA FOR $f^{-1}(x)$

$\boxed{\text{SET } y = f(x)}$

$$y = x^2-6x+11$$

$\boxed{\text{SOLVE FOR } x}$

$$y = x^2-6x+9-9+11$$

$$y = (x-3)^2+2$$

$$y-2 = (x-3)^2$$

$$\sqrt{y-2} = \sqrt{(x-3)^2}$$

$$\sqrt{y-2} = |x-3|$$

$$\sqrt{y-2} = |x-3| = -(x-3) \qquad x<3$$

L-R neg

$$\sqrt{y-2} = -x+3$$

$$x = -\sqrt{y-2} + 3$$

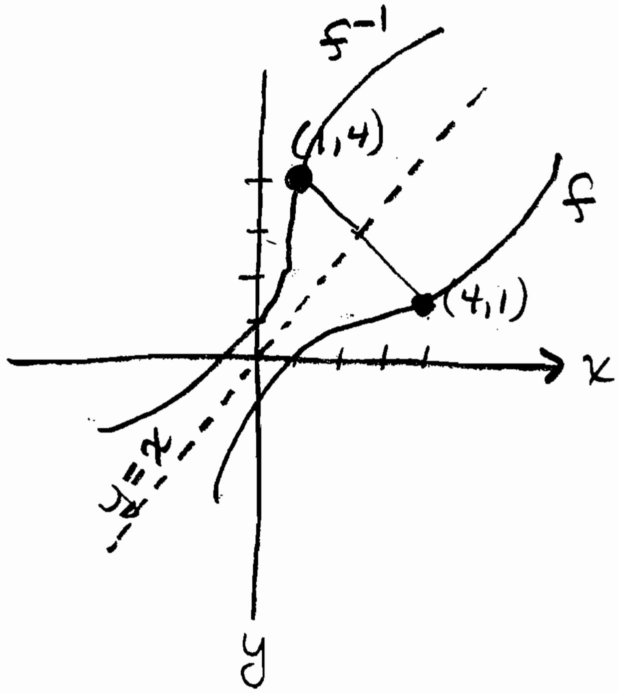
$$\boxed{x \text{ is } f^{-1}(y)}$$

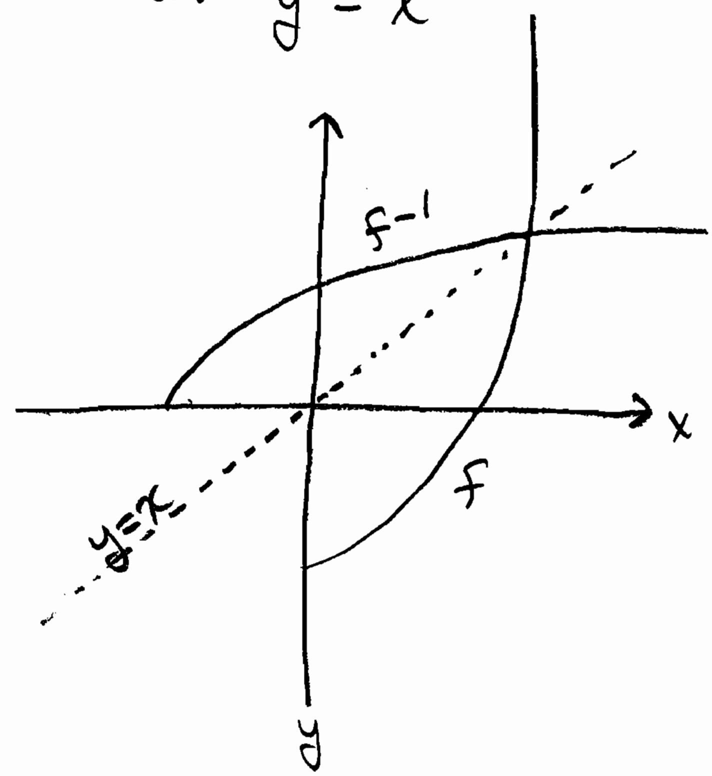
$$f^{-1}(y) = -\sqrt{y-2} + 3$$

THE PROBLEM ASKED TO FIND $f^{-1}(x)$

$$f^{-1}(x) = -\sqrt{x-2} + 3$$

15. THE GRAPHS OF f AND f^{-1} ARE MIRROR REFLECTIONS ABOUT $y = x$





13 - 277

G. HOMEWORK (OIS)

1. SUPPOSE f IS A 1-1 FUNCTION

   a. $f(8) = 1/2$, $f^{-1}(1/2) =$ ___ $(\_,\_) \in f$

   b. $f^{-1}(5) = 3$, $f(3) =$ ___, $(\_,\_) \in f$, $(\_,\_) \in f^{-1}$

   c. $f^{-1}(f(2)) =$ ___ d. $f(f^{-1}(6)) =$ ___

2. WHICH OF THE FOLLOWING IS 1-1 (I.E. INVERTIBLE) FOR SOME YOU MAY HAVE TO SKETCH A GRAPH.

   a. $f = \{(7,3), (2,6), (5,3)\}$ b. $h = \{(5,4), (4,5)\}$

   c. $f(x) = x^2 - 6x + 11$ d. $f(x) = -2x^2 + 4x + 5$

   e. $f(x) = x^2 + 8x + 5 \quad x < -4$ f. $f(x) = |x|$

   g. $f(x) = 2|x-3| + 1$ h. $f(x) = 2|x-3| + 1 \quad x > 3$

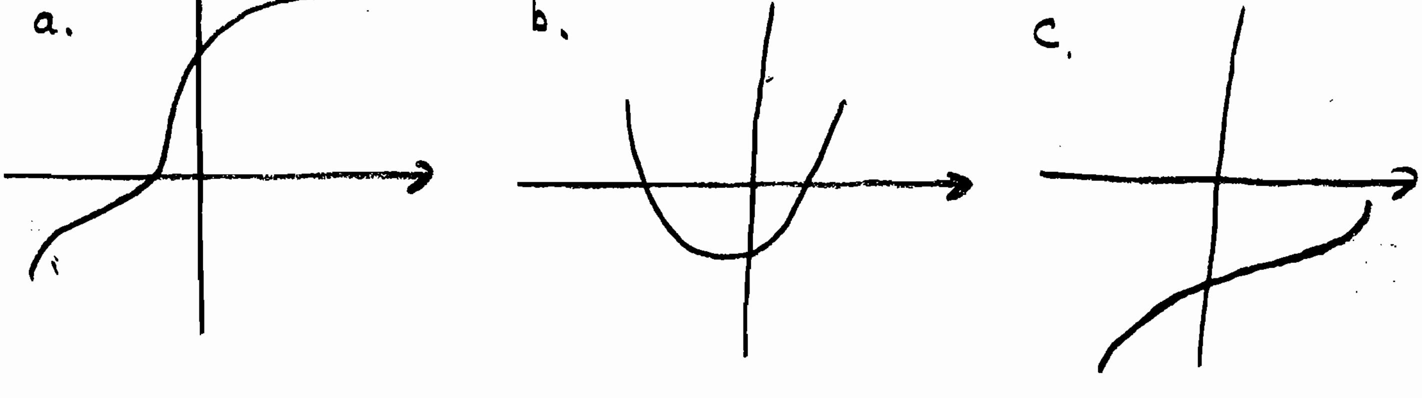
3. EACH OF THE FOLLOWING IS 1-1. FIND FORMULAS FOR $f^{-1}(x)$

   a. $f(x) = 3x + 5$ b. $f(x) = \dfrac{2x+3}{x-1}$

   c. $f(x) = x^2 + 10x + 3 \quad x < -5$

   d. $f(x) = -2x^2 + 4x + 5 \quad x > 1$

4. FOR EACH OF THE FOLLOWING THAT IS 1-1 SKETCH THE GRAPH OF f^{-1} ON SAME AXIS SYSTEM



[CHAPTER 14]

EXPONENTIAL AND LOGARITHM FUNCTIONS

A. $e \cong 2.71828\ldots$ IRRATIONAL

B. FORM OF AN EXPONENTIAL FUNCTION

$$f(x) = a^x \quad \text{(DEF.)}$$

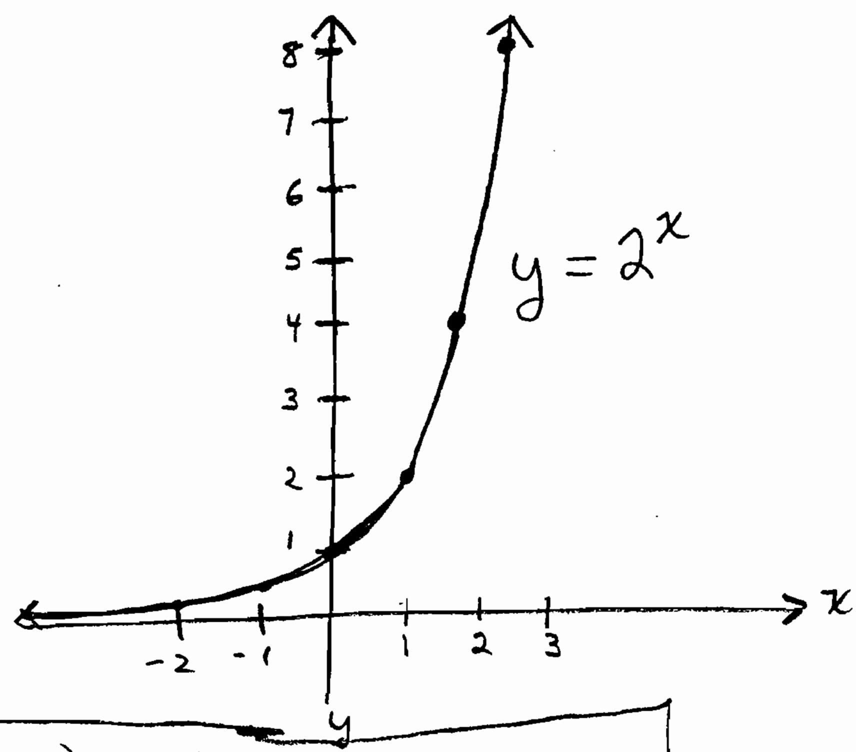
C. EXAMPLES OF EXPONENTIAL FUNCTIONS

$$f(x) = 2^x \qquad f(x) = \left(\tfrac{1}{3}\right)^x \qquad f(x) = e^x$$

D. GRAPH OF $f(x) = a^x$ $a > 1$ FOR $a = 2$

$f(x) = 2^x$

x	y
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
-1	$2^{-1} = \frac{1}{2}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$



NOTE: $\text{dom}(f) = (-\infty, \infty)$

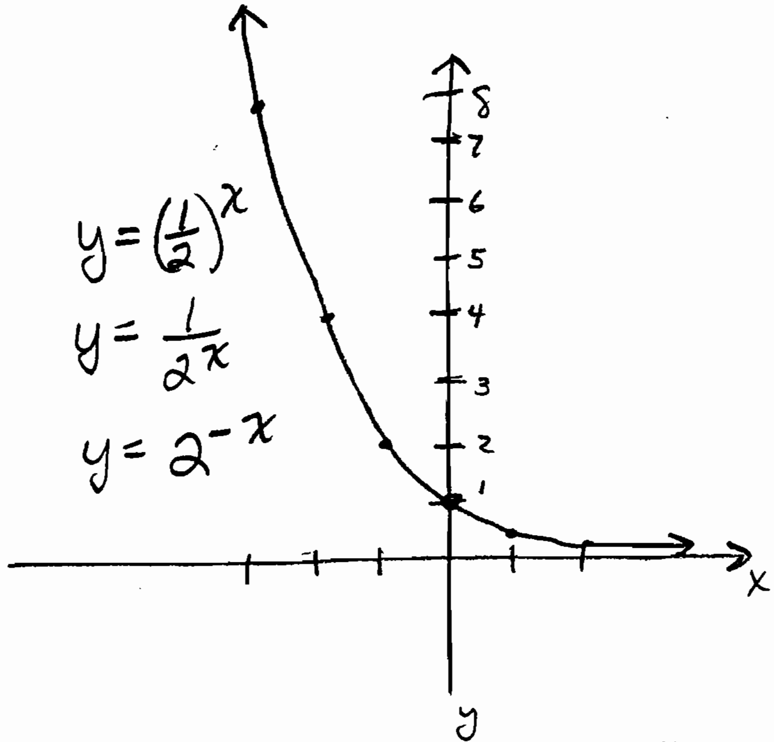
$\text{ran}(f) = (0, \infty)$

$f(0) = 1$ $\qquad$ f IS 1-1

E. GRAPH OF $f(x) = a^x$ $0 < a < 1$ FOR $a = \frac{1}{2}$

$f(x) = \left(\frac{1}{2}\right)^x$

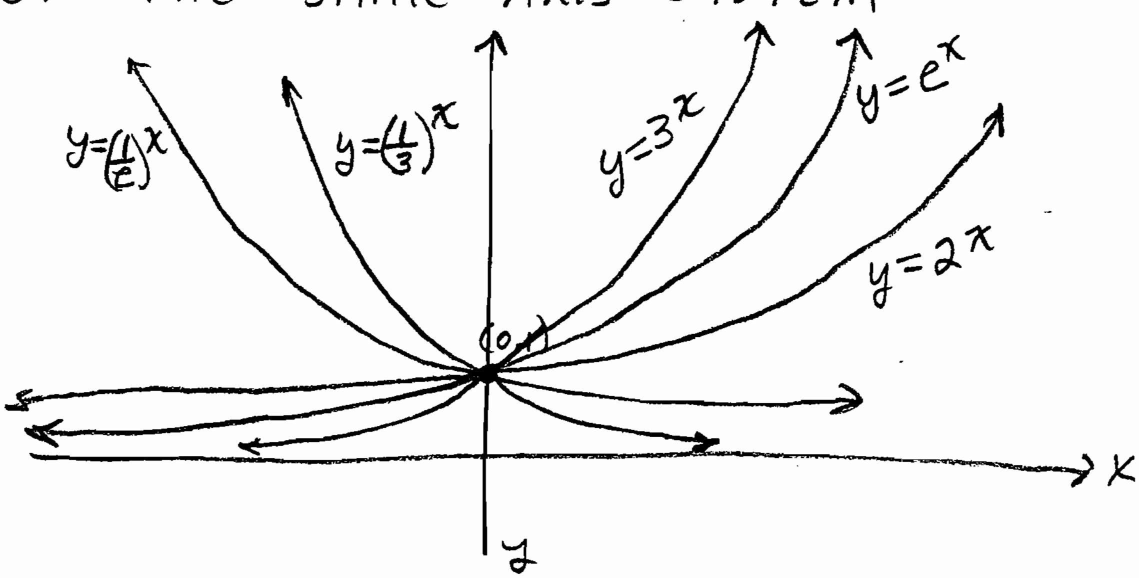
x	y
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$
−1	$\left(\frac{1}{2}\right)^{-1} = \frac{1}{\frac{1}{2}} = 2$
−2	$\left(\frac{1}{2}\right)^{-2} = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$
−3	$\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{8}} = 8$



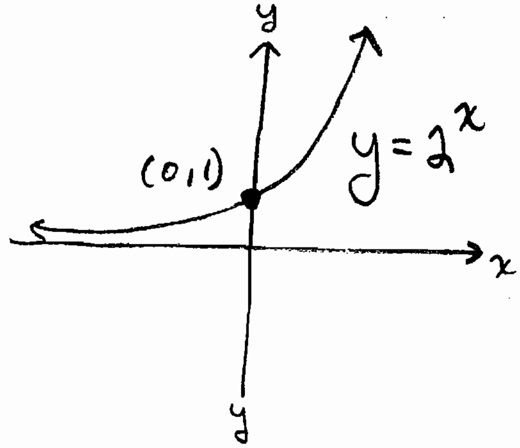
NOTE: IF $g(x) = 2^x$, $g(-x) = 2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x = f(x)$

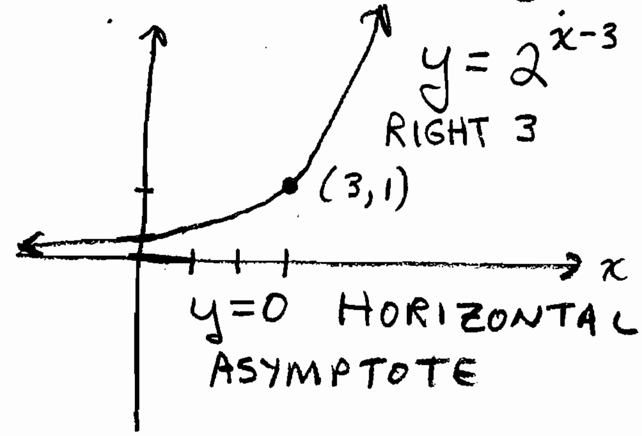
GRAPHS OF $g(x) = 2^x$ AND $g(-x) = f(x) = \left(\frac{1}{2}\right)^x$ ARE REFLECTION OF EACH OTHER ABOUT THE y-AXIS

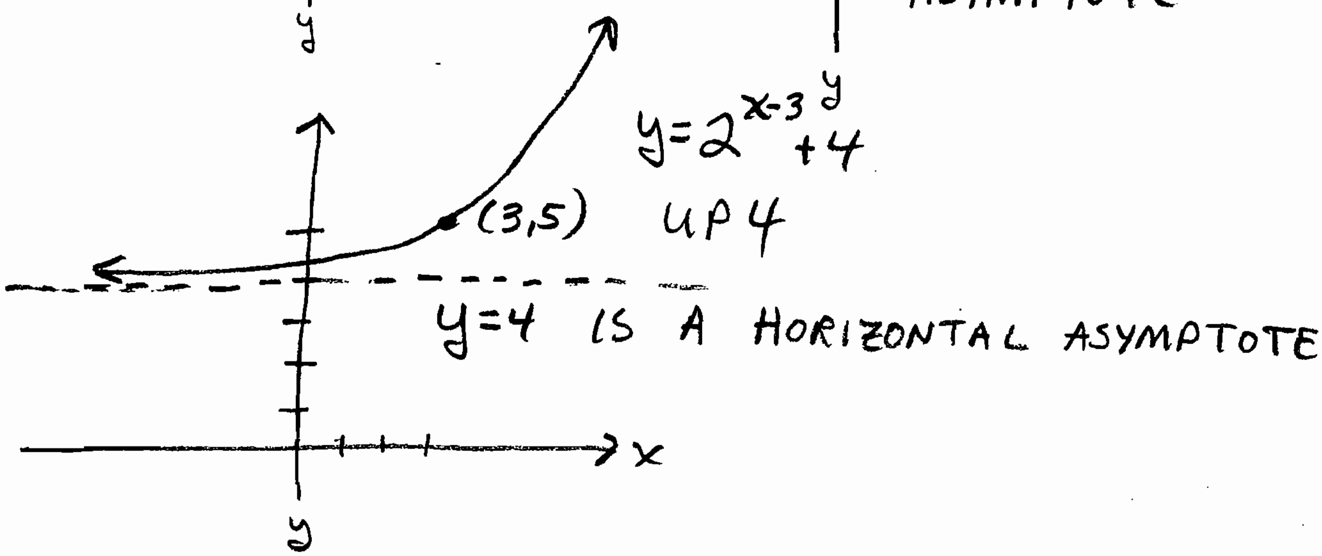
F. SEVERAL EXPONENTIAL FUNCTION GRAPHS ON THE SAME AXIS SYSTEM



F. SKETCH THE GRAPH OF $y = 2^{x-3} + 4$ AS A SEQUENCE OF TRANSLATIONS, REFLECTIONS, AND STRETCH/COMPRESS. SHOW THE FATE OF (0,1) AS IT MOVES THROUGH THE PROCESS

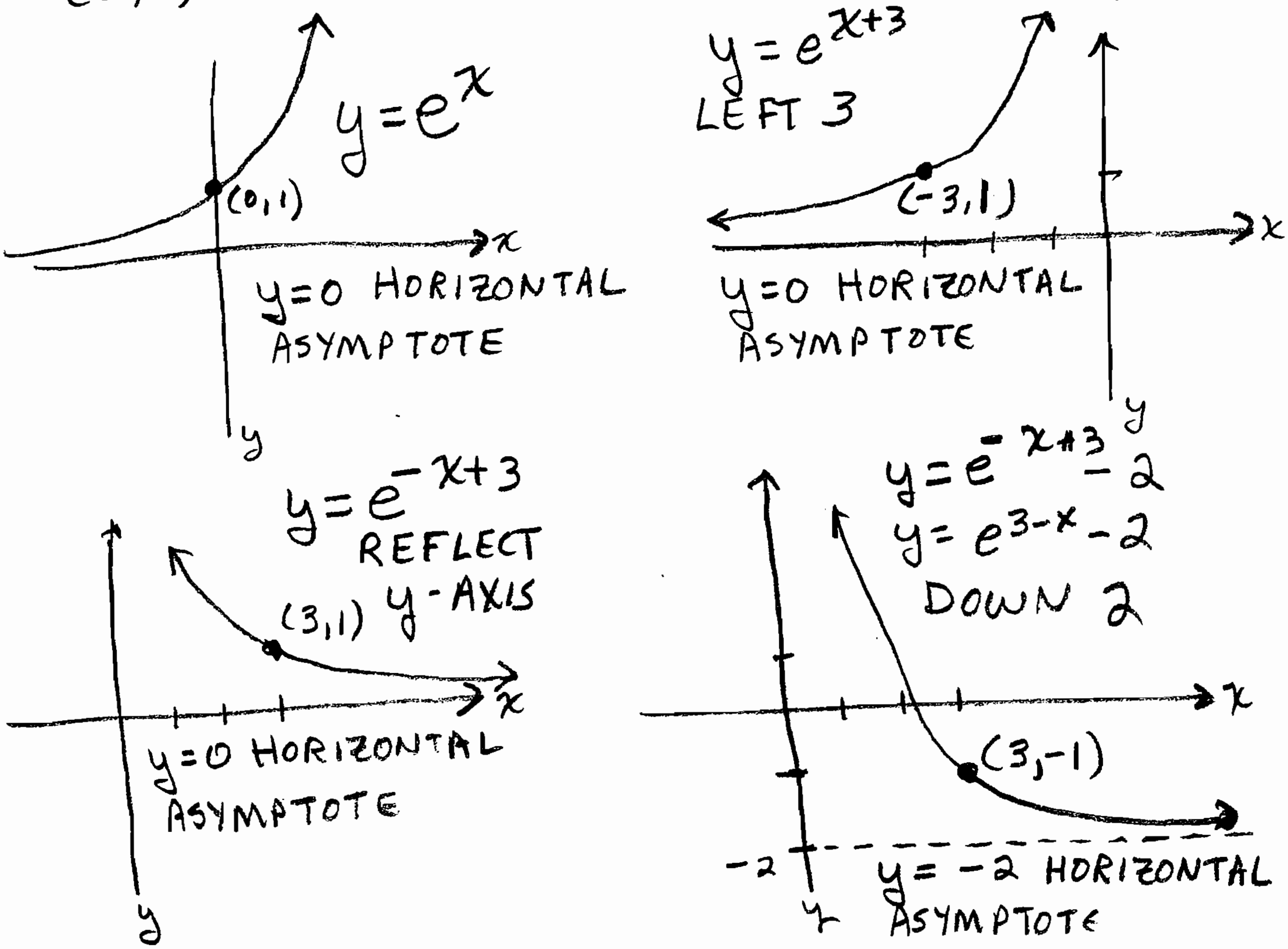






NOTE: IN THE FIRST TWO STAGES $y = 0$, THE x-AXIS, WAS A HORIZONTAL ASYMPTOTE. WHEN THE GRAPH WAS TRANSLATED UP 4, SO WAS THE HORIZONTAL ASYMPTOTE. IT MOVED FROM $y = 0$ TO $y = 4$.

G. SKETCH THE GRAPH OF $y = e^{3-x} - 2$ AS A SEQUENCE OF TRANSLATIONS, REFLECTIONS, AND STRETCH/COMPRESS. SHOW THE FATE OF $(0,1)$ AS IT MOVES THROUGH THE PROCESS.



<u>NOTE</u>: THE GRAPH OF $f(-x) = e^{-x+3}$ IS THE REFLECTION OF THE GRAPH OF $f(x) = e^{x+3}$ (SEE STAGES 2 AND 3).

H. HOMEWORK (OIS)

1. THE VALUE OF e APPROXIMATED TO ONE DECIMAL POINT IS ______.

2. SKETCH THE GRAPH OF EACH OF THE FOLLOWING. PLOT 5 POINTS FOR EACH ONE.

   a. $y = 4^x$ b. $y = \left(\frac{1}{e}\right)^x$

   c. $y = 3^{\frac{1}{2}x}$ d. $y = e^{-2x}$

3. SKETCH THE GRAPH OF EACH OF THE FOLLOWING AS A SEQUENCE OF TRANSLATIONS, REFLECTIONS, AND STRETCH/COMPRESS. SHOW THE OUTCOME OF (0,1) AS IT MOVES THROUGH THE PROCESS. NAME THE HORIZONTAL ASYMPTOTE AT EACH STAGE.

   a. $y = 2^{x+3} - 1$ b. $f(x) = e^{2-x}$

   c. $y = 3^{1-x} + 2$ d. $f(x) = 2^{3x-4} + 1$

   e. $y = \left(\frac{1}{2}\right)^{2-x} + 1$

I. LOGARITHMIC FUNCTIONS

1. $\log_a$ IS READ "LOG TO THE BASE a"
   IT IS THE BASE a LOGARITHM FUNCTION

2. <u>DEFINITION</u>: $\log_a(x) = y$ IFF $a^y = x$

   $\log_a(x)$ IS READ "LOG TO THE BASE a OF x" JUST LIKE $f(x)$ IS READ "f OF x" SINCE $\log_a$ IS A FUNCTION

   TRADITIONALLY, MANY TIMES THE PARENTHESES ARE LEFT OFF AND THE NOTATION IS $\log_a x$

3. <u>EXAMPLES</u>:

   a. $\log_3 9 = \boxed{2}$ SINCE $3^{\boxed{2}} = 9$

   b. $\log_3 81 =$

   $\log_3 81 = \boxed{4}$ SINCE $3^{\boxed{4}} = 81$

   c. $\log_3 \frac{1}{9} =$

   $\log_3 \frac{1}{9} = \boxed{-2}$ SINCE $3^{\boxed{-2}} = \frac{1}{3^2} = \frac{1}{9}$

d. $\log_3 1 =$

$\log_3 1 = \boxed{0}$ SINCE $3^{\boxed{0}} = 1$

e $\log_5 \boxed{} = 3$

$\log_5 \boxed{125} = 3$ SINCE $5^3 = \boxed{125}$

f. NOTE: FOR $a > 0$ $\log_a 1 = 0$
SINCE $a^0 = 1$

g. COMMON LOG = LOG TO BASE 10 (DEF.)

FOR COMMON LOG, THE SUBSCRIPT 10 CAN BE LEFT OFF: $\log_{10} x = \log x$

$\log x$ CAN BE READ "THE COMMON LOG OF x" OR "log TO THE BASE TEN OF x".

i. $\log 10 = 1$ SINCE $10^1 = 10$

ii. $\log 100 =$

$\log_{10} 100 = \boxed{2}$ SINCE $10^{\boxed{2}} = 100$

h. <u>NATURAL</u> <u>LOGARITHM</u> (DEF.) = LOG TO BASE e

NOTATION $\ln x = \log_e x$

$\ln x$ IS READ "THE NATURAL LOG OF x" OR "log TO THE BASE e OF x"

i. $\ln e = \boxed{1}$ SINCE $e^{\boxed{1}} = e$

ii. $\ln e^2 =$

$$\ln e^2 = \log_e e^2 = \boxed{2} \quad \text{SINCE } e^{\boxed{2}} = e^2$$

iii $\ln \frac{1}{e} = x$ WHAT IS x

$$\ln \frac{1}{e} = \log_e\left(\frac{1}{e}\right) = \boxed{-1} \quad \text{SINCE } e^{\boxed{-1}} = \frac{1}{e}$$

iv. IN EVALUATING NATURAL LOGS IT CAN BE HELPFUL TO VISUALIZE AN INVISIBLE e AS THE BASE

$$\ln e^2 = \ln_e e^2 = 2$$

VISUALIZE

J. LOGARITHMIC FORM VS EXPONENTIAL FORM

LOG FORM		EXPONENTIAL FORM
$\log_3 9 = 2$		$3^2 = 9$
$\log 100 = 2$		$10^2 = 100$
$\ln \frac{1}{e} = -1$		$e^{-1} = \frac{1}{e}$
	$\xleftarrow{?}$	$5^3 = 125$
	$\xleftarrow{?}$	$3^0 = 1$
	$\xleftarrow{?}$	$e^2 = e^2$
	$\xleftarrow{?}$	$2^{-3} = \frac{1}{8}$
$\ln x = y$		$e^y = x$
$\log 2z = x$		$10^x = 2z$
	$\xleftarrow{?}$	$5^{2x} = m$
	$\longleftarrow$	$a^b = c$

K. HOMEWORK (OIS)

1. EVALUATE EACH OF THE FOLLOWING

a. $\log_2 8 =$ b. $\log_2\left(\frac{1}{8}\right)$

c. $\ln\sqrt{e} =$ d. $\log\sqrt[3]{10}$

e. $\ln 1 =$ f. $\log\frac{1}{1000}$

2. WHAT IS x?

a. $\log_2 x = 5$ b. $\log_7 x = 0$

c. $\log_x 16 = 2$ d. $\log_x e^3 = 3$

e. $\frac{\log_2 8}{\log_2 4} = x$ f. $\log_x \frac{1}{125} = -3$

3. WRITE EACH EXPONENTIAL FORM IN LOG FORM.

a. $5^4 = 625$ b. $2^{-4} = \frac{1}{16}$

c. $7^0 = 1$ d. $e^{1/2} = \sqrt{e}$

4. WRITE EACH LOG FORM IN EXPONENTIAL FORM.

a. $\log\frac{1}{\sqrt{10}} = -\frac{1}{2}$ b. $\log_2 32 = 5$

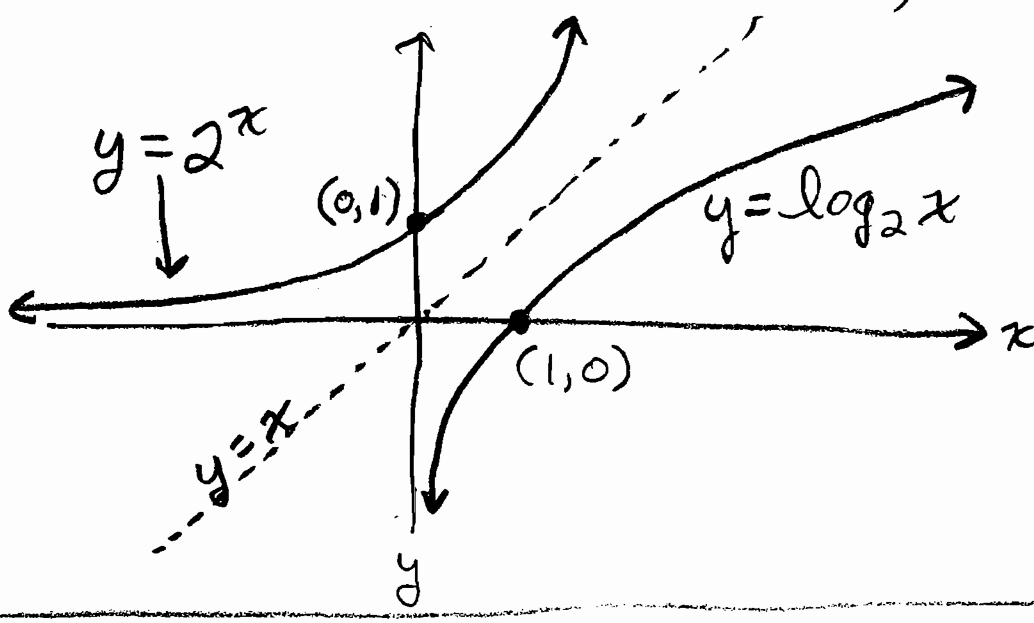
L. LOG GRAPHS

1. LET $g(x) = \log_2 x = y \quad f(y) = 2^y = x$

$(x, y) \in g$ IFF $(y, x) \in f$

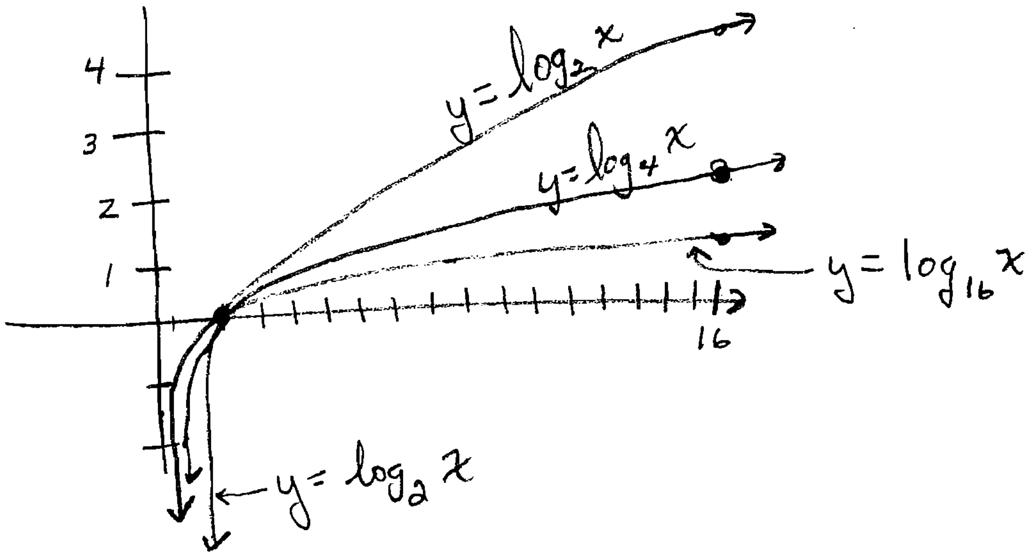
f IS AN EXPONENTIAL FUNCTION. f IS 1-1. f AND g ARE INVERSES OF EACH OTHER. <u>LOG FUNCTIONS</u> AND <u>EXPONENTIAL FUNCTIONS</u> <u>ARE INVERSES OF EACH OTHER</u>

2. GRAPHS OF $y = 2^x$ AND $y = \log_2 x$ (INVERSES OF EACH OTHER)

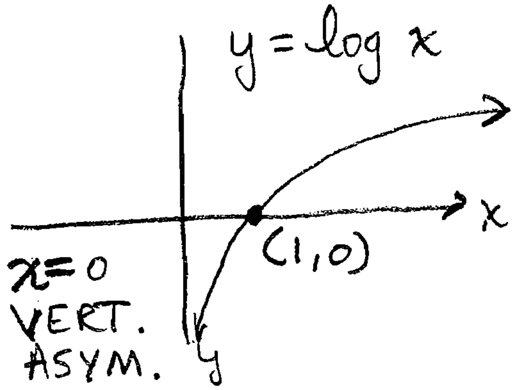


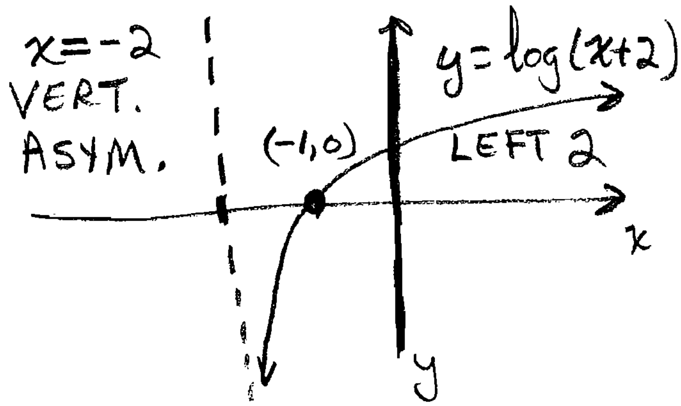
$\boxed{\begin{array}{l} \underline{\text{NOTE:}} \ \mathrm{dom}(\log_2) = (0, \infty) \\ \mathrm{ran}(\log_2) = (-\infty, \infty) \\ \log_2 1 = 0 \end{array}}$

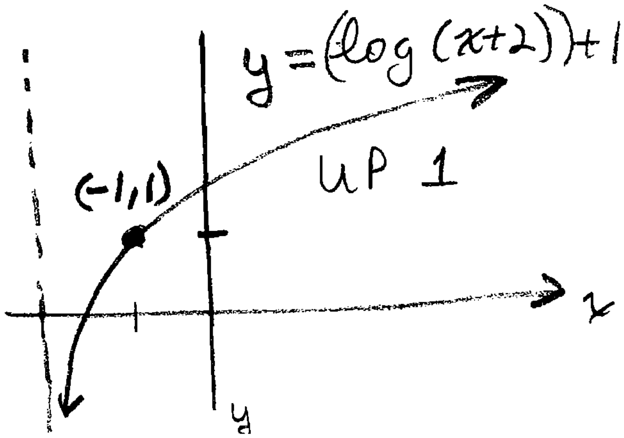
M. FOR $a>1$, $\log_a$ GRAPHS HAVE THE SAME BASIC SHAPE



N. GRAPH $y=(\log(x+2))+1$ AS A SEQUENCE OF REFLECTIONS, TRANSLATIONS, STRETCH\COMPRESS NAME VERTICAL ASYMPTOTE. SHOW THE FATE OF (1,0) THROUGH THE PROCESS.







$x=-2$ VERTICAL ASYMPTOTE

O. FINDING THE DOMAIN OF LOG FUNCTIONS

1. RECALL $\text{dom}(\log) = (0,\infty) = \{x \mid x > 0\}$
2. LET $h(x) = \log_2(x^2-9)$ FIND $\text{dom}(h)$

$$x^2 - 9 > 0$$
$$x^2 > 9$$
$$\sqrt{x^2} > 3$$
$$|x| > 3$$
$$x > 3 \quad \text{OR} \quad x < -3$$
$$\text{dom}(h) = (-\infty, -3) \cup (3, \infty)$$

<u>OBSERVATION</u>: LET $f(x) = \log_2 x$
$g(x) = x^2 - 9$.

$(f \circ g)(x) = f(g(x)) = f(x^2-9) = \log_2(x^2-9)$
$= h(x)$. SO $h(x) = \log_2(x^2-9)$ IS A COMPOSITION OF FUNCTIONS.

P. LOG PROPERTIES

1. $$\boxed{\log MN = \log M + \log N}$$

a. EXAMPLES

$$\log 5x = \log 5 + \log x$$

$$\log 10x = \log 10 + \log x = 1 + \log x$$

$$\ln 3y = \ln 3 + \ln y$$

$$\log_2 5xy = \log_2 5 + \log_2 xy = \log_2 5 + \log_2 x + \log_2 y$$

b. PROOF OF $\log MN = \log M + \log N$

LET $\log M = x$ AND $\log N = y$

$10^x = M$ AND $10^y = N$

$M \cdot N = 10^x \cdot 10^y = 10^{x+y}$ EXP. FORM

$$\log_{10} MN \underset{\text{FORM}}{\overset{\text{LOG}}{=}} x+y = \log M + \log N$$

$$\log MN = \log M + \log N$$

c. FORMULA TRUE FOR ALL $a > 0$

$$\log_a MN = \log_a M + \log_a N$$

2. $$\boxed{\log\left(\frac{M}{N}\right) = \log M - \log N}$$

a. WRITE AS A SUM OR DIFFERENCE OF LOGS.

$$\log \frac{5}{x} = \log 5 - \log x$$

$$\log_2 \frac{1}{8} = \log_2 1 - \log_2 8 = 0 - 3 = -3$$

$$\log_2 \frac{xy}{5} = \log_2 xy - \log_2 5$$

$$= \log_2 x + \log_2 y - \log_2 5$$

$$\ln \frac{(x+1)}{yz} = \ln(x+1) - \ln yz$$

$$= \ln(x+1) - [\ln y + \ln z]$$

$$= \ln(x+1) - \ln y - \ln z$$

b. WRITE AS A SINGLE LOG

$$\log 3 - \log y = \log \frac{3}{y}$$

$$\ln 2 - \ln x + \ln z - \ln w =$$

$$\ln 2 + \ln z - (\ln x + \ln w) =$$

$$\ln 2z - \ln(xw) = \ln \frac{2z}{xw}$$

3. $\boxed{\log M^N = N \log M}$ $\left(= \log(M^N)\right.$ NOT $\left.(\log M)^N\right)$

a. EXAMPLES:

$$\log 10^x = x \log 10 = x \cdot 1 = x$$

$$\log \sqrt{x} = \log x^{1/2} = \tfrac{1}{2} \log x$$

$$\ln\left(\frac{\sqrt[3]{xy}}{e}\right) = \ln\left(\frac{(xy)^{1/3}}{e}\right) = \ln(xy)^{1/3} - \ln e$$

$$= \left(\tfrac{1}{3} \ln xy\right) - 1 = \tfrac{1}{3}(\ln x + \ln y) - 1$$

$$5 \log_2(x+1) = \log_2(x+1)^5$$

b. PROOF OF $\log M^N = N \log M$

LET $\log M = x$

$$10^x = M$$

$$10^{xN} = (10^x)^N = M^N$$

$$10^{Nx} = M^N \leftarrow \text{EXPONENTIAL FORM}$$

$$\log_{10} M^N = N x = N \log M \quad \text{LOG FORM}$$

$$\log M^N = N \log M$$

c. ALSO TRUE FOR OTHER BASES BESIDES 10

$$\log_a M^N = N \log_a M$$

4. $\boxed{\log_a a^x = x}$

a. EXAMPLES:

$\log_2 2^x = x$ $\qquad$ $\log_3 3^{x^2+1} = x^2+1$

$\ln e^x = x$ $\qquad$ $\log 10^{x^2+1} = x^2+1$

b. NOTE: $f(x) = a^x$ EXPONENTIAL FUNCTION

$f^{-1}(x) = \log_a x$ LOG FUNCTION

$\log_a a^x = f^{-1}(f(x)) = x$

5. $\boxed{a^{\log_a x} = x}$ $\qquad$ $e^{\ln \square} = \square$

a. EXAMPLES:

$2^{\log_2 x} = x$ $\qquad$ $e^{\ln(x^2+1)} = x^2+1$

$10^{\log 5x} = 5x$ $\qquad$ $3^{\log_3(2x+3)} = 2x+3$

b. NOTE: $f(x) = a^x$ EXPONENTIAL FUNCTION

$f^{-1}(x) = \log_a x$ LOG FUNCTION

$a^{\log_a x} = f(f^{-1}(x)) = x$

c. NOTE (IF YOU CAN): SINCE

$\log_a x = \log_a x$, $\qquad$ $a^{\log_a x} = x$

LOG FORM $\qquad$ EXPONENTIAL FORM

6. BASE CHANGE FORMULA

$$\boxed{\log_a M = \frac{\log_b M}{\log_b a}}$$

a. EXAMPLES $\log_7 x = \dfrac{\log_2 x}{\log_2 7} =$

$$\frac{\log_3 x}{\log_3 7} = \frac{\log_5 x}{\log_5 7} = \frac{\log_{10} x}{\log_{10} 7} = \frac{\log x}{\log 7}$$

$$= \frac{\log_e x}{\log_e 7} = \frac{\ln x}{\ln 7} = \left(\frac{1}{\ln 7}\right) \ln x = \log_7 x$$

b. MOST CALCULATORS ONLY HAVE ln OR log FUNCTIONS, SO IF ASKED TO FIND $\log_7 3$, FIND $\dfrac{\ln 3}{\ln 7}$ (PART a. $x = 3$)

c. SPECIAL CASE PROOF: LET $\log_7 x = y$

$7^y = x \;.\!\rightarrow\; \log_2 7^y = \log_2 x$

$y \log_2 7 = \log_2 x \;.\!\rightarrow\; (\log_7 x)\log_2 7 = \log_2 x$

$$\boxed{\log_7 x = \frac{\log_2 x}{\log_2 7}}$$

Q. USE LOG PROPERTIES TO WRITE AS SUM/DIFFERENCE OF TERMS LIKE $a \log b$ (WHERE $b \neq y^n$ $n \neq 1$)

$$\ln \frac{\sqrt{xy}}{e^2 z} = \ln\sqrt{xy} - \ln e^2 z$$

$$= \ln(xy)^{1/2} - (\ln e^2 + \ln z) =$$

$$\tfrac{1}{2}\ln xy - (2\ln e + \ln z) =$$

$$\tfrac{1}{2}(\ln x + \ln y) - 2\ln e - \ln z =$$

$$\tfrac{1}{2}\ln x + \tfrac{1}{2}\ln y - 2\ln e - \ln z$$

R. WRITE AS A SINGLE LOG

$$2\log x^3 - 5\log(x^2+1) + \tfrac{1}{2}\log(x^4+1) =$$

$$\log(x^3)^2 - \log(x^2+1)^5 + \log(x^4+1)^{1/2} =$$

$$\log x^6 + \log(x^4+1)^{1/2} - \log(x^2+1)^5 =$$

$$\log x^6(x^4+1)^{1/2} - \log(x^2+1)^5 =$$

$$\log \frac{x^6(x^4+1)^{1/2}}{(x^2+1)^5} = \log \frac{x^6\sqrt{x^4+1}}{(x^2+1)^5}$$

S. **JUGULAR PROBLEM # 11 : UNKNOWN IN EXPONENT : SOLVE $2^{3x-1} = 7^x$**

TAKE A LOGARITHM OF BOTH SIDES

$$\ln 2^{3x-1} = \ln 7^x$$

SOLVE FOR x

$$(3x-1)\ln 2 = x \ln 7$$

$$3x \ln 2 - \ln 2 = x \ln 7$$

$$3x \ln 2 - x \ln 7 = \ln 2$$

$$x(3\ln 2 - \ln 7) = \ln 2$$
$$x(\ln 2^3 - \ln 7) = \ln 2$$
$$x(\ln 8 - \ln 7) = x \ln \tfrac{8}{7} = \ln 2$$

$$x = \frac{\ln 2}{\ln 8/7} \qquad \text{SOL. SET} = \left\{ \frac{\ln 2}{\ln \frac{8}{7}} \right\}$$

(DO NOT WRITE THIS AS $\ln 2 - \ln 8/7$. THERE IS A DIFFERENCE BETWEEN

$\dfrac{\ln 2}{\ln 8/7}$ AND $\ln\left(\dfrac{2}{8/7}\right)$

CONTINUED)

IT DOES NOT MATTER WHICH BASE IS USED ON THE LOGARITHM TO SOLVE THIS PROBLEM. TO ILLUSTRATE WE WILL REWORK IT WITH THE BASE 10 LOGARITHM.

$$2^{3x-1} = 7^x$$

TAKE A LOGARITHM OF BOTH SIDES

$$\log 2^{3x-1} = \log 7^x$$

SOLVE FOR x

$$(3x-1)\log 2 = x \log 7$$

$$3x \log 2 - \log 2 = x \log 7$$

$$3x \log 2 - x \log 7 = \log 2$$

$$x(3\log 2 - \log 7) = \log 2$$

$$x(\log 2^3 - \log 7) = \log 2$$

$$x(\log 8 - \log 7) = \log 2$$

$$x\left(\log \tfrac{8}{7}\right) = \log 2$$

$$x = \frac{\log 2}{\log 8/7} \qquad \text{SOL. SET} = \left\{ \frac{\log 2}{\log \frac{8}{7}} \right\}$$

14 - 299

T. HOMEWORK (OIS)

1. SKETCH THE GRAPH OF $y = \log_3 x$
PLOT THE POINTS WHERE $x = \frac{1}{3}, 1, 3, 9$

2. SKETCH THE GRAPHS OF EACH OF THE FOLLOWING AS A SEQUENCE OF REFLECTIONS, TRANSLATIONS, STRETCH\COMPRESS . NAME THE VERTICAL ASYMPTOTES. SHOW THE FATE OF (1,0) THROUGH THE PROCESS.

a. $y = (\ln(x-3)) - 2$

b. $y = (2\log(x-1)) + 3$

c. $y = -1 - 3\log(2-x)$

3. FIND THE DOMAIN OF EACH OF THE FOLLOWING.

a. $f(x) = \ln 2x - 1$ b. $f(x) = \ln(2x^2 - 5)$

4. LET $h(x) = \ln(5x^2 - 1)$. FIND FUNCTIONS f, g SUCH THAT $h(x) = (f \circ g)(x)$

5. SUPPOSE YOU HAVE A CALCULATOR THAT HAS ONLY THE FUNCTIONS ln AND log. WHAT WOULD YOU EVALUATE TO EVALUATE EACH OF THE FOLLOWING?

a. $\log_5 3$ b. $\log_2 3.2$

6. USE LOG PROPERTIES TO WRITE AS SUM/DIFFERENCE OF TERMS LIKE $a \log b$ (WHERE $b \neq y^n$ $n \neq 1$)

a. $\log_5 xy^2$

b. $\log 3x^2\sqrt{y}$

c. $\ln \sqrt[3]{\frac{xy^2}{z}}$

d. $\ln \frac{\sqrt[3]{xz}}{5y^2}$

7. WRITE AS A SINGLE LOG

a. $\ln x - 3\ln y$

b. $\frac{1}{2}\ln x + 4\ln y - 2\ln z$

c. $\frac{1}{3}\ln x - 2\ln y + 3\ln z$

d. $5\ln(x^2+1) - \frac{1}{2}\ln z + 3\ln x - \frac{1}{4}\ln y$

8. UNKNOWN IN THE EXPONENT; SOLVE FOR x

a. $3^{5x-2} = 4^x$

b. $2^{3x+1} = 5^{2x}$

c. $e^{12x-1} = 10^{3x}$

[CHAPTER 15]

SYNTHETIC DIVISION AND POLYNOMIAL FACTORING

A. SYNTHETIC DIVISION: $\dfrac{\text{POLYNOMIAL}}{x - c}$

1. FIND $\dfrac{2x^3 - 5x^2 + 4x - 8}{x - 3}$ BY SYNTHETIC DIVISION

PUT C DOWN WHEN DIVIDING BY $x - c$

PUT COEFFICIENTS OF x^3 x^2 x AND CONSTANT DOWN IN ORDER

$$\begin{array}{r|rrrr} 3 & 2 & -5 & 4 & -8 \\ & & 6 & 3 & 21 \\ \hline & 2 & 1 & 7 & \boxed{13} \end{array}$$

1st BRING 2 DOWN

2nd MULTIPLY 3(2)

3rd ADD

4th MULTIPLY 3(1)

... CONTINUE TO THE END.

THIS IS THE REMAINDER WHEN DIVIDING BY $x - 3$

ANSWER: $\dfrac{2x^3 - 5x^2 + 4x - 8}{x - 3} =$

$$2x^2 + 1x + 7 + \frac{13}{x - 3}$$

2. SYNTHETIC DIVISION WHERE ONE OF THE POWERS OF x IS MISSING (ALSO, CAREFUL WITH THE PLUS SIGN WHEN DIVIDING) FIND $\dfrac{-4x^3+15x+4}{x+2}$

$x+2 = x-(-2)$

NOTE: $-4x^3+15x+4 = -4x^3+0x^2+15x+4$

$$\begin{array}{r|rrrr} -2 & -4 & 0 & 15 & 4 \\ & & 8 & -16 & +2 \\ \hline & -4 & 8 & -1 & \boxed{6} \end{array}$$

$$\frac{-4x^3+15x+4}{x+2} = -4x^2+8x-1 + \frac{6}{x+2}$$

3. NOTE: REMAINDER $=0$ WHEN $x-c$ IS A FACTOR. CONSIDER $\dfrac{x^2-x-6}{x-3}$

$$\begin{array}{r|rrr} 3 & 1 & -1 & -6 \\ & & 3 & 6 \\ \hline & 1 & 2 & \boxed{0} \end{array}$$

THIS MEANS $x-3$ IS A FACTOR

$$\frac{x^2-x-6}{x-3} = x+2, \text{ SO } x^2-x-6 = (x-3)(x+2)$$

SYNTHETIC DIVISION CAN HELP YOU FACTOR

4. SUPPOSE YOU FORGOT HOW TO FACTOR $8x^3-27$ BUT REMEMBER $2x-3$ IS A FACTOR. DIVIDE USING SYNTHETIC DIVISION.

$$\frac{8x^3-27}{2x-3} = \frac{8x^3-27}{2(x-3/2)} = \frac{\frac{1}{2}(8x^3-27)}{x-3/2} = \frac{4x^3-\frac{27}{2}}{x-3/2}$$

$$\begin{array}{r|rrrr} \frac{3}{2} & 4 & 0 & 0 & -\frac{27}{2} \\ & & 6 & 9 & \frac{27}{2} \\ \hline & 4 & 6 & 9 & \boxed{0} \end{array}$$

NOTE: A CO-EFFICIENT OF 1 IS NEEDED FOR SYNTHETIC DIVISION

$$\frac{8x^3-27}{2x-3} = 4x^2+6x+9$$ SO

$$8x^3-27 = (2x-3)(4x^2+6x+9)$$

B. <u>REMAINDER</u> <u>THEOREM</u>: IF $f(x)$ IS A POLYNOMIAL, THEN $f(c)$ IS THE <u>REMAINDER</u> WHEN YOU DIVIDE BY $x-c$.

ILLUSTRATION: EARLIER WE SAW

$$f(x) = 2x^3-5x^2+4x-8$$

$$f(3) = 2(3^3)-5(3^2)+4(3)-8 = 13$$

NOW DIVIDE $f(x) = 2x^3 - 5x^2 + 4x - 8$ BY $x-3$

$$\begin{array}{r|rrrr} 3 & 2 & -5 & 4 & -8 \\ & & 6 & +3 & 21 \\ \hline & 2 & +1 & 7 & \boxed{13} \end{array} = f(3)$$

REMAINDER

REASON THIS WORKS:

$f(x) = 2x^3 - 5x^2 + 4x - 8 = (x-3)(2x^2 + x + 7) + 13$

$f(3) = (3-3)(2(3^2) + 3 + 7) + 13 = 0 + 13 = \underline{13}$

C. YOU CAN USE SYNTHETIC DIVISION TO EVALUATE A FUNCTION AT C; JUST GET THE REMAINDER WHEN DIVIDING BY $x - c$.

FOR $f(x) = 2x^5 - 25x^4 + 11x^3 + 14x^2 - 26x + 30$ FIND $f(12)$ (I.E. FIND THE REMAINDER WHEN DIVIDING BY $x - 12$)

$$\begin{array}{r|rrrrrr} 12 & 2 & -25 & 11 & 14 & -26 & 30 \\ & & 24 & -12 & -12 & 24 & -24 \\ \hline & 2 & -1 & -1 & 2 & -2 & \boxed{6} \end{array}$$

$f(12) = 6$.

THIS IS MUCH EASIER THAN CALCULATING $f(12) = 2(12^5) - 25(12^4) + 11(12^3) + 14(12^2) - 26(12) + 30$ TO GET 6.

D. FACTOR THEOREM. GIVEN POLYNOMIAL $f(x)$. $x-c$ IS A FACTOR OF $f(x)$ IFF $f(c)=0$.

1. SUPPOSE $x-c$ IS A FACTOR OF $f(x)$

   $f(x) = (x-c)(\text{SOMETHING})$

   $f(c) = (c-c)(\text{SOMETHING}) = 0$

2. SUPPOSE $f(c)=0$. BY THE REMAINDER THEOREM, 0 IS THE REMAINDER WHEN DIVIDING BY $x-c$, SO $x-c$ IS A FACTOR.

3. IS $x-1$ A FACTOR OF x^6-1?

   LET $f(x) = x^6-1$. $f(1) = 1^6-1 = 0$

   YES, $x-1$ IS A FACTOR.

4. IS $x+1$ A FACTOR OF $x^{10}+1$?

   LET $f(x) = x^{10}+1$. $f(-1) = (-1)^{10}+1 = 1+1 \neq 0$.

   NO, $x-(-1)$ IS NOT A FACTOR OF $x^{10}+1$.

   THIS IS MUCH QUICKER THAN DOING THE FOLLOWING SYNTHETIC DIVISION

$$
\begin{array}{r|rrrrrrrrrrr}
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
 & & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
\hline
 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & \boxed{2}
\end{array}
$$

E. FUNDAMENTAL THEOREM OF ALGEBRA BUILDUP. LET $f(x) = x^6 - 1$

$$x^6 - 1 = (x^3 - 1)(x^3 + 1)$$
$$= (x-1)(x^2+x+1)(x+1)(x^2-x+1)$$

NOTE SO FAR: THIS POLYNOMIAL COULD BE BROKEN DOWN INTO LINEAR AND QUADRATIC FACTORS (THAT IS BECAUSE ALL POSITIVE DEGREE POLYNOMIALS CAN)

$x^2 + x + 1 = 0$ $\quad x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$$x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

SIMILARILY FOR $x^2 - x + 1 = 0$ $\quad x = \dfrac{1 \pm i\sqrt{3}}{2}$

$$x^6 - 1 = (x-1)\cdot\left(x - \left[\frac{-1+i\sqrt{3}}{2}\right]\right)\cdot\left(x - \left[\frac{-1-i\sqrt{3}}{2}\right]\right)\cdot(x+1)\cdot$$
$$\left(x - \left[\frac{1+i\sqrt{3}}{2}\right]\right)\cdot\left(x - \left[\frac{1-i\sqrt{3}}{2}\right]\right)$$

SO THIS POLYNOMIAL CAN BE BROKEN DOWN INTO A PRODUCT OF LINEAR FACTORS OVER THE COMLEX NUMBERS - THIS WILL BE TRUE FOR ALL POSITIVE DEGREE POLYNOMIALS

F. **FUNDAMENTAL THEOREM OF ALGEBRA (IN DISGUISE): EVERY POLYNOMIAL OF POSITIVE DEGREE CAN BE BROKEN DOWN INTO A PRODUCT OF LINEAR FACTORS OVER THE COMPLEX NUMBERS.**

1. THE FUNDAMENTAL THEOREM OF ALGEBRA WAS JUST ILLUSTRATED FOR $f(x) = x^6 - 1$. IT WAS FACTORED COMPLETELY.

2. WHEN THE HOMEWORK ASKS YOU TO ILLUSTRATE THE FUNDAMENTAL THEOREM OF ALGEBRA FOR A PROBLEM, IT MEANS TO FACTOR COMPLETELY INTO A PRODUCT OF LINEAR FACTORS.

3. THE REASON THE WORDS "POSITIVE DEGREE" ARE IN THE THEOREM IS A <u>ZERO</u> DEGREE POLYNOMIAL, LIKE $f(x) = 7$, CANNOT BE WRITTEN AS A PRODUCT OF <u>LINEAR</u> FACTORS.

G. HOMEWORK (OIS)

1. USE SYNTHETIC DIVISION TO PERFORM EACH OF THE FOLLOWING DIVISIONS. WRITE YOUR ANSWER IN BOTH OF THE FOLLOWING FORMS $\frac{T}{B} = Q + \frac{R}{B}$ $\quad T = B \cdot Q + R$

   a. $\dfrac{5x^3 - 7x^2 + 3x - 1}{x - 2}$ b. $\dfrac{4x^3 - 5x^2 + 2x + 7}{x + 3}$

   c. $\dfrac{2x^7 - 3x^2 + 4}{x + 2}$ d. $\dfrac{3x^3 - 5x^2 + 2}{2x + 1}$

2. FIND $f(c)$ FOR EACH $f(x)$ BELOW USING SYNTHETIC DIVISION.

   a. $f(x) = 2x^2 - 17x + 10$
   $f(8) =$ $\quad c = 8$

   b. $f(x) = x^2 - 8x - 10$
   $f(9) =$ $\quad c = 9$

   c. $f(x) = x^4 - 145x^3 + 143x^2 + 145x - 146$
   $f(144) =$ $\quad c = 144$

3. IS $x - 2$ A FACTOR OF $x^8 - 256$?

4. ILLUSTRATE THE FUNDAMENTAL THEOREM OF ALGEBRA BY FACTORING INTO PRODUCTS OF LINEAR FACTORS

   a. $x^4 - 1$ b. $x^7 - 8x^4 - x^3 + 8$

[CHAPTER 16]

SOLVING SYSTEMS OF LINEAR EQUATIONS

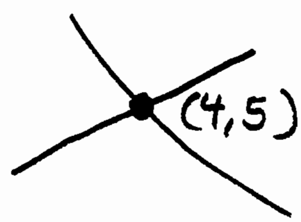
A. EXAMPLE OF A SYSTEM OF 2 EQUATIONS IN 2 UNKNOWNS

$$2x + 3y = 23$$
$$3x - 4y = -8$$

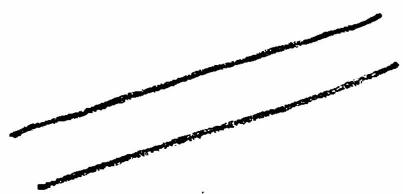
B. GRAPHS OF LINEAR EQUATIONS ARE LINES. GRAPH FOR 2 EQUATIONS CAN:

DEFINITIONS

1. INTERSECT IN EXACTLY ONE POINT. THE LINEAR SYSTEM IS INDEPENDENT



2. BE PARALLEL
NO SOLUTION
INCONSISTENT



3. BOTH EQUATIONS HAVE THE SAME GRAPH.
DEPENDENT
INFINITELY MANY SOLUTIONS

C. REDUCTION METHOD FOR SOLVING A LINEAR SYSTEM OF 2 EQUATIONS, 2 UNKNOWNS.

1. AN INDEPENDENT SYSTEM

a. $2x+3y=23$ } GIVEN
b. $3x-4y=-8$ }

c. $-6x-9y=-69$ MULTIPLY a BY -3

d. $6x-8y=-16$ MULTIPLY b BY 2

e. $-17y=-85$ ADD c AND d. THIS REDUCES TO A SIMPLE EQUATION

f. $y=\frac{-85}{-17}=5$ DIVIDE e BY -17

g. $2x+3(5)=23$ SUBSTITUTE 5 FOR y IN EQUATION a

h. $2x+15=23$ g

i $2x=8$ h

j $x=4$ i, DIVIDE BY 2

SOLUTION SET $\{(4,5)\}$

NOTE: YOU MULTIPLY EACH EQUATION BY A NUMBER SO THAT WHEN THE EQUATIONS ARE ADDED A VARIABLE CANCELS

2. AN <u>INCONSISTENT</u> SYSTEM

a. $3x + 4y = 2$
b. $6x + 8y = -1$
} GIVEN

c. $-6x - 8y = -4$ MULTIPLY a BY -2

d. $0 = -5$ ADD LINES b AND c

WE ASSUMED (x, y) WAS A SOLUTION TO THE GIVEN SYSTEM AND GOT A CONTRADICTION. HENCE, NO SOLUTION

<u>INCONSISTENT</u> SOL. SET $= \{\ \}$

3. A <u>DEPENDENT</u> SYSTEM

a. $10x + 4y = 8$
b. $15x + 6y = 12$
} GIVEN

c. $-30x - 12y = -24$ MULTIPLY a BY -3

d. $30x + 12y = 24$ MULTIPLY b BY 2

e. $0 = 0$ ADD LINES c AND d

SINCE BOTH SIDES OF THE EQUATION EQUAL 0, ONE EQUATION WAS JUST -1 TIMES THE OTHER, HENCE, THE SAME LINE.

<u>DEPENDENT</u>

SOLUTION SET $\{(x, y) \mid 10x + 4y = 8\}$

AN INFINITE SET

D. SUBSTITUTION METHOD FOR SOLVING A LINEAR SYSTEM OF 2 EQUATIONS, 2 UNKNOWNS

1. AN INDEPENDENT SYSTEM

a. $2x + 3y = 23$
b. $3x - 4y = -8$

GIVEN

c. $2x = 23 - 3y$
d. $x = \dfrac{23 - 3y}{2}$

SOLVE EQUATION a FOR x

e. $3\left(\dfrac{23-3y}{2}\right) - 4y = -8$
f. $\dfrac{69 - 9y}{2} - 4y = -8$
g. $\dfrac{69 - 9y - 8y}{2} = -8$
h. $69 - 9y - 8y = -16$
i. $69 - 17y = -16$
j. $-17y = -85$
k. $y = \dfrac{-85}{-17} = 5$

SUBSTITUTE $x = \dfrac{23 - 3y}{2}$ IN FOR x IN THE OTHER EQUATION (b) AND SOLVE FOR y.

l. $2x + 3(5) = 23$
m $2x + 15 = 23$
n $2x = 8$
o $x = 4$

SUBSTITUTE $y=5$ IN EQUATION a AND SOLVE FOR x

SOLUTION SET $\{(4,5)\}$

2. AN <u>INCONSISTENT</u> SYSTEM (SUBSTITUTION METHOD)

a. $3x + 4y = 2$
b. $6x + 8y = -1$
GIVEN

c. $3x = 2 - 4y$
d. $x = \dfrac{2 - 4y}{3}$
SOLVE FOR x IN EQUATION a

e. $6\left(\dfrac{2-4y}{3}\right) + 8y = -1$
f. $2(2-4y) + 8y = -1$
g. $4 - 8y + 8y = -1$
<u>SUBSITUTE</u> $x = \dfrac{2-4y}{3}$ IN <u>OTHER</u> EQUATION (b)

h. $4 = -1$ A CONTRADICTION WAS ARRIVED AT ASSUMING (x, y) WAS A SOLUTION TO THE GIVEN SYSTEM. HENCE, <u>NO</u> <u>SOLUTION</u>

INCONSISTENT

3. A <u>DEPENDENT</u> SYSTEM OF EQUATIONS ATTEMPTED TO BE SOLVED BY THE SUBSTITUTION METHOD WOULD YIELD $0 = 0$ AS IN THE REDUCTION METHOD.

E. HOMEWORK (OIS)

SOLVE EACH OF THE FOLLOWING BY (1) REDUCTION METHOD (2) SUBSTITUTION METHOD.

1. $6x - 7y = 51$
   $5x + 2y = 19$

2. $12x + 4y = 11$
   $3x - 8y = -4$

3. $5x - 3y = 2$
   $-\frac{5}{3}x = -y - \frac{2}{3}$

4. $y = \frac{1}{7} + \frac{2}{7}x$
   $4x - 14y = 6$

F. AN INDEPENDENT SYSTEM OF
3 EQUATIONS 3 UNKNOWNS
(REDUCTION METHOD)

$$\left.\begin{array}{ll} a. & 2x-5y+4z=11 \\ b. & 3x+4y-2z=4 \\ c. & 5x-2y-4z=-4 \end{array}\right\} \text{GIVEN}$$

ELIMINATE x AND REDUCE TO 2-EQUATIONS, 2 UNKNOWNS.

d. $6x-15y+12z=33$ MULTIPLY a BY 3

e. $-6x-8y+4z=-8$ MULTIPLY b BY -2

f. $\boxed{-23y+16z=25}$ ADD d AND e

g. $15x+20y-10z=20$ MULTIPLY b BY 5

h. $-15x+6y+12z=12$ MULTIPLY c BY -3

i. $\boxed{26y+2z=32}$ ADD g AND h

NOW SOLVE THE SYSTEM OF 2-EQUATIONS, 2-UNKNOWNS (f, i) AS PREVIOUSLY LEARNED. (ELIMINATE z)

j. $-208y-16z=-256$ MULTIPLY i BY -8

k. $-231y=-231$ ADD f AND j

l. $y=1$

NOW SUBSTITUTE BACK INTO ξ OR i

m. $26(1)+2z=32$ SUBSTITUTE $y=1$ IN i

n. $2z=6$ m

o. $z=3$

NOW SUBSTITUTE BACK INTO a, b, OR c

p. $2x-5(1)+4(3)=11$ SUBSTITUTE $y=1$ AND $z=3$ INTO a

q. $2x-5+12=11$ p

r. $2x=11-12+5=4$ q

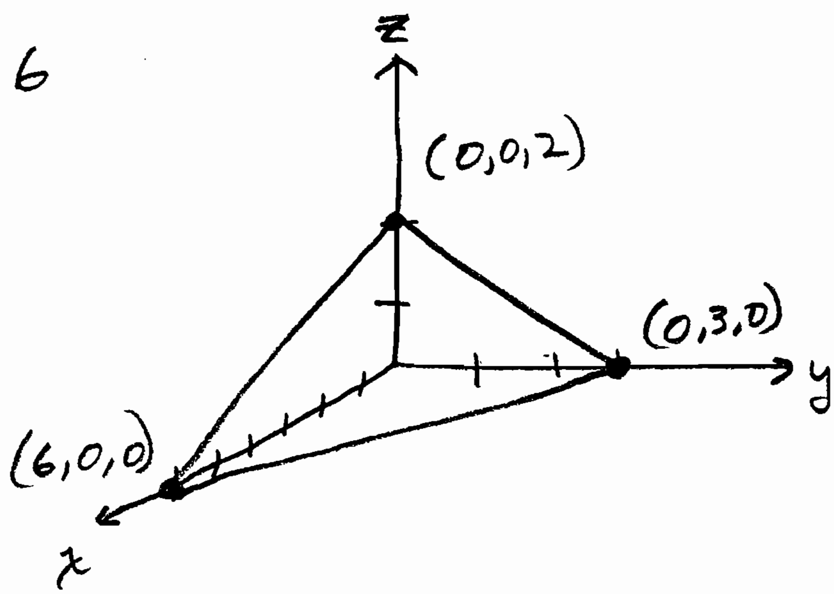
s. $x=2$ r

SOLUTION SET $\{(2,1,3)\}$

G. THE GRAPH OF A LINEAR EQUATION IN 3-UNKNOWNS IS A PLANE

$x+2y+3z=6$

x	y	z
0	0	2
0	3	0
6	0	0



H. WHAT CAN HAPPEN TO 3-PLANES

1. THEIR INTERSECTION CAN BE EXACTLY ONE POINT (THE LINEAR SYSTEM WE JUST LOOKED AT WAS AN EXAMPLE OF THIS) <u>INDEPENDENT</u>

2. THEIR INTERSECTION CAN BE EXACTLY A LINE. <u>DEPENDENT</u>

3. THEIR INTERSECTION CAN BE EXACTLY A PLANE. <u>DEPENDENT</u>

4. THERE CAN BE NO POINT THAT SATISFIES ALL 3 EQUATIONS FOR THE PLANES. <u>INCONSISTENT</u>

I. FINDING THE SOLUTION WHEN 2 PLANES INTERSECT IN A LINE.

$$\left.\begin{array}{ll} a. & 4x - y - 4z = 11 \\ b. & 4x - 3y + 4z = 25 \end{array}\right\} \text{GIVEN}$$

c. $8x - 4y = 36$ ELIMINATE A VARIABLE ADD a AND b

$$\left.\begin{array}{lll} d. & 8x - 36 & = 4y \\ e. & 2x - 9 & = y \end{array}\right\} \text{SOLVE FOR } y$$

f. $4x-(2x-9)-4z=11$

g. $4x-2x+9-4z=11$

h. $2x-2=4z$

i. $\dfrac{2x-2}{4}=z$

j. $\dfrac{2(x-2)}{4}=\dfrac{x-2}{2}=z$

SOLVE FOR z BY SUBSTITUTING $y=2x-9$ INTO AN ORIGINAL EQUATION (a)

SOLUTION $\left\{\left(x, 2x-9, \dfrac{x-2}{2}\right) \middle| x \text{ IS A REAL}\right\}$

$= \left\{(x,y,z) \middle| x \text{ IS A REAL, } y=2x-9, \text{ AND } z=\dfrac{x-2}{2}\right\}$

PLAN: ELIMINATE ONE VARIABLE

SOLVE FOR SECOND VARIABLE IN NEW EQUATION

SOLVE FOR ELIMINATED VARIABLE IN AN ORIGINAL EQUATION

THIS PLAN WILL NOW ALSO BE FOLLOWED WHEN 3 PLANES INTERSECT IN A LINE. (SEE NEXT EXAMPLE)

J. DEPENDENT SYSTEM, 3 EQUATIONS, 3 UNKNOWNS, INTERSECTION EXACTLY A LINE.

a. $4x - y - 4z = 11$
b. $8x - 3y - 4z = 29$ GIVEN
c. $4x - 3y + 4z = 25$

ELIMINATE z IN a, b

d. $-4x + y + 4z = -11$ MULTIPLY a BY -1

e. $4x - 2y = 18$ ADD b AND d

ELIMINATE z IN b, c

f. $12x - 6y = 54$ ADD b AND c

g. $-12x + 6y = -54$ MULTIPLY e BY -3

h. $0 = 0$ THUS, e, f MULTIPLES OF EACH OTHER

WE NOW FINISH CARRYING OUT THE PLAN FROM THE PREVIOUS PAGE, WE HAVE ELIMINATED ONE VARIABLE (e, f)
NOW SOLVE FOR SECOND VARIABLE IN EITHER NEW EQUATION (e, f)

i. $4x - 18 = 2y$ e

j. $\dfrac{4x - 18}{2} = y$ i

k. $\dfrac{2(2x-9)}{2} = 2x - 9 = y$ j

NOW SOLVE FOR THE ELIMINATED VARIABLE IN AN ORIGINAL EQUATION (a, b, c)

l. $4x-(2x-9)-4z=11$

m. $4x-2x+9-4z=11$

n. $2x-2=4z$

o. $\frac{2(x-1)}{4}=z$

p. $\frac{(x-1)}{2}=z$

SOLVE FOR z BY SUBSTITUTING $y=2x-9$ INTO AN ORIGINAL EQUATION (a)

SOLUTION $\left\{\left(x, 2x-9, \frac{x-1}{2}\right) \middle| x \text{ IS A REAL}\right\}$

K. DEPENDENT SYSTEM, 3 EQUATIONS, 3 UNKNOWNS, INTERSECTION EXACTLY A PLANE

a. $x+y+z=1$

b. $2x+2y+2z=2$

c. $3x+3y+3z=3$

GIVEN

d. $-2x-2y-2z=-2$ MULTIPLY a BY -2

e. $0=0$ ADD b AND d

f. $-3x-3y-3z=-3$ MULTIPLY a BY -3

g. $0=0$ ADD a AND c

EACH EQUATION IS A MULTIPLE OF THE OTHER

SOLUTION $\{(x, y, z) \mid x+y+z=1\}$

L. <u>INCONSISTENT</u> SYSTEM, 3 EQUATIONS, 3 UNKNOWNS. (I.E. NO SOLUTION)

$$\left.\begin{array}{ll} a. & 4x - y - 4z = 11 \\ b. & 4x - 2y = 17 \\ c. & 4x - 3y + 4z = 25 \end{array}\right\} \text{ GIVEN}$$

d. $8x - 4y = 36$ ADD a AND c

e. $-8x + 4y = -34$ MULTIPLY b BY -2

f. $0 = 2$ A CONTRADICTION WAS ARRIVED AT ASSUMING (x, y, z) WAS A SOLUTION TO THE GIVEN SYSTEM. HENCE, <u>NO</u> <u>SOLUTION</u>

<u>INCONSISTENT</u>

M. HOMEWORK (OIS) SOLVE EACH OF THE FOLLOWING.

1. $$\begin{aligned} 2x - 5y + 3z &= -22 \\ 7x + y - 2z &= -9 \\ 5x + 3y + 4z &= -5 \end{aligned}$$

2. $$\begin{aligned} 2x + 3y - 4z &= -6 \\ 4x - 6y + 5z &= 10 \\ 6x \quad\quad - 7z &= -11 \end{aligned}$$

3. $$\begin{aligned} 4x - 3y + 2z &= 8 \\ 3x - 5y + 3z &= -2 \\ -14x + 16y - 10z &= 10 \end{aligned}$$

4. $$\begin{aligned} 2x + y - 2z &= 7 \\ 4x - 10y + 2z &= -40 \\ 6x - 5y - 2z &= -15 \end{aligned}$$

5. $$\begin{aligned} 4x + 2y - 4z &= 36 \\ 4x - 8y + 4z &= -8 \\ 8x - 21y + 12z &= -38 \end{aligned}$$

16 - 322A

LINEAR INEQUALITIES

N. EXAMPLES $2x+3y>6$

$4x+6y<-2$

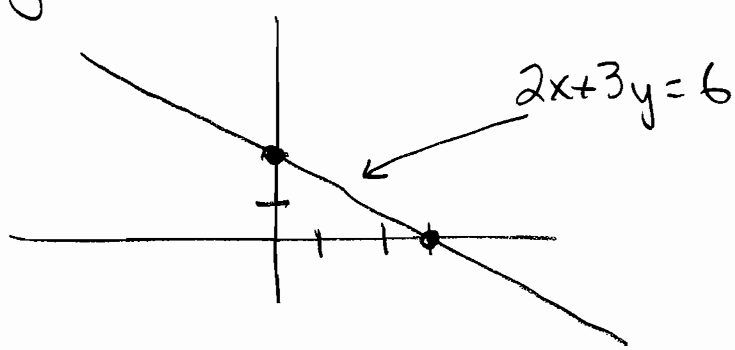
$3x+2y-6z \leq 40$

O. HOW TO GRAPH A LINEAR INEQUALITY : $2x+3y>6$

1. CHANGE TO A LINEAR <u>EQUALITY</u> AND GRAPH

$$2x+3y=6$$

x	y
0	2
3	0



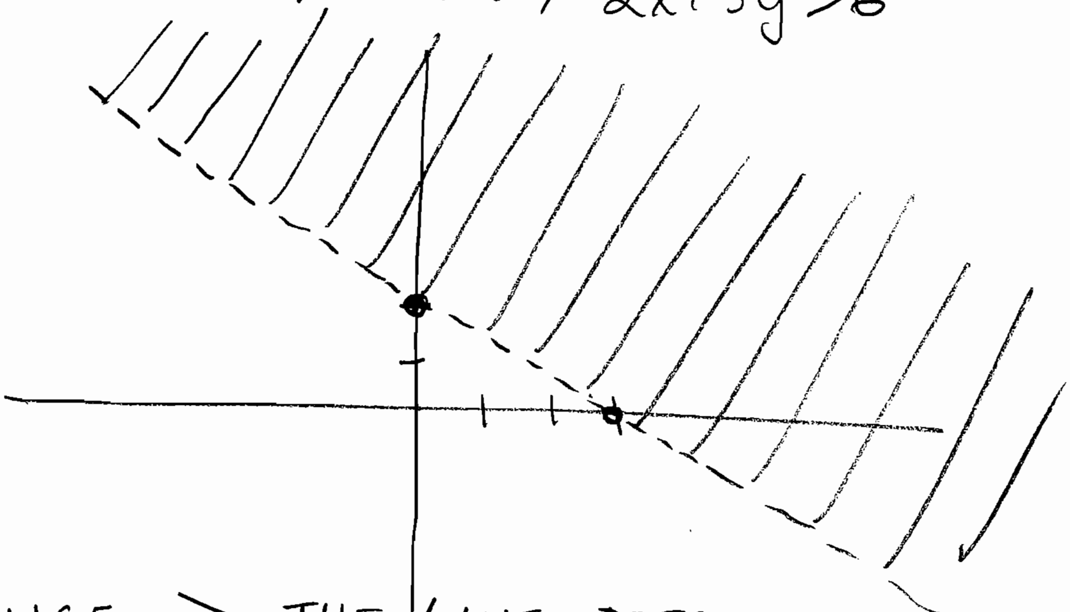
2. PICK A POINT NOT ON THE LINE AND SEE IF IT SATISFIES THE INEQUALITY.

IF YES, ALL POINTS ON THAT SIDE OF THE LINE SATISFY THE INEQUALITY. IF NO, ALL POINTS ON THE OTHER SIDE OF THE LINE SATISFY THE INEQUALITY.

PICK $(0,1)$

$2x+3y = 2(0)+3(1) = 3 \not> 6$ FALSE,

DOES NOT SATISFY $2x+3y>6$



SINCE $>$ THE LINE DOES NOT SATISFY

INDICATE THE SOLUTION WITH PERPENDICULAR LINES AND A DASHED LINE TO INDICATE POINTS ON THE LINE DO NOT SATISFY THE INEQUALITY.

16 - 322C

P. GRAPHING A SOLUTION TO A SYSTEM OF LINEAR INEQUALITIES

$$2x+3y>6$$
$$4x-y\le 8$$
$$x\ge 0$$

GRAPH EACH INEQUALITY

INTERSECT THE GRAPHS

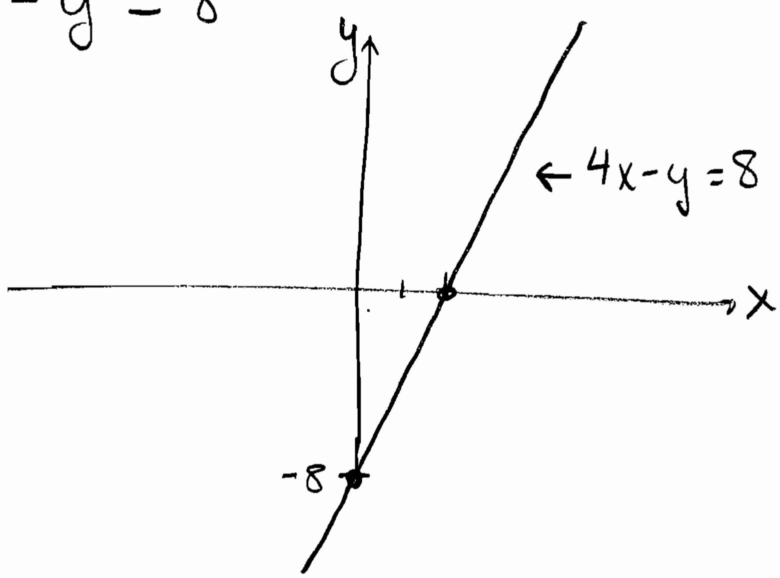
1. GRAPH $2x+3y>6$

JUST DONE

2. GRAPH $4x-y\le 8$

$$4x-y=8$$

x	y
0	-8
2	0

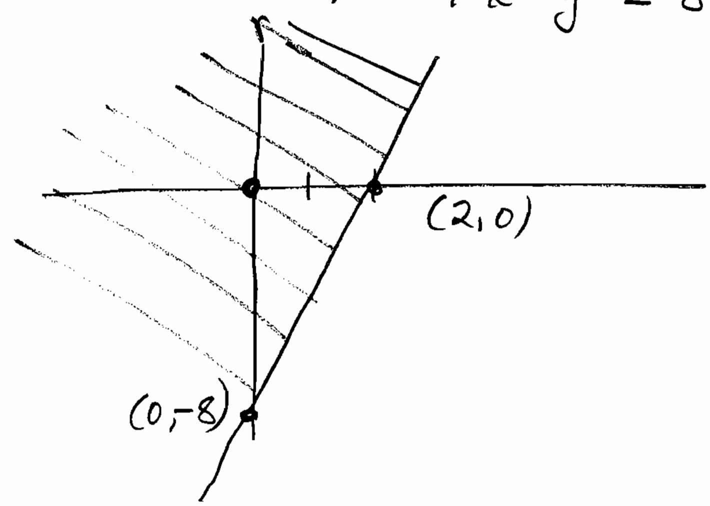


16 − 322D

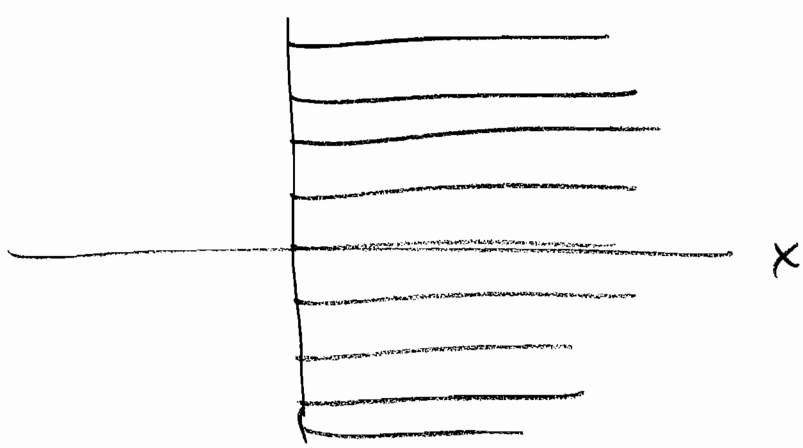
PICK POINT $(0,0)$

$4x - y = 4(0) - 0 = 0 \leq 8$ TRUE,

DOES SATISFY $4x - y \leq 8$

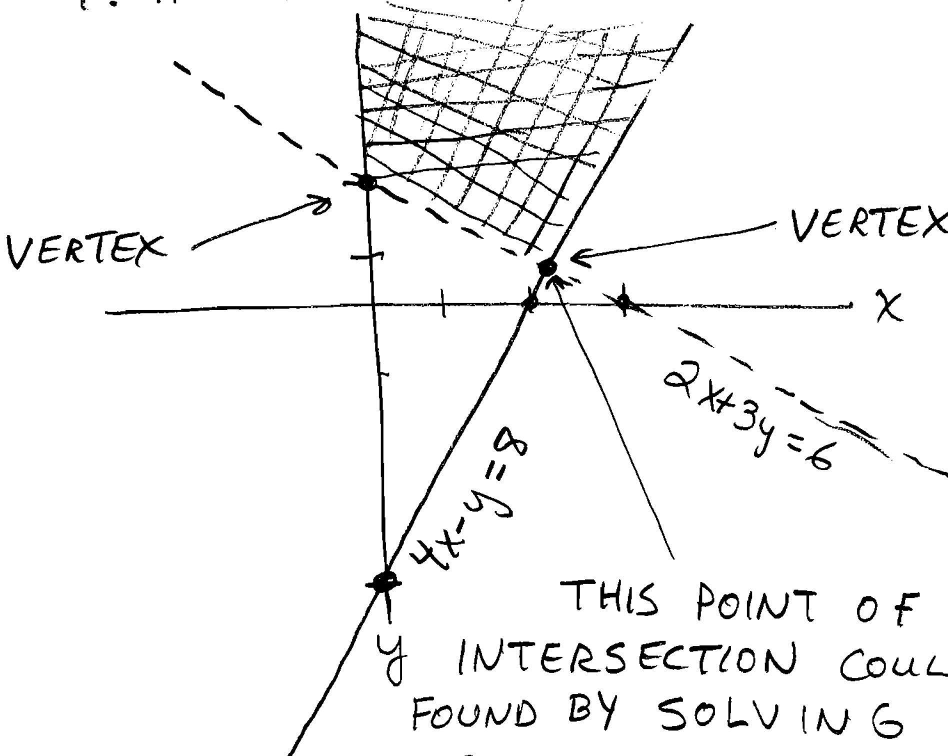


3. GRAPH $x \geq 0$



16-322E

4. INTERSECT THE GRAPHS



THIS POINT OF INTERSECTION COULD BE FOUND BY SOLVING

$2x+3y=6$

$4x-y=8$

SIMULTANEOUSLY

16 - 322F

Q.

HOMEWORK

1. GRAPH THE SOLUTION TO

$$2x - y \leq 2$$

2. GRAPH THE SOLUTION TO

$$2x - y \leq 2$$
$$-3x + 2y < 12$$
$$2x + y \geq 0$$

AND FIND THE VERTICES FOR THE SOLUTION REGION.

SELECTED ANSWERS

ANSWERS TO ABOUT HALF OF THE PROBLEMS.

(2, J, 1, a) "THE SET WHOSE ELEMENTS ARE 3, 4, AND 5".

(2, J, 1, c) "THE SET WHOSE ONLY ELEMENT IS 7".

(2, J, 1, e) "THE SET OF ALL x SUCH THAT x IS AN ELEMENT OF N AND x IS LESS THAN 5".

(2, J, 2, a) $\{0, 1, 2, 3\}$

(2, J, 3, a) $\{x \mid x \in W \text{ AND } x < 5\}$

(2, J, 5) $\sqrt{2}$ (2, J, 6, a) $.\overline{714285}$

(2, J, 7, b) $\frac{2834}{49950}$ (2, J, 8, a) I, Q, R

(2, J, 8, c) W, I, Q, R (2, J, 8, e) Q, R

(2, J, 10, a) FALSE (2, J, 10, c) FALSE

(2, J, 10, e) TRUE (2, J, 10, g) TRUE

(2, J, 10, i) FALSE (2, J, 10, k) FALSE

(2, J, 10, m) TRUE

(2, W, 1) 8 (2, W, 2) $H \cap K = \{7\}$ $H \cap \{5\} = \phi$

(2, W, 4, a) COMMUTATIVE LAW OF MULTIPLICATION

(2, W, 4, c) COMMUTATIVE LAW OF MULTIPLICATION

(2, W, 4, e) DISTRIBUTIVE PROPERTY

17-324

(2, W, 4, g) DISTRIBUTIVE PROPERTY

(2, W, 4, i) MULTIPLICATIVE IDENTITY

(2, W, 5) MULTIPLICATIVE INVERSE OF $7/9$ IS $9/7$

(3, J, 1, a) $-xyzw$

(3, J, 2) $\Big|\underbrace{3zw}_{\substack{pos \cdot pos \cdot neg \\ NEG}}\Big| = -3zw$

$$|z-w| = \Big|\underbrace{z+(-w)}_{\substack{pos + pos \\ pos}}\Big| = z-w$$

(3, J, 4, a) 1 (3, J, 4, c) 14

(3, J, 4, e) 3 (3, J, 4, g) -24

(4, L, 1) $-5x+6$ (4, L, 3) -1

(4, L, 5) -122 (4, L, 7) 1

(4, L, 8, a) $\frac{4}{x^6}$ (4, L, 8, c) $\frac{3^5 \cdot 2^2 x^4}{y^3}$

(4, L, 9, a) $5^{\boxed{-6}} x^{\boxed{21}} y^{\boxed{-51}}$

(4, L, 11) $\{6, 8\}$

17-325

(4,Q,1,a) 7.43×10^{-4} (4,Q,1,c) 2.537×10^{8}

(4,Q,1,d) 3.0×10^{2} (4,Q,2,a) 3

(4,Q,2,c) -2 (4,Q,2,e) 2

(4,Q,2,g) 8 (4,Q,2,i) 16

(4,Q,2,k) -32 (4,Q,2,m) 8

(4,Q,2,o) UNDEFINED (4,Q,2,q) $|x|^3$

(4,Q,3,a) NOT DEFINED. $\sqrt{x}$ UNDEFINED

(4,Q,3,b) DEFINED $-xy>0$, $-x>0$, $y>0$

(4,Q,4,a) $\frac{23}{20}$ (4,Q,4,c) $\frac{11}{5}$

(4,Q,4,e) $\frac{2-5x}{x}$ (4,Q,4,g) $\frac{3x-2}{3}$

(4,Q,5,b) $\frac{2}{x^{4/3}y^{3/80}}$ (4,Q,6) $3^{\boxed{1}}x^{\boxed{\frac{17}{40}}}y^{\boxed{-\frac{4}{3}}}$

(4,V,1,a) $xy^3\sqrt[3]{x^2y}$ (4,V,1,c) $\frac{2\sqrt[5]{x^3}}{x^2}$

(4,V,1,e) $\frac{3\sqrt[4]{8x^3y}}{4xy^2}$ (4,V,1,g) $\frac{5\sqrt[7]{x}}{x}$

(4,V,2,a) $(10-x)\sqrt{2}$ (4,V,2,c) $(x^2-2)\sqrt{x}$

(5, E, 1, a) YES

(5, E, 1, c) NO

(5, E, 1, e) YES

(5, E, 1, g) YES

(5, E, 3) x^3+2

(5, E, 5) $2x$

(5, E, 6, a) $-x^2-13x+3$

(5, E, 6, c) $6x^2-17x-14$

(5, E, 6, e) $25x^2-4$

(5, E, 6, g) $16x^2+56x+49$

(5, E, 6, i) $4x^2+20x+25$

(5, E, 6, k) $4x^2-25$

(5, E, 6, m) $25x^{2/7}-9$

(5, E, 6, o) $x-8\sqrt{x}+16$

(5, E, 6, q) $x+8\sqrt{x}+16$

(5, E, 7, a) $6x^4+2x^3-37x^2+43x-14$

(5, E, 8, a) $(x-4)^2 = x^2-8x+16$

(5, E, 9, a) $(4x+3)(4x-3)$

(5, H, 1, a) $-5(\sqrt{3}+2)$

(5, H, 2, a) $\dfrac{5x^7}{3x^2} = \dfrac{5}{3}x^5$ $\qquad 5x^7 = \left(\frac{5}{3}x^5\right)(3x^2)$

(5, H, 2, c) $\dfrac{2x^5-9x^3+7x}{3x^2} = \dfrac{2}{3}x^3-3x+\dfrac{7x}{3x^2}$

$$2x^5-9x^3+7x = \left(\tfrac{2}{3}x^3-3x\right)(3x^2)+7x$$

(5, H, 2, e) $\dfrac{x^4+16}{x+2} = x^3-2x^2+4x-8+\dfrac{32}{x+2}$

$$x^4+16 = (x^3-2x^2+4x-8)(x+2)+32$$

(5, J, 1, a) $5x^2(x^5-2)$ (5, J, 1, c) $x^{1/2}(7x^2+4x+1)$

(5, J, 2, a) $(x^3+6)(x+3)$ (5, J, 2, c) $(2x-3)(x^5+2x)$

(5, J, 2, e) $(3+4x^5)(2x^3-5x^2)$ (5, J, 2, g) $(b-p)(3w+6q)$

(5, L, a) $(2x-4)(x+1)$ (5, L, c) $(x+3)(x-3)$

(5, L, e) IRREDUCIBLE (5, L, g) IRREDUCIBLE

(5, L, i) $(2x^5-1)(3x^5-7)$ (5, L, k) $(4y^2-3)^2$

(5, θ, 1, a) $(4x+3)(2x+5)$ (5, θ, 1, c) $(4y-3)^2$

(5, θ, 2, a) $(6y^3-2x)(6y^3+2x)$ (5, θ, 2, c) $(3x+5)^2$

(5, θ, 2, e) $(x+3)(x^2-3x+9)$ (5, θ, 2, g) $(x+3)(x-3)(x^2+9)$

(5, θ, 2, i) $(3x^2y-5)(9x^4y^2+15x^2y+25)$

(5, θ, 2, k) $(5y+6)^2$

(5, θ, 4) $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

(5, θ, 5, a) $(x+2)^3$

(5, T, 1, a) $-3(x+2)(x-2)(x^2+4)$

(5, T, 1, c) $(x-1)(x^2+x+1)(x+1)(x^2-x+1)(x^2+1)(x^4-x^2+1)$

(5, T, 1, e) $(x-2)(x+2)(x-2)(x-1)$

(5, T, 2, a) $\dfrac{x-1}{x-3}$ (5, T, 2, c) $\dfrac{x+4}{x-1}$

(5, T, 2, e) $\dfrac{2}{x^2-x+1}$

(5, Z, 1) $2^7 \cdot 3^{10} \cdot 5^3 \cdot 7^9$ (5, Z, 3) $(x+1)^2(2x-3)^5$

(5, Z, 4, a) $\dfrac{6y-5x^2}{10x^3y^3}$ (5, Z, 4, c) $\dfrac{2x-11}{(x+2)^2(2x-1)^2}$

(5, Z, 4, e) $\dfrac{x^2+2x+4}{(x+1)^3(x-2)}$ (5, Z, 4, g) $\dfrac{4x-5}{3x-2}$

(5, Z, 4, i) $\dfrac{(x+5)(x+2)(x+3)}{(x-1)(x+1)(-x+2)}$ (5, Z, 4, k) $\dfrac{-x^2y^2}{y-x}$

(5, Z, 4, m) $\dfrac{-(x-y)^2}{x^4y^4} = \dfrac{-(y-x)^2}{x^4y^4} = \dfrac{(y-x)(x-y)}{x^4y^4}$

(6, I, 1) i (6, I, 3) $\frac{1}{2}i$

(6, I, 5) PROOF START: ASSUME $z = a+bi$ IS A COMPLEX NUMBER (SHOW $z \cdot \bar{z}$ IS A REAL NUMBER)

$z \cdot \bar{z} = (a+bi)(\overline{a+bi}) = \ldots$

RECALL a AND b ARE REAL

(6, I, 6, a) $\frac{5}{4} + \left(\frac{-11}{2}\right)i$ (6, I, 6, c) $\frac{-3}{10} + \left(\frac{-13}{6}\right)i$

(6, I, 6, e) $5+0i$ (6, I, 6, f) $\frac{77}{30} + \frac{5}{6}i$

(6, I, 7, a) $-1+3i$ (6, I, 7, c) $2+39i$

(6, I, 7, e) $21+27i$

(6, L, 1, a) $\frac{23}{41} + \frac{2}{41}i$ (6, L, 1, c) $-2+\frac{7}{2}i$

(6,L,1,e) $\left(\frac{\sqrt{6}-\sqrt{21}}{5}\right)+\left(\frac{-3-\sqrt{14}}{5}\right)i$

(6,L,1,g) $\frac{\sqrt{6}+\sqrt{42}}{10}+\left(\frac{\sqrt{14}-3\sqrt{2}}{10}\right)i$

(6,L,2,b) $-25-3\sqrt{5}+5\sqrt{30}+3\sqrt{6}$

(6,L,3) SUBSTITUTE THOSE NUMBERS FOR x, MULTIPLY OUT, AND SEE IF EACH EQUALS 0.

(7,E,1,a) IDENTITY (7,E,1,c) CONDITIONAL

(7,E,2,b) IF REALS ONLY, YES EQUIVALENT. IF COMPLEX, $\pm 2, \pm 2i$ SATISFY $x^4=16$ BUT NOT $x^2=4$.

(7,E,3,a) $\{11/3\}$ (7,E,3,c) $\{0\}$

(7,E,3,e) $\left\{\frac{279}{560}\right\}$ (7,E,3,g) $\left\{\frac{84}{13}\right\}$

(7,E,3,i) $\left\{\frac{p-2}{2}\right\}$

(7,G,1) $\{9/4\}$ (7,G,3) $\{+\sqrt{8}, -\sqrt{8}\}$

(7,G,5) NO SOLUTION (7,G,7) $\{2\}$

(7,K,1) $x^2+12x+36=(x+6)^2$

(7,K,3,a) $\{+5,-5\}$ (7,K,3,c) $\{-1,5\}$

(7,K,3,e) $\{-\frac{1}{2}+2i, -\frac{1}{2}-2i\}$

(7,K,4,a) $\{\frac{1}{2},-6\}$ (7,K,4,c) $\{4,5\}$

(7,K,4,e) $\{-5,3\}$

(7,K,5,a) $\{-5,3\}$ (7,K,5,c) $\{\frac{1}{2},-6\}$

(7,K,5,e) $\left\{\frac{-1-2i\sqrt{5}}{3}, \frac{-1+2i\sqrt{5}}{3}\right\}$

(7,K,6,a) $1(x-[-2+2i])(x-[-2-2i])$

(7,K,6,c) $2\left(x-\left[\frac{3-3i\sqrt{7}}{4}\right]\right)\left(x-\left[\frac{3+3i\sqrt{7}}{4}\right]\right)$

(7,M,1,a) $b^2-4ac=64>0$ 2 <u>REAL</u> SOLUTIONS

(7,M,1,c) $b^2-4ac=169>0$ 2 <u>REAL</u> SOLUTIONS

(7,M,1,g) $b^2-4ac=0=0$ 1 REAL SOL. MULTIPLICITY 2

(7,M,3) $3\left(x-\left[\frac{-1+2i\sqrt{5}}{3}\right]\right)\left(x-\left[\frac{-1-2i\sqrt{5}}{3}\right]\right)$

(7, P, 1, a) $\{-2, +2, -\frac{1}{2}\}$ (7, P, 1, c) $\{-3i, +3i, \frac{3}{2}\}$

(7, P, 1, e) $\{-\frac{2}{3}, -\frac{22}{23}\}$

(7, P, 2, a) $\{5\}$ (7, P, 2, c) $\{1\}$

(7, P, 2, e) NO SOLUTION $\{\ \}$

(7, S, 1) $\{1, 2\}$ (7, S, 3) $\{64, +1\}$

(7, S, 5) $\{+3, -3, +4, -4\}$ (7, S, 7) $\sqrt[3]{16}$

(7, S, 9) $\{+2\sqrt{3}, -2\sqrt{3}\}$

(7, S, 11) $(x^2+6)^2 = x^8$

$$\pm(x^2+6) = x^4$$

$$x^4 = x^2+6 \quad \text{OR} \quad x^4 = -(x^2+6)$$

$$x^4 - x^2 - 6 = 0 \quad \text{OR} \quad x^4 = -x^2 - 6$$

$$\vdots$$

SOLUTION SET $\{\pm\sqrt{3}\}$

(7, S, 13) For #11 $\left\{\pm\sqrt{3}, \pm i\sqrt{2}, \pm\sqrt{\frac{-1\pm i\sqrt{23}}{2}}\right\}$

(8, M, 1) 70 FEET (8, M, 3) $(\frac{23}{2})^2$

(8, Q, 1) $\frac{12}{7}$ HOURS (8, Q, 3) 250 MILES

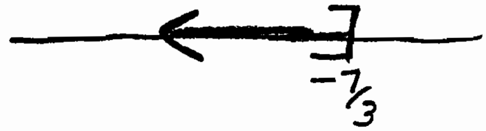
(8, Q, 5) $\frac{134}{5}$ M.P.H. (8, Q, 7) $x = 100\left(\sqrt{3/2} - 1\right)$

(9, E, 1, a) $[-2, 3]$ (9, E, 1, c) $(-\infty, -10)$

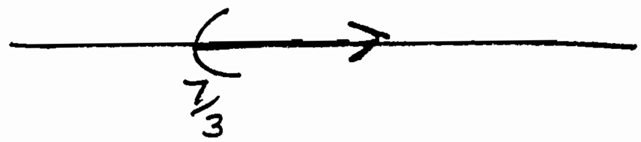
(9, E, 2, a) $\{x \mid -7 < x < 3\}$ (9, E, 2, c) $\{x \mid x \geq -4\}$

(9, E, 2, e) $\{x \mid 2 < x \leq 3\}$

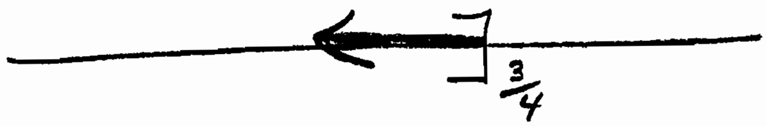
(9, E, 3, a) $\left(-\infty, -\frac{7}{3}\right]$



(9, E, 3, c) $\left(\frac{7}{3}, \infty\right)$

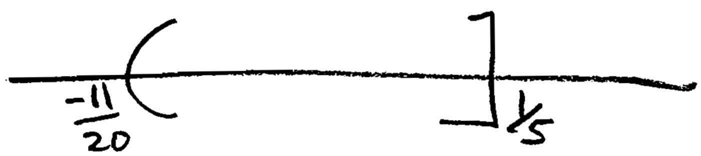


(9, E, 3, e) $\left(-\infty, \frac{3}{4}\right]$

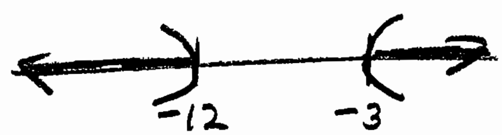


(9, E, 3, g) $\left[\frac{103}{71}, +\infty\right)$

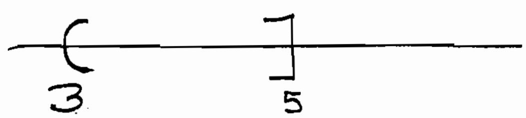
(9, E, 3, i) $\left(-\frac{11}{20}, \frac{1}{5}\right]$



(9, E, 3, k) $(-\infty, -12) \cup (-3, +\infty)$

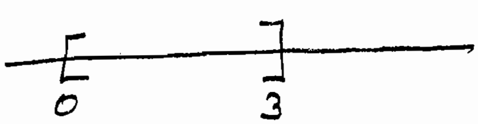


(9, I, 1, a)



$(3, 5]$

(9, I, 1, c)

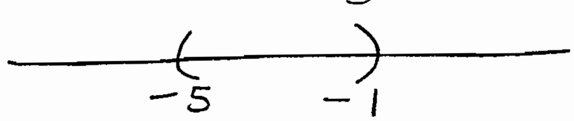


$[0, 3]$

(9, I, 2, a) $(-\infty, -5) \cup (5, +\infty)$

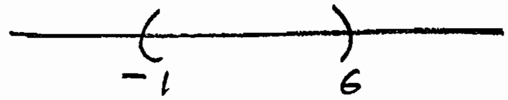
(9, I, 2, c) $(-3, 3)$

(9, I, 2, e) $(-5, -1)$

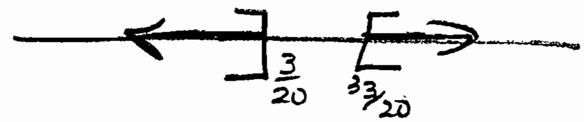


17-333

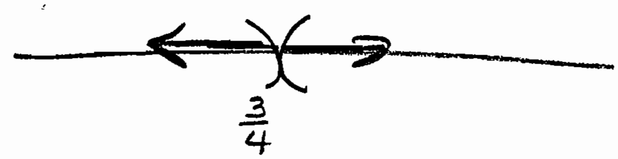
$(9, I, 2, g)$ $(-1, 6)$



$(9, I, 2, i)$ $(-\infty, \frac{3}{20}] \cup [\frac{33}{20}, \infty)$

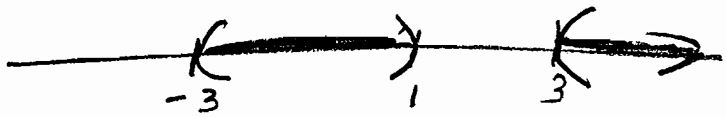


$(9, I, 2, k)$ $(-\infty, \frac{3}{4}) \cup (\frac{3}{4}, +\infty)$



$(9, K, 1, b)$ $[-\frac{5}{2}, 3)$ $(9, K, 1, d)$ $[-2, \frac{5}{2}) \cup [3, \infty)$

$(9, K, 2)$ $(-3, 1) \cup (3, \infty)$



$(10, F, 1)$ $(+2, -3)$ $(10, F, 3)$ NO

$(10, F, 5, a)$ $+7$ $(10, F, 7, b)$ $\sqrt{37}$

$(10, F, 8)$ HINT: USE PYTHAGOREAN THEOREM

$(10, H, 1, a)$ $-28/5$ $(10, H, 1, c)$ 0

$(10, H, 2, a)$ $\frac{11}{48}$ $(10, H, 4)$ $-5/8$

$(10, J, 1)$ $x + y = 1$ $(10, J, 3)$ $2x + y = 4$

$(10, J, 5)$ $3x - 4y = -15$ $(10, J, 7)$ $5x + 2y = 29/2$

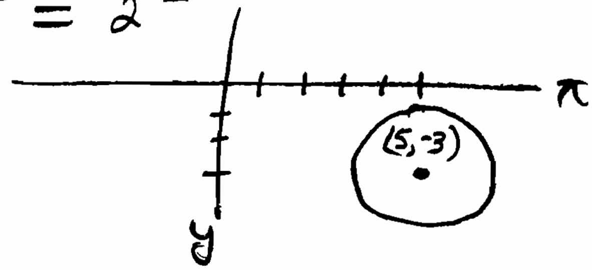
(11, I, 1, a) $(x-2)^2+(y-3)^2 = 6^2$

(11, I, 1, c) $(x-[-\tfrac{2}{3}])^2+(y-[-\tfrac{1}{5}])^2=(\tfrac{1}{7})^2$

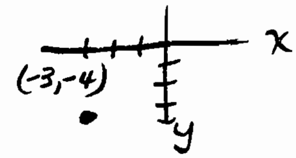
(11, I, 2, a) $(x-5)^2+(y-[-3])^2 = 2^2$

CENTER $(5,-3)$

RADIUS 2



(11, I, 2, c) $(x-[-3])^2+(y-[-4])^2 = 0$ POINT GRAPH

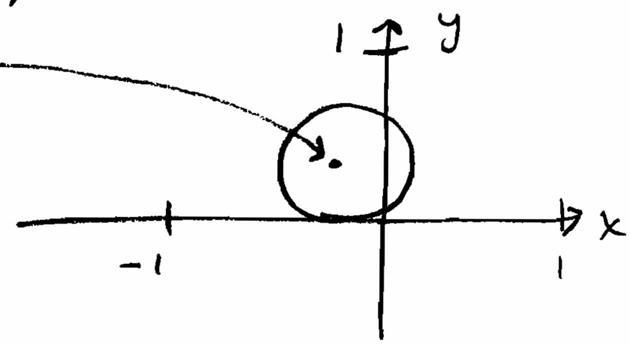


(11, I, 2, e)

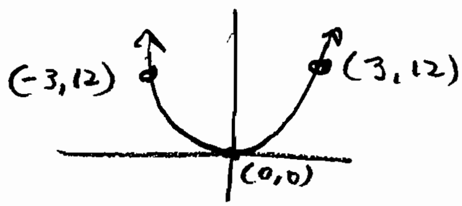
$(x-[-\tfrac{1}{5}])^2+(y-\tfrac{1}{3})^2=(\tfrac{1}{3})^2$

CENTER $(-\tfrac{1}{5}, \tfrac{1}{3})$

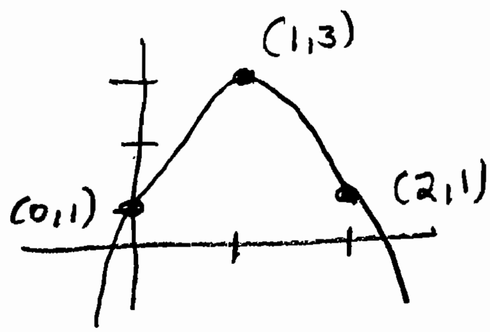
RADIUS $\tfrac{1}{3}$



(11, 5, 1) vertex $(0,0)$;
axis of symm $x=0$
symmetric partners
$(3,12)$, $(-3,12)$
$y=\frac{4}{3}(x-0)^2+0$ ← st. form



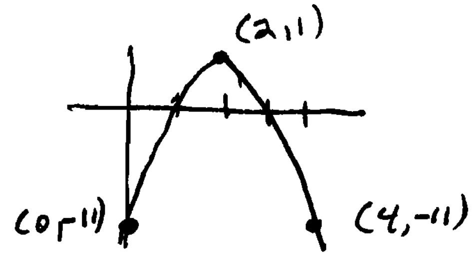
(11, 5, 3) vertex $(1,3)$
axis of symm $x=1$
$y=-2(x-1)^2+3$ st. form
$(0,1)$, $(2,1)$ symm. partners



(11, S, 5) vertex $(2,1)$ axis. of sym $x=2$

st. form $y=-3(x-2)^2+1$

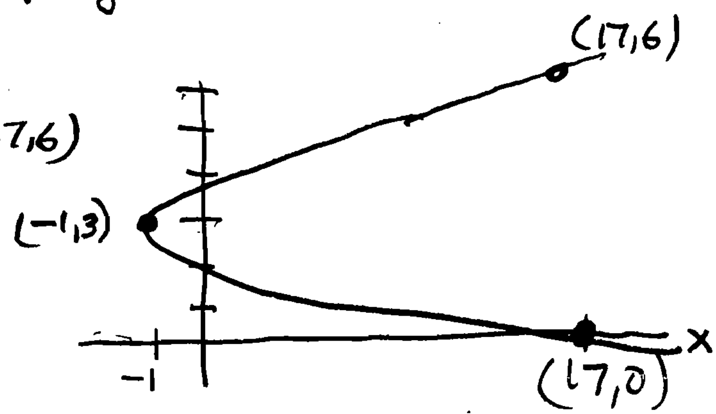
symm. partners $(0,-11)$, $(4,-11)$



(11, S, 7) vertex $(-1,3)$ axis. of sym. $y=3$

st. form $x=2(y-3)^2-1$

Symm. partners $(17,0)$, $(17,6)$



(12, H, 1, a) YES dom(f) $= \{2,3,5\}$ ran(f) $= \{8,9\}$

(12, H, 1, c) NO (12, H, 2, b) 89

(12, H, 2, d) $3x^2+6xh+3h^2-2x-2h+4$

(12, H, 3, b) 1 (12, H, 3, d) $\dfrac{1}{\sqrt{2x+2h+3}}$

(12, H, 3, e) $$\frac{-2}{\sqrt{2x+3}\,\sqrt{2x+2h+3}\,\left(\sqrt{2x+3}+\sqrt{2x+2h+3}\right)}$$

(12, H, 4, a) YES x INDEPENDENT y DEPENDENT

(12, H, 4, c) NO (12, H, 4, e) YES x INDEP. y DEP

(12, H, 5, a) NO (12, H, 5, c) NO

(12, H, 5, e) YES

(12, M, 1, a) $\{x \mid x \neq 2\}$ (12, M, 1, c) $[\frac{1}{5}, \infty)$

(12, M, 1, e) $\{x \mid x \neq 2 \text{ AND } x \neq -2\}$

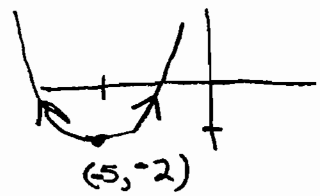
(12, M, 1, g) $\{x \mid x \neq \frac{-1+\sqrt{5}}{2} \text{ AND } x \neq \frac{-1-\sqrt{5}}{2}\}$

(12, M, 1, i) $(-\infty, \infty)$

(12, M, 2, a) DOMAIN $= (-\infty, \infty)$ RANGE $= [1, \infty)$

(12, M, 2, c) DOMAIN $= (-2, \infty)$ RANGE $= [-1, \infty)$

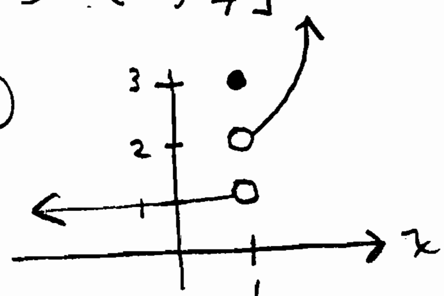
(12, M, 3, b) DOMAIN $= (-\infty, \infty)$
RANGE $= [-2, \infty)$



(12, M, 4, b) $(-\infty, 2]$ (12, M, 4, d) $\{y \mid y \neq \frac{3}{2}\}$

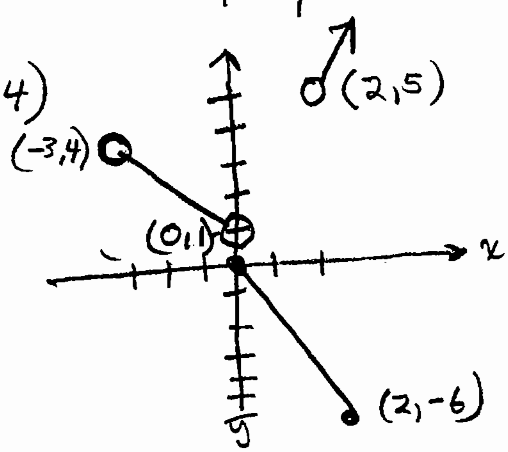
(12, M, 4, f) $(0, \frac{1}{4}]$

(12, Θ, 2)



$\text{dom}(f) = (-\infty, \infty)$
$\text{ran}(f) = \{1\} \cup (2, \infty)$

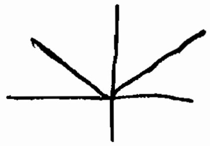
(12, Θ, 4)



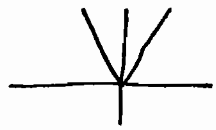
$\text{dom}(f) = (-3, \infty)$
$\text{ran}(f) = [-6, 0] \cup (1, 4) \cup (5, \infty)$

(12, T, 4)

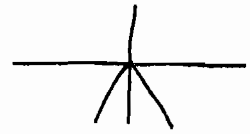
$y = |x|$



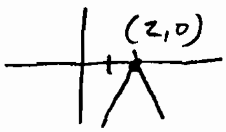
$y = 3|x|$
NARROW



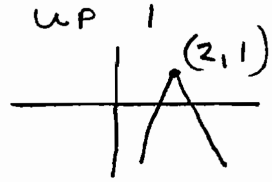
$y = -3|x|$
REFLECT X-AXIS



$y = -3|x-2|$
RIGHT 2

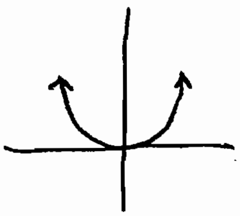


$y = -3|x-2| + 1$
UP 1

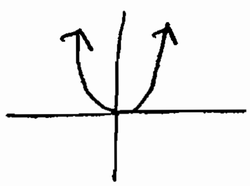


(12, T, 8) $y = -2\left(x - \frac{3}{4}\right)^2 + \frac{49}{8}$

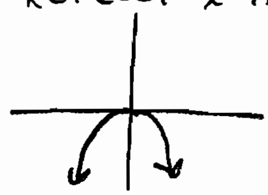
$y = x^2$



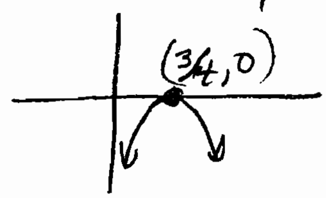
$y = 2x^2$
NARROW



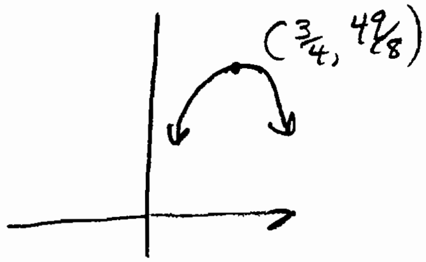
$y = -2x^2$
REFLECT X-AXIS



$y = -2\left(x - \frac{3}{4}\right)^2$
RIGHT 3/4

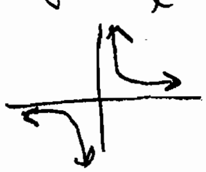


$y = -2\left(x - \frac{3}{4}\right)^2 + \frac{49}{8}$
UP 49/8



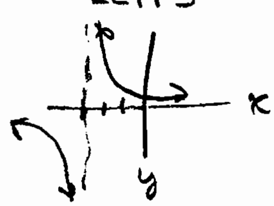
(12, Y, 1, a) $y = \frac{2}{x+3} - 1$ $(0, -\frac{1}{3})$ Y-INTER. $(-1, 0)$ X-INTER

$y = \frac{1}{x}$

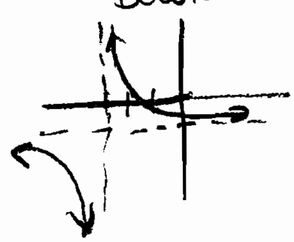


$y = \frac{2}{x}$
NARROW

$y = \frac{2}{x+3}$
Left 3



$y = \frac{2}{x+3} - 1$
DOWN 1



$x = -3$
VERT. ASYM.
$y = -1$
HOR. ASYM.

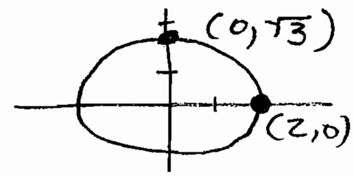
17-338

(12, Y, 2, a) ODD, SYMMETRIC ABOUT ORIGIN, NOT EVEN
NOT SYMMETRIC ABOUT y-AXIS

(12, Y, 2, c) NEITHER EVEN NOR ODD. NOT SYMMETRIC ABOUT y-AXIS. NOT SYMMETRIC ABOUT ORIGIN

(12, Y, 3, a) ORIGIN (12, Y, 3, c) x-AXIS, y-AXIS

(12, Y, 4, a) $\frac{x^2}{4}+\frac{y^2}{3}=1$



(12, Y, 6, c) $\frac{(y-1)^2}{4}-\frac{(x+3)^2}{2}=1$

(13, E, 1) $f+g=\{(3,9),(7,2)\}$ $fg=\{(3,20),(7,0)\}$

(13, E, 2, a) $(f+g)(x)=\sqrt{4-x}+\frac{x}{(4-x)^{1/3}}$ $(fg)(x)=\frac{x\sqrt{4-x}}{(4-x)^{1/3}}$

$\left(\frac{f}{g}\right)(x)=\frac{(4-x)^{5/6}}{x}$

(13, E, 2, d) $\operatorname{dom}(g\pm h)=\operatorname{dom}(g\cdot h)=\operatorname{dom}\left(\frac{g}{h}\right)=\{x \mid x\neq 0 \text{ AND } x\neq 4\}$

(13, E, 3, a) $(f\circ h)(x)=\sqrt{4-3\sqrt{2-x}}$

(13, E, 3, b) $\operatorname{dom}(f\circ h)=\left[\frac{2}{9}, 2\right]$

(13, E, 4, b) $f(x)=x^3+4x^2-\pi$ $g(x)=2x-3$

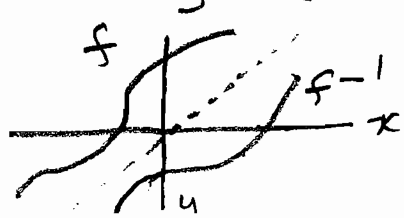
(13, G, 1, a) $f^{-1}\left(\frac{1}{2}\right)=8$ $\left(8,\frac{1}{2}\right)\in f$ (13, G, 1, c) 2

(13, G, 2, a) NOT 1-1 (13, G, 2, c) NOT 1-1

(13, G, 2, e) 1-1 (13, G, 2, g) NOT 1-1

(13, G, 3, a) $f^{-1}(x)=\frac{x-5}{3}$ (13, G, 3, c) $f^{-1}(x)=-\sqrt{x+22}-5$

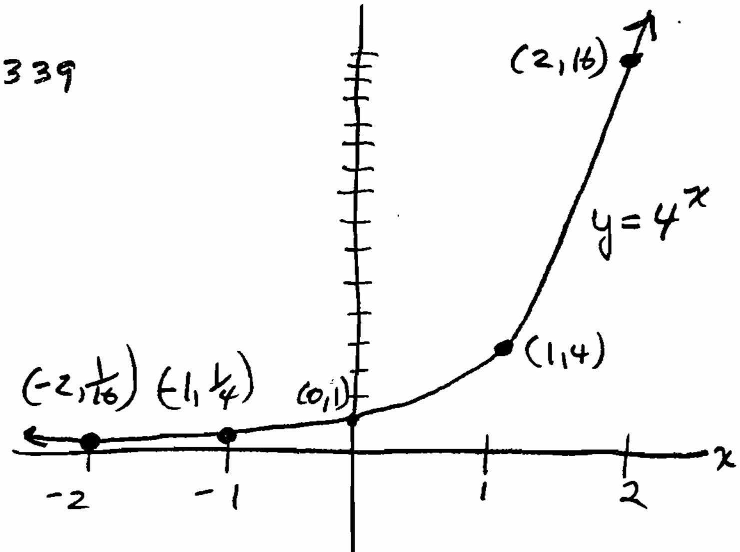
(13, G, 4, a)



17-339

(14, H, 1) 2.7 (14, H, 2, a)

x	y
0	1
1	4
2	16
−1	1/4
−2	1/16

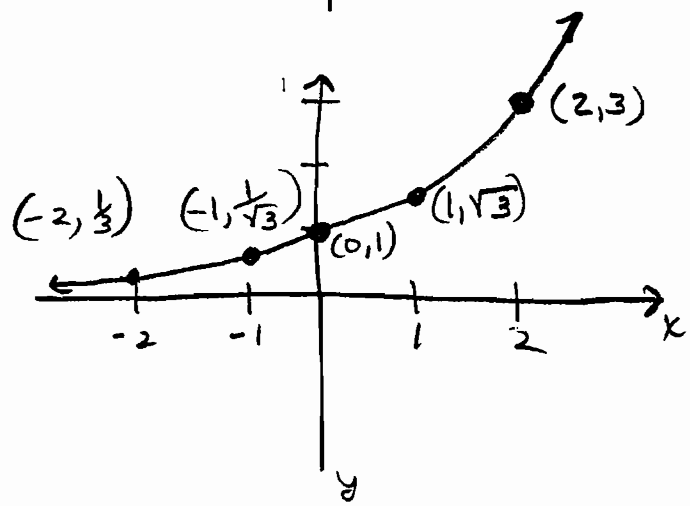


(14, H, 2, c)

$y = 3^{\frac{1}{2}x} = \left(3^{\frac{1}{2}}\right)^x$

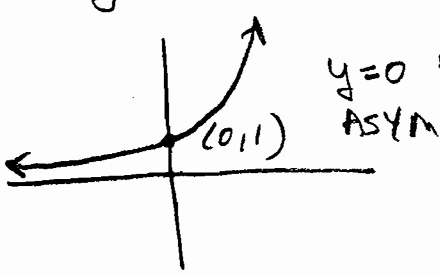
$y = (\sqrt{3})^x$

x	y
0	1
1	$\sqrt{3}$
2	3
−1	$1/\sqrt{3}$
−2	1/3



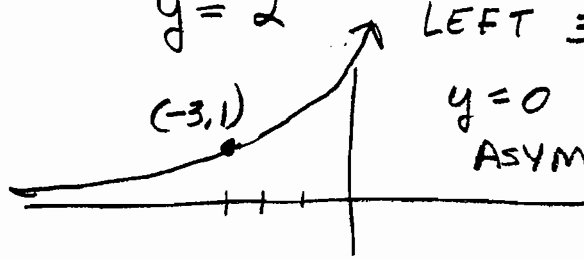
(14, H, 3, a)

$y = 2^x$



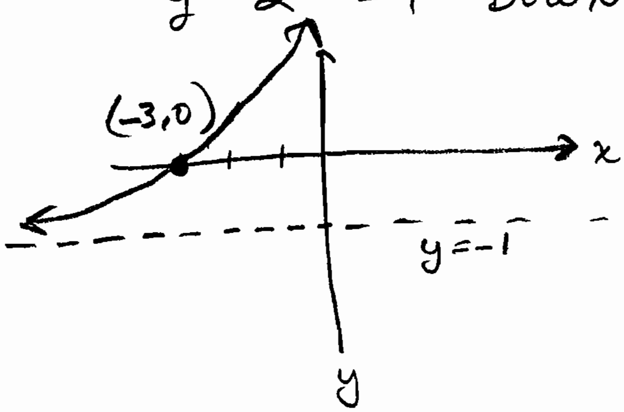
y=0 HOR. ASYM

$y = 2^{x+3}$ LEFT 3



y=0 HOR. ASYM.

$y = 2^{x+3} - 1$ DOWN 1

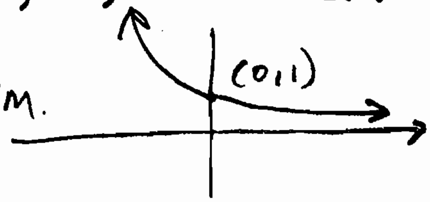


y=-1 HOR. ASYM.

17-340

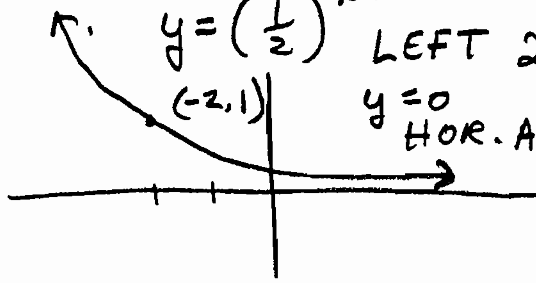
(14, H, 3, e) $y=\left(\frac{1}{2}\right)^x$

y = 0
HOR. ASYM.



$y=\left(\frac{1}{2}\right)^{x+2}$ LEFT 2

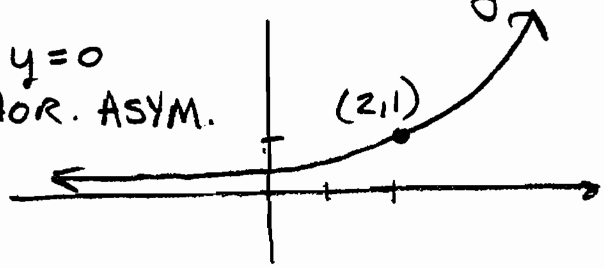
y = 0
HOR. ASYM.



$y=\left(\frac{1}{2}\right)^{-x+2}=\left(\frac{1}{2}\right)^{2-x}$

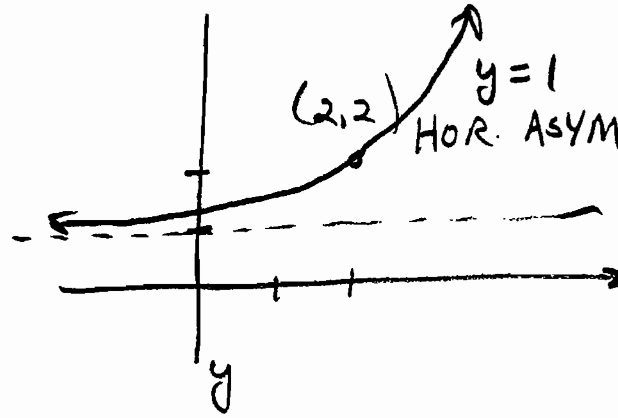
REFLECT ABOUT y-AXIS

y = 0
HOR. ASYM.



$y=\left(\frac{1}{2}\right)^{2-x}+1$ UP 1

y = 1
HOR. ASYM.



(14, K, 1, a) 3

(14, K, 1, c) $\frac{1}{2}$

(14, K, 1, e) 0

(14, K, 2, a) 32

(14, K, 2, c) 4

(14, K, 2, e) $\frac{3}{2}$

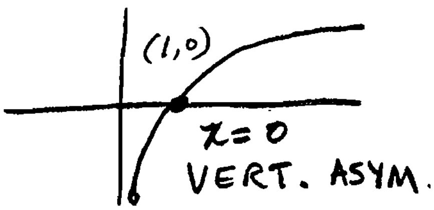
(14, K, 3, a) $\log_5 625 = 4$

(14, K, 2, c) $\log_7 1 = 0$

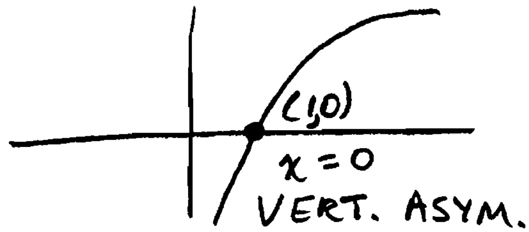
(14, K, 4, b) $2^5 = 32$

(14, T, 2, b)

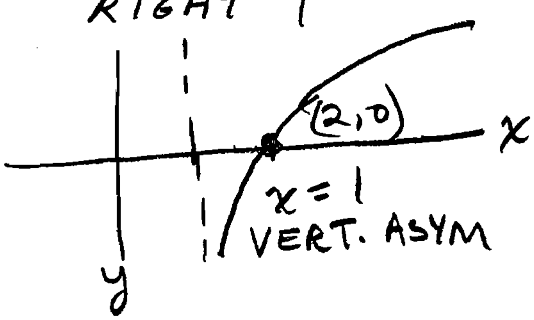
$y = \log x$



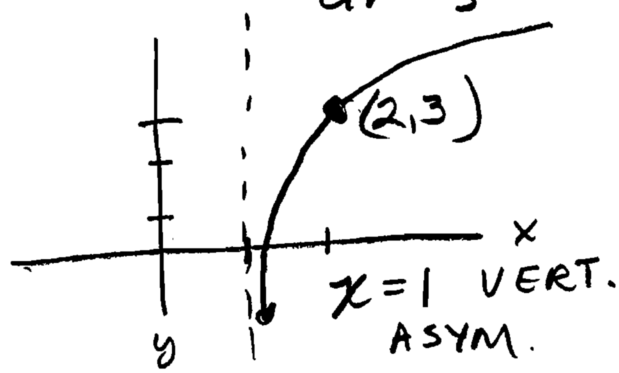
$y = 2\log x$ NARROW



$y = 2\log(x-1)$
RIGHT 1



$y = 2\log(x-1) + 3$
UP 3



(14, T, 3, a) $(\frac{1}{2}, \infty)$ (14, T, 5, a) $\frac{\ln 3}{\ln 5}$

(14, T, 6, a) $\log_5 x + 2\log_5 y$

(14, T, 6, c) $\frac{1}{3}\ln x + \frac{2}{3}\ln y - \frac{1}{3}\ln z$

(14, T, 7, a) $\ln \frac{x}{y^3}$ (14, T, 7, c) $\ln \frac{z^3\sqrt[3]{x}}{y^2}$

(14, T, 8, b) $\frac{-\ln 2}{\ln \frac{8}{25}}$

17-342

(15, G, 1, a) $\frac{5x^3-7x^2+3x-1}{x-2} = 5x^2+3x+9+\frac{17}{x-2}$

$5x^3-7x^2+3x-1 = (x-2)(5x^2+3x+9)+17$

(15, G, 1, c) $\frac{2x^7-3x^2+4}{x+2} = 2x^6-4x^5+8x^4-16x^3+32x^2-67x+134-\frac{264}{x+2}$

$2x^7-3x^2+4=(x+2)(2x^6-4x^5+8x^4-16x^3+32x^2-67x+134)-264$

(15, G, 2, a) 2 (15, G, 2, c) -2

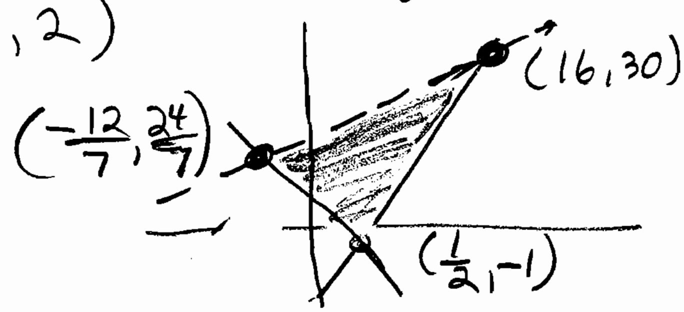
(15, G, 4, a) $x^4-1=(x+1)(x-1)(x+i)(x-i)$

(16, E, 1) $\{(5,-3)\}$ (16, E, 3) DEPENDENT
$\{(x,y) \mid 5x-3y=2\}$

(16, M, 1) $\{(-2,3,-1)\}$ (16, M, 3) NO SOLUTION
SOLUTION SET ϕ

(16, M, 5) $\{(x, \frac{4x-14}{3}, \frac{5x-34}{3}) \mid x \text{ IS REAL}\}$

(16, Q, 2)



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Dugopolski, Mark College Algebra
Addison-Wesley Publishing Company, 1995
ISBN 0-201-52618-2

BASIC KNOWLEDGE QUESTIONS

18-343

BASIC QUESTIONS

CHAPTER 2

1. HOW IS $\{2,3\}$ READ?

2. HOW IS $\{x \mid x<3\}$ READ?

3. NAME A RATIONAL NUMBER THAT IS NOT AN INTEGER.

4. WHAT TYPE OF DECIMAL NUMBERS ARE IRRATIONAL NUMBERS?

5. NAME A SET T SUCH THAT $T \subseteq \{1,2,3\}$

6. $A = \{2, 3, 4, 5\}$ $B = \{3, 5, 6, 7\}$

   $A \cap B =$ $A \cup B =$

7. STATE THE PROPERTY COMPLETELY

   a. $(x+2y)+z = (x+y2)+z$

   b. $(x+2y)+z = x+(2y+z)$

   c. $3(x+y) = 3x+3y$

CHAPTER 3

1. WRITE WITHOUT ABSOLUTE VALUE SIGNS

   $x > 5$ $y < -2$

   a. $|3x| =$

   b $|2y| =$

2. $-(a-b)=$ 18 – 344

3. $2+4\cdot 3=$

CHAPTER 4

1. $-3^2=$
2. $(-4)^2=$
3. $(5^2+2^3)^0=$
4. $x^2\cdot x^3=x^{\square}$
5. $(x^2)^3=x^{\square}$
6. $\dfrac{x^6}{x^{10}}=x^{\square}$
7. PUT IN SCIENTIFIC NOTATION
   .000213
8. $8^{1/3}=$
9. $\sqrt{9}=$
10. $\sqrt[4]{x}$ NAME A VALUE FOR x THAT MAKES THIS UNDEFINED FOR THE REALS
11. $8^{2/3}=$
12. $\sqrt{x^2}=$

13. NAME SPECIFIC VALUES FOR a AND b SO THAT $\sqrt{ab} = \sqrt{a}\sqrt{b}$ IS NOT TRUE FOR THE REALS.

14. WRITE AS A SINGLE FRACTION: $\frac{2}{y} + \frac{x}{q}$

15. WRITE AS A SINGLE FRACTION: $\dfrac{\frac{2}{x}}{\frac{3}{y}}$

16. TO RATIONALIZE THE DENOMINATOR IN $\dfrac{5}{\sqrt[7]{x^2y^4}}$, MULTIPLY TOP AND BOTTOM BY

CHAPTER 5

1. GIVE AN EXAMPLE OF A POLYNOMIAL EXPRESSION.

2. GIVE AN EXAMPLE OF AN EXPRESSION THAT IS <u>NOT</u> A POLYNOMIAL

3. GIVE AN EXAMPLE OF A CUBIC BINOMIAL.

4. GIVE AN EXAMPLE OF A QUADRATIC MONOMIAL

5. $(3x-2)(4x-5)=$

6. $(x+y)^2=$

7. $(x+y)(x-y) =$

8. FOR LONG DIVISION, WHICH IS THE FIRST TERM AT THE TOP?

$$\begin{array}{r|l} & \boxed{} \\ 3x^2+5 & \overline{15x^6+2x^5-3x^4+5x^3-4x^2+2x-3} \end{array}$$

9. $5x^3(2x-1)-7(2x-1) = (2x-1)(\underline{\qquad})$

10. FACTOR $x^2-5x+6 = (\quad)(\quad)$

11. FACTOR $a^3-b^3 = (\quad)(\qquad)$

12. $\operatorname{lcm}(2^8 \cdot 3^5 \cdot 5^4,\ 3^7 \cdot 5^3) =$

13. SMALLEST COMMON DENOMINATOR FOR

$$\frac{5x}{(x+1)^3(y+3)^2} + \frac{4x}{(x+1)^2(y+3)^4}$$ IS

14. GIVE AN EXAMPLE OF A COMPLEX FRACTION.

15. TRUE OR FALSE $\dfrac{x^5+y^4}{2\cancel{x^3}} = \dfrac{x^2+y^4}{2}$

CHAPTER 6

1. $i^2 =$

2. GIVE AN EXAMPLE OF A COMPLEX NUMBER THAT IS NOT REAL

3. GIVE AN EXAMPLE OF A PURE IMAGINARY NUMBER

4. $\overline{3-2i}$

a. HOW IS THIS READ?

b. WHAT IS IT EQUAL TO?

5. $\sqrt{-9} =$

6. TO PUT $\frac{5+3i}{2-7i}$ IN $a+bi$ FORM, MULTIPLY TOP AND BOTTOM BY ________.

CHAPTER 7

1. SOLVE $x+2=5$

2. NAME AN IDENTITY

3. NAME A CONDITIONAL EQUATION

4. NAME AN INCONSISTENT EQUATION FOR THE REALS.

5. TO SOLVE FOR x, NAME A REASONABLE NEXT LINE

$$3x-5=5x+7$$

6. NAME A REASONABLE NEXT LINE TO SOLVE

$$(x-3)^2=25$$

$$\sqrt{(x-3)^2}=\sqrt{25}$$

$$|x-3|=5$$

7. NEXT LINE IN SOLVING
$(x-3)(x+2)=0$

8. COMPLETE THE SQUARE
$x^2-8x+ \_\_\_ = 5+ \_\_\_$

9. SOLVE BY QUADRATIC FORMULA
$2x^2+3x+4=0$

10 WHAT IS THE DISCRIMINANT FOR
$2x^2+3x+4=0$

11. WHEN THE DISCRIMINANT IS LESS THAN 0, HOW MANY AND WHAT TYPES OF SOLUTIONS ARE THERE?

12. $a+bc = c(\qquad)$

13 WHAT CAN CAUSE AN EXTRANEOUS SOLUTION TO APPEAR.

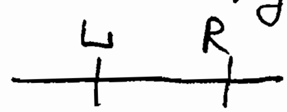
CHAPTER 8

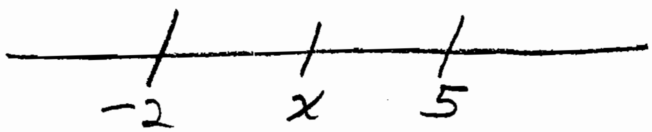
1. NAME THE SEVEN STEPS FOR SOLVING A WORD PROBLEM.

2. SAM CAN DO THE WHOLE JOB IN 5 HOURS. HOW MUCH OF THE JOB CAN SAM DO IN <u>ONE</u> HOUR?

18-349

CHAPTER 9

1. TRUE OR FALSE $5 \in (5,7)$
2. SOLVE FOR x: $-2x < 6$
3. WRITE $2 < x < 5$ AS TWO INEQUALITIES
4. REMOVE ABSOLUTE VALUE SIGNS $|x| < 5$
5. REMOVE ABSOLUTE VALUE SIGNS $|y| > 3$
6. SUPPOSE $L < R$ 

   a. $L - R$ IS NEGATIVE POSITIVE ZERO ?
   b. $R - L$ IS NEGATIVE POSITIVE ZERO ?
7. 

   $x - 5$ IS NEGATIVE POSITIVE ZERO ?

   $x - (-2) = x + 2$ IS NEGATIVE POSITIVE ZERO ?

CHAPTER 10

1. NAME A POINT IN THE SECOND QUADRANT.
2. ANOTHER NAME FOR THE x-COORDINANT IS ___.
3. ANOTHER NAME FOR THE y-COORDINANT IS ___.
4. FIND THE DISTANCE BETWEEN $(2,3)$ AND $(4,9)$.
5. WHAT IS THE MIDPOINT BETWEEN 8 AND 21 ON THE NUMBER LINE?.

18 - 350

6. WHAT IS THE MIDPOINT BETWEEN (2,3) AND (4,9)?

7. WHAT IS THE SLOPE OF THE LINE BETWEEN (2,3) AND (4,9)?

8. WHAT IS THE SLOPE OF A HORIZONTAL LINE?

9. TELL ABOUT THE SLOPE FOR A VERTICAL LINE.

10. A LINE HAS SLOPE $3/2$

a. WHAT IS THE SLOPE OF A LINE PARALLEL TO IT?

b. WHAT IS THE SLOPE OF A LINE PERPENDICULAR TO IT?

11. GIVE AN EQUATION FOR THE LINE THROUGH THE POINT (2,3) WITH SLOPE 4.

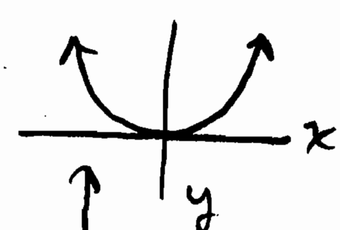
12. WHAT IS THE SLOPE OF THE LINE WITH EQUATION $y = 5x - 6$?

13. GIVE AN EQUATION FOR THE HORIZONTAL LINE THROUGH THE POINT (2,3)

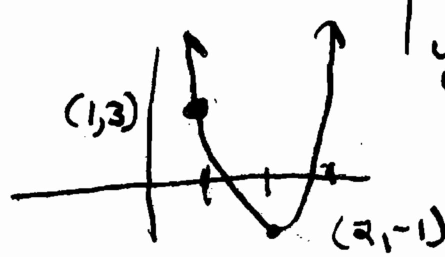
CHAPTER 11

1 GIVE AN EQUATION FOR THE CIRCLE WITH CENTER (2,3) WITH RADIUS 4.

2. NAME AN EQUATION WHOSE GRAPH IS AT THE RIGHT



3. THIS HAS VERTEX (2,-1)
(1,3) IS ON THE GRAPH.
NAME A SYMMETRIC PARTNER

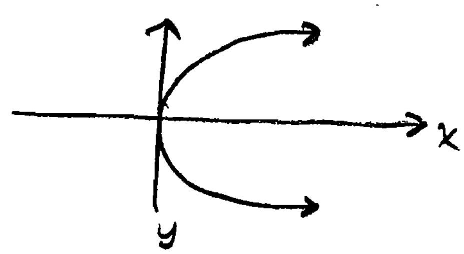


4. $x = -3y^2 + 7y + 6$ opens ______

18-351

CHAPTER 12

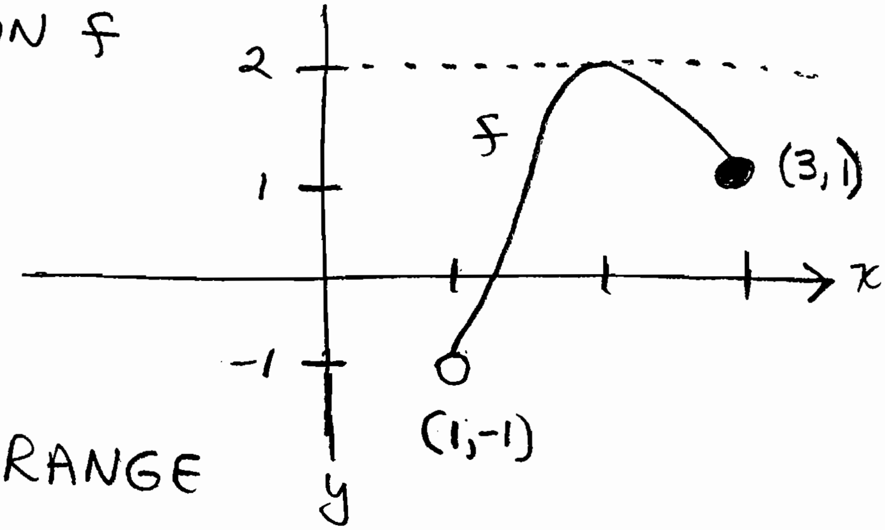
1. DEFINITION OF A FUNCTION:
2. $f = \{(1,5), (2,6), (3,5)\}$ IS A FUNCTION
   a. $dom(f) =$
   b. $ran(f) =$
   c. $f(2) =$
3. SUPPOSE h IS A FUNCTION AND $h(2)=3$. NAME AN ORDERED PAIR IN h.
4. SUPPOSE $m(x) = x^2$ $\quad m(x+1) =$
5. SUPPOSE z IS THE INDEPENDENT VARIABLE AND w IS THE DEPENDENT VARIABLE IN $3z+2w=1$. IS z ASSOCIATED WITH THE FIRST OR SECOND TERMS OF THE SET OF ORDERED PAIRS THAT MAKE UP THE ASSOCIATED FUNCTION?
6. IS THE GRAPH AT THE RIGHT THE GRAPH OF A FUNCTION?



7. NAME THE DOMAIN FOR THE FUNCTION DEFINED BY $f(x) = \frac{1}{x-2}$.

18-352

8. FOR THE FUNCTION f AT THE RIGHT



a. $dom(f) =$

b. $ran(f) =$

9. WHAT IS THE RANGE FOR THE FUNCTION DEFINED BY $h(x) = x^2 + 1$?

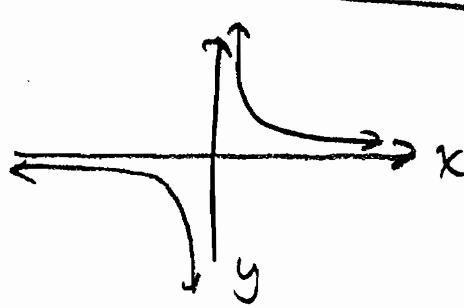
10. FOR $g(x) = \begin{cases} 3x & \text{IF } x > 1 \\ 2x & \text{IF } x \leq 1 \end{cases}$ $g(-4) =$ ___

11. THE GRAPH OF $y = -2x^2$ IS THE GRAPH OF $y = 2x^2$ REFLECTED ABOUT ________.

12. THE GRAPH OF $y = -2(x-3)^2$ IS THE GRAPH OF $y = -2x^2$ TRANSLATED ________

13. THE GRAPH OF $y = -2(x-3)^2 + 1$ IS THE GRAPH OF $y = -2(x-3)^2$ TRANSLATED ________

14. NAME AN EQUATION WHOSE GRAPH IS AT THE RIGHT.



15. NAME ALL HORIZONTAL ASYMPTOTES FOR THE GRAPH IMMEDIATELY ABOVE.

16. TO GET SYMMETRY ABOUT THE y-AXIS FOR $y = f(x)$, SUBSTITUTE _____ FOR _____ AND GET AN EQUIVALENT EQUATION

17. TO GET SYMMETRY ABOUT THE x-AXIS IN AN EQUATION INVOLVING x AND y, SUBSTITUTE _____ FOR _____ AND GET AN EQUIVALENT EQUATION

CHAPTER 13

1. $\mathrm{dom}(f+g) =$

2. FORMULA FOR $(g \circ h)(x) =$

3. SUPPOSE f IS A FUNCTION SUCH THAT f^{-1} IS A FUNCTION

   a. $(2,3) \in f$ MEANS _____ $\in f^{-1}$

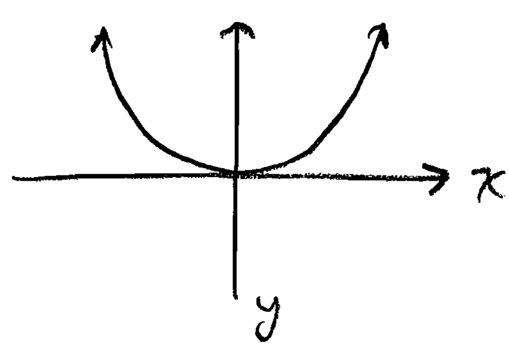
   b. $f(7)=4$ MEANS $f^{-1}(4) =$

   c. $f^{-1}(f(2)) =$

4. FILL IN THE BLANK SO THAT h IS NOT 1-1

   $h = \{(1,3), (2,5), (4,\_\_)\}$

5. IS THE GRAPH AT THE RIGHT THE GRAPH OF A 1-1 FUNCTION? WHY?

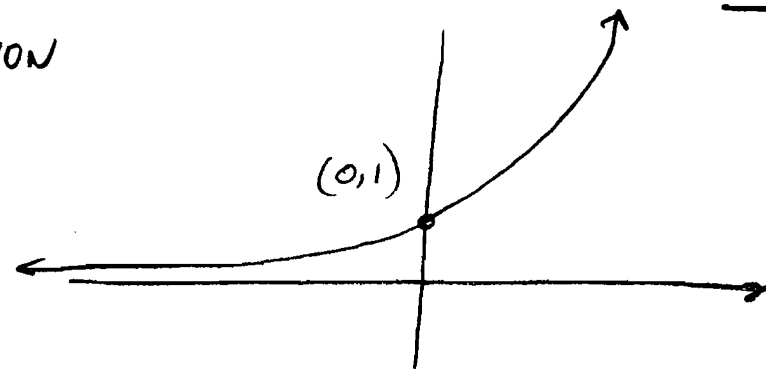


6. THE GRAPHS OF f AND f^{-1} ARE MIRROR REFLECTIONS ABOUT THE LINE ______.

CHAPTER 14

1. e APPROXIMATED TO 1 DECIMAL PLACE IS ___.

2. GIVE AN EQUATION WHOSE GRAPH IS THE GRAPH AT THE RIGHT

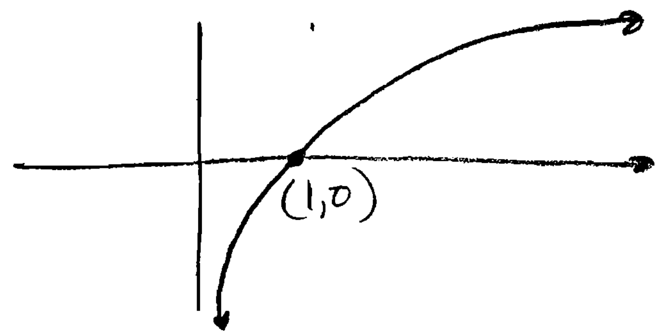


3. $\log_3 9 =$

4. WHAT IS THE BASE FOR $\ln$?

5. PUT $\log x = y$ IN EXPONENTIAL FORM.

6. GIVE AN EQUATION WHOSE GRAPH IS THE GRAPH AT THE RIGHT.



7. $\log MN =$

8. $\log \frac{M}{N} =$

9. $\log M^N =$

10. $e^{\ln x} =$

11. CHANGE TO BASE e LOG : $\log_5 x$

18-355

CHAPTER 15

1. FOR SYNTHETIC DIVISION OF $\dfrac{3x^2-5x+7}{x-2}$ WHAT GOES HERE ?

$$
\begin{array}{r|rrr}
\underline{\quad} & 3 & -5 & 7 \\
 & & 6 & 2 \\
\hline
 & 3 & 1 & \boxed{9}
\end{array}
$$

2. CONSIDER THE SYNTHETIC DIVISION BELOW

$$
\begin{array}{r|rrrr}
7 & 2 & -15 & 8 & -7 \\
 & & 14 & -7 & 7 \\
\hline
 & 2 & -1 & 1 & \boxed{0}
\end{array}
$$

a. WHAT IS $\dfrac{2x^3-15x^2+8x-7}{x-7}$?

b. WHAT IS $f(7)$ WHERE $f(x)=2x^3-15x^2+8x-7$?

3. ACCORDING TO THE FUNDAMENTAL THEOREM OF ALGEBRA (IN DISGUISE), EVERY POLYNOMIAL OF POSITIVE DEGREE CAN BE BROKEN DOWN INTO ______________________

18-356

CHAPTER 16

1. A SYSTEM OF EQUATIONS THAT HAS EXACTLY <u>ONE</u> POINT IN THE SOLUTION SET IS _ ________________ SYSTEM

2. A SYSTEM OF EQUATIONS THAT HAS NO SOLUTION IS _ ________________ SYSTEM

TG - 357

TRUTH GEMS

THE ANSWER IS IN

THE BACK OF THE BOOK!

TRUTH GEM

BE IN THE WILL OF GOD FOR WHAT YOU DO

A. ROM 15:32 ... that I may come to you with joy by the WILL OF GOD, and may be refreshed together with you.

B. I come to you in the WILL OF GOD with joy and we will have refreshing math.

C. Being in the WILL OF GOD taking this course and faithfully, wisely studying you will flourish.

D. HEB. 10:36 For you have need of endurance, so that after you have done the WILL OF GOD you may receive the promise.

TRUTH GEM

WHAT A TEST TESTS

A. (DEUT. 8:2) AND YOU SHALL REMEMBER THAT THE LORD YOUR GOD LED YOU ALL THE WAY THESE FORTY YEARS IN THE WILDERNESS, TO HUMBLE YOU AND TEST YOU TO KNOW WHAT WAS IN YOUR HEART, WHETHER YOU WOULD KEEP HIS COMMANDMENTS OR NOT.

B. SOME CONSIDER A TEST A BRAIN DUMP TO REVEAL WHAT WAS IN YOUR BRAIN AT THE TIME. BUT FOR THOSE WHO HAVE EYES TO SEE A TEST REVEALS WHAT IS IN THE HEART. A TEST WILL REVEAL

1. A HEART THAT IS ON FIRE TO BE ESTABLISHED IN WHAT IS IN THE WILL OF GOD TO BE LEARNED

   OR

2. A HEART THAT IS REGULARLY UNFAITHFUL IN STUDYING WISELY, ONLY SPURTS RIGHT BEFORE A TEST.

C. HEARTS CAN CHANGE. (PS 51:10) CREATE IN ME A CLEAN HEART, O GOD, AND RENEW A STEADFAST SPIRIT WITHIN ME.

TRUTH GEM

WISDOM

A. PR 1:7 WISDOM IS THE PRINCIPAL THING; THEREFORE GET WISDOM. AND IN ALL YOUR GETTING, GET UNDERSTANDING.

B. DEFINITIONS:

1. KNOWLEDGE: FACTS, GAINED INFORMATION
2. UNDERSTANDING: WHY A FACT IS A FACT.
3. WISDOM: BEING LED BY THE SPIRIT. KNOWING WHAT TO DO AT ANY MOMENT.

C. PRAY FOR WISDOM IN FAITH: JAMES 1:5
IF ANY OF YOU LACKS WISDOM, LET HIM ASK OF GOD, WHO GIVES TO ALL LIBERALLY AND WITHOUT REPROACH, AND IT WILL BE GIVEN HIM.

ONE
NOT DIS-INTEGRATED

A. I THESS 5:23 NOW MAY THE GOD OF PEACE HIMSELF SANCTIFY YOU COMPLETELY; AND MAY YOUR WHOLE SPIRIT, SOUL, AND BODY BE PRESERVED BLAMELESS AT THE COMING OF OUR LORD JESUS CHRIST.

B. MK 12:29 "... THE FIRST OF ALL THE COMMANDMENT IS: HEAR, O ISRAEL, THE LORD OUR GOD, THE LORD IS ONE.

C. YOU ARE ONE WHEN OUT OF LOVE OF GOD YOU FOCUS SPIRIT, SOUL, AND BODY ON THE GOD-LED TASK AT HAND ... THIS CLASS, IN THIS COURSE, NOW

- NOT BODY HERE AND MIND ELSEWHERE
- NOT BODY AND MIND HERE, BUT HEART NOT IN IT
- NOT ABSENT IN THE BODY BUT WITH US IN SPIRIT

} DIS-INTEGRATED

D. DESIRE AND ADMIRE BEING ONE.

TG-5

TRUTH GEM

BEGIN

A. ACTS 1:1 "... of all that Jesus <u>BEGAN</u> both to do and teach."

B. MK 4:1 "And again He <u>BEGAN</u> to teach by the sea."

C. For a task to be accomplished, you must <u>BEGIN</u>.

D. <u>BEGINNINGS</u>:

1. <u>BEGIN</u> to s<u>ee</u> yourself as a faithful, good math student
2. <u>See</u> changes you need to make, and <u>BEGIN</u> on those changes.
3. <u>See</u> and learn <u>BEGINNINGS</u> of different problem types.

TG-6

TRUTH GEM

PRESS ON

A. PLP 3:12 NOT THAT I HAVE ALREADY ATTAINED... BUT I PRESS ON, THAT I MAY LAY HOLD OF THAT FOR WHICH CHRIST JESUS HAS ALSO LAID HOLD OF ME.

B. LIKE A DISTANCE RUNNER OR CRUISE CONTROL

1. CONSTANCY, STEADFASTNESS
2. PATIENCE POWER

GOOD WORDS

C. PRESS ON CYCLE DONE WITH A GOOD ATTITUDE (NOT AT LAST MINUTE)

HEAR THE NEW → STUDY → DO HOMEWORK → REVIEW → HEAR THE NEW

D. HEB 6:12 ... THAT YOU DO NOT BECOME SLUGGISH (LAZY), BUT IMITATE THOSE WHO THROUGH FAITH AND PATIENCE INHERIT THE PROMISES.

TG-7

TRUTH GEM

COMPLETE IT

A. PLP 1:6 "... being confident of this very thing, that He who has BEGUN a good work in you will COMPLETE IT until the day of Jesus Christ.

B. THINGS GOD BEGAN, IN HIS WILL, GOD GIVES ABILITY AND PROVISION TO COMPLETE.

1. SEE THESE THINGS THROUGH TO THE END.
2. VICTORY IS SWEET

C. THERE CAN BE BARRIERS TO BREAK THROUGH AT THE END.

1. LIKE A TAPE AT THE END OF A RACE.
2. LIKE THE SOUND BARRIER.

TG-8

TRUTH GEM

BE ESTABLISHED

A. Ps 90:17 And let the beauty of the Lord our God be upon us, and ESTABLISH the work of our hands for us;

B. Be established in the BEGIN - PRESS ON - COMPLETE IT cycle for working problems

C. Be established in knowing how to work certain problem types

D. Hebrew: Established = koon : things brought into incontrovertible existence like:

1. Your nature to faithfully study
2. Your ability to work certain problem types

E. To learn better how to be ESTABLISHED

1. Is 54:14a In righteousness you shall be ESTABLISHED
2. We will learn of righteousness

TG-9

TRUTH GEM

RIGHTEOUSNESS

A. HEB 9:28a For He will finish the work and cut it short in RIGHTEOUSNESS;

B. RIGHTEOUSNESS = RIGHT STANDING WITH GOD BY FAITH.

PLP 3:9 and be found in Him, not having my own righteousness, which is from the law, but that which is through faith in Christ, the righteousness which is from God by faith.

C. BEING IN THE WILL OF GOD FOR WHAT YOU DO, IN RIGHT STANDING WITH GOD, THERE IS GREAT LIBERTY AND SPEEDUP IN WHAT YOU DO. (RIGHTEOUSNESS ENHANCED ACCELERATED LEARNING)

TRUTH GEM

RIGHTEOUSNESS - SHALOM

A. IS 32:17 THE WORK OF RIGHTEOUSNESS (RIGHT STANDING) WILL BE PEACE (SHALOM: NOTHING MISSING, NOTHING BROKEN, COMPLETENESS, HEALTH, PROSPERITY, SAFETY, PEACE)

B. THE SHALOM LEARNING ENVIRONMENT: TEACHER & STUDENT AT PEACE AS THEY TEACH, STUDY, TAKE TESTS, & GRADE PAPERS.

C. "LET RIGHTEOUSNESS WORK FOR YOU" (ED TAYLOR)

D. SPEEDUP BY LEARNING IN A STATE OF SHALOM.

TRUTH GEM

RIGHTEOUSNESS - BOLD

A. PR. 28:1 THE WICKED FLEE WHEN NO ONE PURSUES, BUT THE RIGHTEOUS ARE AS BOLD AS A LION

B. PR. 30:30 A LION, WHICH IS MIGHTY AMONG BEASTS AND DOES NOT TURN AWAY FROM ANY;

C. LEARNING IS SLOWED DOWN BY WIMPILY, TIMIDLY TURNING AWAY FROM SOME MATH PROBLEMS.

D. LEARNING SPEEDUP BY BOLDLY (IN ACCORDANCE WITH YOUR RIGHTEOUS NATURE) TAKING ON THINGS ON YOUR PATH AND OVERCOMING.

E. NEXT, BOLDNESS AND HUMILITY, NOT CONTRADICTARY.

TRUTH GEM

RIGHTEOUSNESS: BOLD & HUMBLE

A. PLP 2:8a He (Jesus) HUMBLED Himself and became obedient..

B. James 4:6-7a "... God resists the proud, but gives grace to the HUMBLE." Therefore submit to God.

C. HUMILITY: Go God's way not your own way.

D. Some problems it takes boldness to solve. A false humility doctrine can cause someone to wimp out and be defeated.

E. There is a difference between boldness and aggression

TRUTH GEM

MADE RIGHTEOUS - NO CONDEMNATION

A. 2 COR 5:21 For he hath made him to be sin for us, who knew no sin; that we might be MADE the RIGHTEOUSNESS of God in him.

B. ROM 8:33-34a Who shall bring a charge against God's elect? It is God who justifies ("righteousifies"). Who is he who condemns?

C. Being condemned, saying you are no good, or no good in math is not of God - can cause some to quit - slows down learning!

D. Being in the will of God, God-esteemed ("much more" than self-esteem), no discouragement = learning speedup!

TG-14

TRUTH GEM

FAITH – NO FEAR

A. MK 5:36b "DO NOT BE AFRAID, ONLY BELIEVE:" —— HAVE FAITH

B. FEAR IS HAVING MORE FAITH IN THE POWER OF THE DEVIL TO DO HARM THAN THE POWER OF <u>GOD TO DO GOOD</u>. Philip Derber

C. FAITH IS LIKE PUTTING IT IN DRIVE,
DOUBT IS LIKE PUTTING IT IN NEUTRAL,
FEAR IS LIKE PUTTING IT IN REVERSE
UNBELIEF IS LIKE PUTTING IT IN PARK.
John Paul

D. ROM 12:21 "DO NOT BE OVERCOME BY EVIL, BUT OVERCOME EVIL WITH GOOD." GOOD OVERCOMES EVIL SO DO NOT FEAR.

E. FEAR ATTRACTS THE FEARED THING. FAITH IS THE SUPERNATURAL CONNECTION THAT RECEIVES THE OVERCOMING GOOD – THE THING BELIEVED FOR ... MATH UNDERSTANDING

TRUTH GEM

GRACE

A. I Pet 4:10-11 part As each one has received a GIFT, minister it to one another as good stewards of the manifold GRACE of God... If anyone ministers let him do it as with the ABILITY which God supplies...

B. DEFINITION: GRACE - God's ability gift to live and function in the gifts and callings.

C. Grace is part of God's supernatural provision to do excellently what God has called us to do

D. 2 Cor 9:8 And God is able to make all GRACE abound toward you, that you, always having all sufficiency in all things, may have an abundance for every good work.

TRUTH GEM

GRACE - WORKS EFFECTIVELY

A. Parts of Gal 2:7-9 ...when they SAW that the gospel for the uncircumcised had been committed to me.. for He who... WORKED EFFECTIVELY in me toward the Gentiles, ... when James, Cephas, and John,... perceived the GRACE that had been given to me.

B. A GRACEFUL PERSON WORKS EFFECTIVELY.

C. GRACE: GOD'S ABILITY GIFT TO LIVE AND FUNCTION IN THE GIFTS AND CALLINGS

D. BEING IN GOD'S WILL FOR TAKING THIS COURSE, THERE IS GRACE (SUPERNATURAL ABILITY TO WORK EFFECTIVELY) FOR YOU TO DO SO WELL IT CAN BE SEEN.

CALLING & DESTINY

A. EPH 1:18 ... the eyes of your understanding enlightened that you may know what is the hope (i.e. <u>destiny</u>) of His <u>calling</u>

B. (Jack Shoup) The <u>calling</u> is the <u>office</u>. The destiny is to be fulfilled in that office. <u>Grace</u> is the provision to fulfill your destiny within and by being in your calling

C. <u>EXAMPLE</u>: Part of my calling is to be a math teacher. Part of my destiny is to "make it plain". Since I have answered the call - there is grace to fulfill it.

Similar example: <u>call</u>: college student
destiny - x = major grade = y

D. You can answer the call and not fulfill your destiny (Jack Shoup)

E. PLP 3:14 I press toward the goal for the prize (<u>destiny</u>) of the upward <u>call</u> of God in Christ Jesus

TRUTH GEM

USE YOUR IRREVOCABLE GIFT

A. I PET 4:10 AS EACH ONE HAS RECEIVED A GIFT, MINISTER IT TO ONE ANOTHER AS GOOD STEWARDS OF THE MANIFOLD GRACE OF GOD

B. ROM 11:29 FOR THE GIFTS AND CALLING OF GOD ARE IRREVOCABLE.

C. EACH OF US HAS AN IRREVOCABLE GIFT

1. IT IS TO BE USED TO HELP US AND OTHERS
2. IT IS IRREVOCABLE SO THAT YOU CAN HAVE GREAT CONFIDENCE AT TIME OF NEED, THE GIFT WILL BE THERE TO BLESS

D. WHEN YOU DISCERN YOUR CALLING AND GIFTS

1. HONOR THE CALL AND GIFTS - DON'T DESIRE SOME OTHER.
2. USE EACH GIFT BY GRACE TO FULFILL THE CALL.

E. BEING IN THE WILL OF GOD FOR BEING HERE BE SURE THERE IS A GIFT YOU HAVE THAT GOD WILL GRACE FOR YOU TO USE TO SUCCEED WITH JOY.

TG-19

TRUTH GEM

NO MATH ANXIETY

A. PLP 4: 6-7 BE ANXIOUS FOR NOTHING, BUT IN EVERYTHING BY PRAYER AND SUPPLICATION, WITH THANKSGIVING, LET YOUR REQUESTS BE MADE KNOWN TO GOD; AND THE PEACE OF GOD, WHICH SURPASSES ALL UNDERSTANDING, WILL GUARD YOUR HEARTS AND MINDS THROUGH JESUS CHRIST

B. THEOREM: TO HAVE NO MATH ANXIETY, HAVE NO ANXIETY!!

C. YOU HAVE NOTHING DOUBTING FAITH, THAT BEING IN GOD'S WILL FOR TAKING THIS COURSE, THERE IS SUPERNATURAL PROVISION FOR YOU TO SUCCEED.

YOU MAKE PRAYER AND SUPPLICATION FOR IT.

YOU DO NOT DOUBT.

GIVE THANKS TO GOD.

PEACE COMES.

SUPERNATURAL PROVISION COMES.

MATH SUCCESS COMES.

NECESSITY OF UNDERSTANDING

A. (MT 13:19) WHEN ANYONE HEARS THE WORD OF THE KINGDOM AND DOES NOT UNDERSTAND IT, THEN THE WICKED ONE COMES AND SNATCHES AWAY WHAT WAS SOWN IN HIS HEART....

B. TO KEEP PATH OF LIFE KNOWLEDGE FROM BEING SNATCHED AWAY, GET UNDERSTANDING OF THAT KNOWLEDGE BY

1. PRAYING FOR WISDOM
2. DOING THE HOMEWORK

↳ (JAMES 1:22-24) BUT BE DOERS OF THE WORD AND NOT HEARERS ONLY. FOR IF ANYONE IS A HEARER OF THE WORD AND NOT A DOER, HE IS LIKE A MAN OBSERVING HIS NATURAL FACE IN A MIRROR; FOR HE OBSERVES HIMSELF, GOES AWAY, AND IMMEDIATELY FORGETS WHAT KIND OF MAN HE WAS.

TG-21

TRUTH GEM

DISCIPLINE

A. 2 Tim 1:7 For God has not given us a spirit of timidity, but of power and love and DISCIPLINE.

B. The undisciplined have problems with math.

C. Regular steady study (in wisdom) prevails, not just a big flurry the night before the test.

D. You will have to keep retaking life's test on discipline until you pass it.

E. For the Christian, DISCIPLINE is a fruit of what we are given.
Gifts must be received.
Gifts must be cultivated.

TG-22

TRUTH GEM

PERFECTED LOVE

A. I JN 4:18 THERE IS NO FEAR IN LOVE; BUT PERFECT LOVE CASTS OUT FEAR, BECAUSE FEAR INVOLVES TORMENT, BUT HE WHO FEARS HAS NOT BEEN MADE PERFECT IN LOVE.

B. PERFECT LOVE CASTS OUT THAT WHICH TORMENTS.

C. MATH TORMENTS MANY BECAUSE OF A LACK OF DISCIPLINE TO STUDY PROPERLY.

D. YOU HAVE TO KEEP RETAKING THE TESTS OF LIFE UNTIL YOU PASS THEM. (P. Derbey)

E. PASS LIFE'S TEST OF DISCIPLINED STUDY BY LETTING PERFECTED LOVE CAST OUT THE TORMENTING LACK OF DISCIPLINE.

F. 2 TIM 1:7 FOR GOD HAS NOT GIVEN US A SPIRIT OF TIMIDITY, BUT OF POWER AND LOVE AND DISCIPLINE (NASB)

TRUTH GEM

EAT THE LABOR OF YOUR HANDS

A. Ps 128:2 WHEN YOU EAT THE LABOR OF YOUR HANDS, YOU SHALL BE HAPPY AND IT SHALL BE WELL WITH YOU.

B. WHEN HARVEST COMES IN, EAT OF IT. WHEN MATH UNDERSTANDING AND GOOD GRADE COME IN AS A HARVEST, SAVOR IT.

C. NOT ONLY IS VICTORY SWEET, BUT PROPERLY ENJOYING IT CAUSES YOU TO BE HAPPY AND IT TO BE WELL WITH YOU

D. GEN 1:31a THEN GOD SAW EVERYTHING THAT HE HAD MADE, AND INDEED IT WAS VERY GOOD.

E. I GET THIS "AAAAHHH!" SATISFACTION OF GOD'S GRACE WORKING THROUGH ME.

TG-24

TRUTH GEM

MEDITATE

A. PS 1:2 BUT HIS DELIGHT IS IN THE LAW OF THE LORD AND IN HIS LAW HE MEDITATES* DAY AND NIGHT.

*PONDERS BY TALKING TO HIMSELF

B. WHAT YOU ARE IN THE WILL OF GOD TO LEARN CAN BE MEDITATED.

C. PICK A DEFINITION, THEOREM, OR PROBLEM DERIVATION.

1. CHEW ON IT, WORD FOR WORD, SEEKING UNDERSTANDING.
2. SPEAK IT OUT SO YOU CAN HEAR IT.
3. WRITE IT DOWN OVER AND OVER.
4. REVIEW IT.

TG-25

TRUTH GEM

MEDITATE STAYING CONNECTED TO THE SOLUTION REALM

A. PS 119:23 PRINCES ALSO SIT AND SPEAK AGAINST ME, BUT YOUR SERVANT MEDITATES ON YOUR STATUTES.

B. LIKE A BASKETBALL PLAYER IS AWARE OF THE BALL AS HE DRIBBLES EVEN THOUGH HE MAY HAVE OTHER THINGS ON HIS MIND, SO I REMAIN AWARE OF GOD AT ALL TIMES, EVEN WHEN TEACHING AND THINKING ABOUT MATH PROBLEMS: THE GOD REALM IS THE SOLUTION REALM

C. SINCE I PRAY FOR WISDOM AND EXPECT GOD TO PROVIDE IT, I STAY CONNECTED TO GOD TO RECEIVE THE WISDOM WHEN IT COMES.

D. IF YOU LOCK IN FOCUSSING ONLY ON THE PROBLEM IT CAN ADDLE YOU. CONNECTION TO GOD PREVENTS THAT.

E. JAMES 1:17 EVERY GOOD AND EVERY PERFECT GIFT IS FROM ABOVE, AND COMES DOWN FROM THE FATHER OF LIGHTS, WITH WHOM THERE IS NO VARIATION OR SHADOW OF TURNING.

TRUTH GEM

MAGNIFY THE SOLUTION AND NOT THE PROBLEM

A. PS 34:3 OH, MAGNIFY THE LORD WITH ME AND LET US EXALT HIS NAME TOGETHER

B. FOCUS ON WHAT YOU SEE IS TRUE AND GOOD AND CAN BE DONE. DO THAT. THE PROBLEM SHRINKS. SEE SOMETHING ELSE THAT IS TRUE AND GOOD AND CAN BE DONE. DO THAT. THE PROBLEM SHRINKS. PRESS ON DOING THIS UNTIL THE PROBLEM IS GONE!

C. PR 3:27 DO NOT WITHHOLD GOOD FROM THOSE TO WHOM IT IS DUE, WHEN IT IS IN THE POWER OF YOUR HAND TO DO SO.

D. JAMES 4:17 THEREFORE TO HIM WHO KNOWS TO DO GOOD AND DOES NOT DO IT, TO HIM IT IS SIN.

TG-27

TRUTH GEM

THE TEACHING OF KNOWLEDGE

A. IS 28:9-10 WHOM WILL HE TEACH KNOWLEDGE? AND WHOM WILL HE MAKE TO UNDERSTAND THE MESSAGE? THOSE JUST WEANED FROM MILK? THOSE JUST DRAWN FROM THE BREASTS?
FOR PRECEPT MUST BE UPON PRECEPT, PRECEPT UPON PRECEPT, LINE UPON LINE, LINE UPON LINE. HERE A LITTLE, THERE A LITTLE.

B. ADMIT THE TRUTH THAT UNDERSTANDING ADVANCED MATH TOPICS REQUIRES GOOD UNDERSTANDING OF PREVIOUS MATH TOPICS THAT THE ADVANCED TOPIC IS BASED ON.
JUGULAR PROBLEM SOLUTIONS NEED PREVIOUS KNOWLEDGE.

C. LOVE ORDER AND FLOW IN IT RATHER THAN FIGHTING AGAINST ORDER.

D. COL 2:5 FOR THOUGH I AM ABSENT IN THE FLESH, YET I AM WITH YOU IN SPIRIT, REJOICING TO SEE YOUR GOOD ORDER AND THE STEADFASTNESS OF YOUR FAITH IN CHRIST.

TG-28

TRUTH GEM

LITTLE BY LITTLE

A. DEUT. 7:22 AND THE LORD YOUR GOD WILL DRIVE OUT THOSE NATIONS BEFORE YOU LITTLE BY LITTLE; YOU WILL BE UNABLE TO DESTROY THEM AT ONCE, (EX 23:29) LEST THE LAND BECOME DESOLATE AND) LEST THE BEASTS OF THE FIELD BECOME TOO NUMEROUS FOR YOU.

B. LEARNING MATHEMATICS IS LIKE POSSESSING NEW TERRITORY. YOU MUST DO IT LITTLE BY LITTLE AND GET ESTABLISHED IN, CULTIVATE, PATROL\GUARD THAT GAINED WISDOM\KNOWLEDGE\UNDERSTANDING

C. TO TRY TO LEARN ALL AT ONCE, YOUR MIND IS STRETCHED TOO THIN (LAND BECOMES DESOLATE) YOU ARE UNABLE TO PATROL\GUARD AGAINST THE BEASTS OF THE FIELD (CONFUSION ATTEMPTS, LACK OF REMEMBERANCE, FEAR OF A HERD OF QUESTIONS COMING AT YOU ON A TEST)

D. DO NOT WAIT UNTIL RIGHT BEFORE THE TEST TO TRY TO LEARN DAYS OF WORK. WISDOM: LITTLE BY LITTLE.

TRUTH GEM

BE STEADFAST IN HOMEWORK

A. I COR 15:58 THEREFORE, MY BELOVED BRETHREN, BE STEADFAST, IMMOVABLE, ALWAYS ABOUNDING IN THE WORK OF THE LORD, KNOWING THAT YOUR LABOR IS NOT IN VAIN IN THE LORD.

B. BEING IN THE WILL OF GOD BY TAKING THIS COURSE MEANS YOU ARE DOING THE WORK OF THE LORD, ... SO... BE STEADFAST

C. DISTRACTIONS, LOWER PRIORITY THINGS WILL TRY TO TAKE YOU AWAY FROM "ALWAYS ABOUNDING" IN STUDY... BE SET. DO NOT BE MOVED. HOLD FAST AND STUDY.

D. PS 16:8 I HAVE SET THE LORD ALWAYS BEFORE ME; BECAUSE HE IS AT MY RIGHT HAND I SHALL NOT BE MOVED.

TG-30
TRUTH GEM

PRIORITIZE

A. NEH 6:3 I AM DOING A GREAT WORK, SO THAT I CANNOT COME DOWN. WHY SHOULD THE WORK CEASE WHILE I LEAVE IT AND GO DOWN TO YOU?

B. HAVE GOD'S PRIORITIES FOR YOUR LIFE CLEAR, DECIDED, SET, ESTABLISHED... A DIVINE ORDER. WHEN SOMETHING COMES UP FOR A DECISION, DISCERN WHAT CATEGORY IT IS IN AND THE DECISION HAS ALREADY BEEN MADE.

C. BEING IN GOD'S WILL FOR TAKING THIS COURSE MEANS THIS COURSE IS A HIGH PRIORITY, SO REGULAR, NONDISTRACTED STUDY TIME IS A HIGH PRIORITY, SO... DO NOT LEAVE IT AND GO DOWN TO DO A LESSER PRIORITY.

TG-31

TRUTH GEM

OVERCOME

A. I JN 5:4 FOR WHATEVER IS BORN OF GOD <u>OVERCOMES</u> THE WORLD. AND THIS IS THE VICTORY THAT HAS OVERCOME THE WORLD— OUR <u>FAITH</u>.

B. BEING IN THE WILL OF GOD FOR TAKING THIS COURSE, AND HENCE DOING A GREAT WORK, YOU WILL BE COME AGAINST TO TRY TO STOP, HINDER, OR HARASS THE WORK OF REGULAR STUDY — <u>OVERCOME</u> IT WITH SUPERNATURAL HELP

C. WITH A BOLD, NONTIMID, STRONG AND COURAGEOUS INNER MAN — SAY NO AND RESIST AND OVERCOME THE OPPOSITION

D. EPH 3:16 ... THAT HE WOULD GRANT YOU, ACCORDING TO THE RICHES OF HIS GLORY, TO BE <u>STRENGTHENED</u> <u>WITH</u> <u>MIGHT</u> <u>THROUGH</u> <u>HIS</u> <u>SPIRIT</u> IN THE INNER MAN

TRUTH GEM

DECISIONS VS. OTSGWFIADI

A. MK 14:36b ... NEVERTHELESS, NOT WHAT I WILL, BUT WHAT YOU WILL.

B. WHEN WHAT IS CALLED A DECISION IS PRESENTED TO YOU (LIKE TO START STUDYING ON DO SOMETHING ELSE - OR - TO KEEP STUDYING LONGER OR STOP). WHAT IS THE BASIS FOR YOUR DECISION?... YOUR OWN UNDERSTANDING? YOUR PLEASURES?

C. PR 3:5-6 TRUST IN THE LORD WITH ALL YOUR HEART AND LEAN NOT ON YOUR OWN UNDERSTANDING; IN ALL YOUR WAYS ACKNOWLEDGE HIM AND HE SHALL DIRECT YOUR PATHS

D. INSTEAD OF DECISION TIME ITS OTSGWFIADI TIME: Opportunity To Seek God's Will, Find It, And Do It.

E. PART OF MT 7:7 ... SEEK AND YOU WILL FIND...

TRUTH GEM

DAVID AND GOLIATH

A. BEFORE GOLIATH'S DEFEAT: PART I SAMUEL 17:11
... THEY WERE DISMAYED AND GREATLY AFRAID.

B. PROPER BATTLE ATTITUDE: I SAM. 17:48b
... DAVID HURRIED AND RAN TOWARD THE ARMY TO MEET THE PHILISTINE.

C. AFTER GOLIATH'S DEFEAT: I SAM. 17:51b
... AND WHEN THE PHILISTINES SAW THAT THEIR CHAMPION WAS DEAD, THEY FLED.

D. THE LOSS OF ONE SOLDIER IN THE PHILISTINE ARMY CAUSED A MAJOR REVERSAL - FROM TAUNTING TO FLEEING ... A REASON: YOU DEFEAT AN ENEMY'S MOST POWERFUL WEAPON, THEIR STRENGTH IS GONE; WHAT THEY TRUSTED IN WAS GONE; THEY FLEE.

E. BY NOW YOU HAVE HAD VICTORY OVER SOME OF THE MOST POWERFUL PROBLEMS IN THE COURSE. SOME OF INTIMIDATION'S STRONGEST WEAPONS - FEAR OF HARD MATH PROBLEMS AND PAST EXPERIENCE OF DIFFICULTY WITH HARD MATH PROBLEMS, HAS BEEN OVERCOME. THIS ENEMY IS FLEEING... PURSUE ... PLUNDER WITH CONTINUED DISCIPLINE & CLARITY

F. PARTS OF I SAM 17:52,53 NOW THE MEN OF ISRAEL AND JUDAH... PURSUED... AND THEY PLUNDERED.

TRUTH GEM

IDOLATRY

A. COL. 3:5b ... PUT TO DEATH YOUR MEMBERS WHICH ARE ON THE EARTH: FORNICATION, UNCLEANNESS, PASSION, EVIL DESIRE, AND <u>COVETEOUSNESS</u>, WHICH IS <u>IDOLATRY</u>.

B. IDOLS: PEOPLE WORSHIPPED <u>IMAGES</u> OF THINGS THAT ARE <u>NOTHING</u> (I COR 8:4b ... WE KNOW THAT AN IDOL IS <u>NOTHING</u>...)

C. MANY PEOPLE <u>COVET</u> THE <u>IMAGE</u> OF BEING EDUCATED, BUT THERE IS NO REALITY, <u>NOTHING</u> TO BACK UP THE IMAGE.

D. SOME WANT TO WORK IN GROUPS AND GET A GROUP GRADE WHEN IN TRUTH THEY DO NOT KNOW IT, OR WRITE A PAPER ON FEELINGS ABOUT MATH RATHER THAN DO MATH. THEY ARE CONTENT WITH THE GRADE, THE IMAGE (THE IDOL) EVEN THOUGH THEY DO NOT KNOW.

E. FROM I JN 5:21 ... KEEP YOURSELF FROM IDOLS. [DESIRE TO KNOW THE MATH, DESIRE TO BE ABLE TO OBJECTIVELY DEMONSTRATE THAT YOU KNOW.]

TG-35

TRUTH GEM

JOY IN RESPONSIBILITY

A. MT 25:21 HIS LORD SAID TO HIM, "WELL DONE, GOOD AND FAITHFUL SERVANT; YOU WERE FAITHFUL OVER A FEW THINGS, I WILL MAKE YOU RULER OVER MANY THINGS. ENTER INTO THE JOY OF YOUR LORD.

B. IT TAKES RESPONSIBILITY TO BE GOOD IN MATH.

C. RESPONSIBILITY IS A GOOD WORD.

D. WHEN GOD PROMOTES YOU AFTER LONG FAITHFUL SERVICE TO MORE RESPONSIBILITY, GOD IS JOYFUL AT HAVING SOMEONE GOOD AND FAITHFUL OVER WHAT GOD CARES VERY MUCH FOR.

1. WE ARE TO ENTER INTO HIS JOY
2. IT IS COMMANDED.
3. GOD MAKES YOU RULER, SO THROUGH HIM YOU ARE ABLE.

E. MANY OF YOU HAVE BEEN GOOD AND FAITHFUL STUDENTS FOR YEARS; YOU HAVE BEEN PROMOTED TO THIS CLASS; ENTER INTO HIS JOY. THIS CLASS IS ALSO A PROVING GROUND FOR FURTHER PROMOTION BY GOD.

TG-36

TRUTH GEM

FLOURISHING BY BEING BLESSED

A. GEN 39:2a, 5b THE LORD WAS WITH JOSEPH AND HE WAS A SUCCESSFUL MAN... THE LORD BLESSED THE EGYPTIAN'S HOUSE FOR JOSEPH'S SAKE; AND THE BLESSING OF THE LORD WAS ON ALL THAT HE HAD IN THE HOUSE AND IN THE FIELD.

B. Num 23:20b ...HE (GOD) HAS BLESSED AND I CANNOT REVERSE IT.

C. BE IN THE WILL OF GOD FOR WHAT YOU DO AND THE IRREVERSIBLE BLESSING OF GOD WILL BE UPON YOU AND WITH YOU TO CAUSE YOU TO FLOURISH, BE SUCCESSFUL. BLESSING IS A SUPERNATURAL FLOURISHING FORCE.

D. OTHERS WILL EVEN BE BLESSED FOR YOUR SAKE.

E. BEING IN THE WILL OF GOD FOR TAKING THIS COURSE, GOD BLESSES YOU AND I BLESS YOU.

TG-37

TRUTH GEM

PROTECTING THE PRECIOUS:
A HEART THAT DESIRES TO STUDY RIGHT THINGS

A. PR 4:23 KEEP YOUR HEART WITH ALL DILIGENCE, FOR OUT OF IT SPRING THE ISSUES OF LIFE.

B. BEING IN THE WILL OF GOD FOR TAKING THIS CLASS, THAT DESIRE TO LEARN EXCELLENTLY BY FLOWING IN STUDYING GRACE IS A PRECIOUS NATURE OF YOUR HEART; GUARD IT; KEEP IT.

C. IN FUTURE TRUTH GEMS WE WILL LEARN WAYS TO PROTECT YOUR HEART'S DESIRE TO BE GENUINE, STUDY RIGHT THINGS, AND EXCEL. IT IS A START TO KNOW IT IS PRECIOUS AND CAN BE KEPT.

D. I JN 5:18b ... BUT HE WHO HAS BEEN BORN OF GOD KEEPS HIMSELF, AND THE WICKED ONE DOES NOT TOUCH HIM.

TRUTH GEM

PROTECTING THE PRECIOUS: A HEART THAT DESIRES TO STUDY RIGHT THINGS (PART 2)
FEED YOUR HEART THE RIGHT TEACHINGS

A. HEB 13:9 DO NOT BE CARRIED ABOUT BY VARIOUS AND STRANGE DOCTRINES. FOR IT IS GOOD THAT THE HEART BE ESTABLISHED BY GRACE, NOT WITH FOODS WHICH HAVE NOT PROFITED THOSE WHO HAVE BEEN OCCUPIED WITH THEM.

B. TEACHINGS ARE FOOD FOR THE HEART. WE ARE TO A CERTAIN EXTENT WALKING TEACHINGS.

C. TO KEEP A PRECIOUS DILIGENT HEART, FEED IT THE RIGHT FOODS (I.E. TEACHINGS): GRACE, RIGHTEOUSNESS, STEADFASTNESS

D. DO NOT LET YOUR HEART EAT JUNK FOOD (DECEPTIVE, UNPROFITABLE TEACHINGS)

E. PR 4:23 KEEP YOUR HEART WITH ALL DILIGENCE; FOR OUT OF IT SPRING THE ISSUES OF LIFE.

TG-39

TRUTH GEM

PROTECTING THE PRECIOUS: A HEART THAT DESIRES TO STUDY RIGHT THINGS (PART 3) HAVE FAITH THAT THE TEACHING THAT "YOU ARE TO STUDY RIGHT THINGS TO LEARN AND DO THEM EXCELLENTLY" IS A TEACHING THAT IS HIGH AND LIFTED UP AND ABOVE REPROACH.

A. EPH 6:16 ... ABOVE ALL, TAKING THE SHIELD OF FAITH WITH WHICH YOU WILL BE ABLE TO QUENCH ALL THE FIERY DARTS OF THE WICKED ONE.

B. HAVING DEEP FAITH IN YOUR HEART AND SPEAKING FROM THE HEART YOUR FAITH IN THE NOBILITY OF WHAT YOU HAVE FAITH IN AND ARE DOING, IS A POWERFUL SHEILDING FORCE FOR YOUR HEART THAT CANNOT BE PENETRATED.

C. THAT WHICH WOULD COME AGAINST THE PRECIOUS DESIRE TO STUDY RIGHT THINGS IS OF THE WORLD.

D. I JN 5:4b AND THIS IS THE VICTORY THAT HAS OVERCOME THE WORLD — OUR FAITH

TG-40

TRUTH GEM

LOVE IS NOT PROVOKED

A. PART OF I COR 13:5 (LOVE)... IS NOT PROVOKED.

B. I USED TO THINK THAT JUST MEANT DO NOT LET PEOPLE PROVOKE YOU. ONE DAY AFTER GETTING PROVOKED AT A COMPUTER, I SAW LOVE IS NOT TO GET PROVOKED PERIOD.

C. LOVE IS AN INNER STATE OF BEING THAT CANNOT BE PROVOKED BY PEOPLE, THINGS, CIRCUMSTANCES... ANYTHING

D. DO NOT GET FRUSTRATED BY ONE HOMEWORK PROBLEM YOU HAVE NOT BEEN ABLE TO WORK YET. DO NOT LET THAT ROB YOU OF WORKING MANY OTHERS. DO NOT LET IT ROB YOU OF JOY OF VICTORY OVER OTHER PROBLEMS

E. THERE ARE NO VICTORIES WITHOUT BATTLES. DO NOT GET FRUSTRATED OVER A PROBLEM. IT IS A VICTORY IN THE MAKING.

TG-41

TRUTH GEM

THE HOLY SPIRIT TEACHES

A. Jn 16:13 HOWEVER, WHEN HE, THE SPIRIT OF TRUTH, HAS COME, HE WILL GUIDE YOU INTO ALL TRUTH; FOR HE WILL NOT SPEAK OF HIS OWN AUTHORITY, BUT WHATEVER HE HEARS HE WILL SPEAK; AND HE WILL TELL YOU THINGS TO COME.

B. THIS IS A WAY GOD TEACHES US WISDOM (MUCH NEEDED FOR THIS COURSE AND LIFE IN GENERAL).

C. WHAT THE HOLY SPIRIT TEACHES IS NOT A MIXTURE OF TRUTH AND ERROR, BUT ALL TRUTH.

D. PRAY FOR WISDOM. SIMPLY ASK, "GOD, WHAT'S TRUE." BE EXPECTANT FOR ILLUMINATION. IT COMES.

E. SOME OF THE TEACHING IS DISCERNED BY DETECTING A RED LIGHT OR GREEN LIGHT TO PURSUE A CERTAIN COURSE.

TG-42

TRUTH GEM

HOLY SPIRIT BAPTISM IMPLIES BOLDNESS

A. ACTS 4:31b ... THEY WERE ALL FILLED WITH THE HOLY SPIRIT AND SPOKE THE WORD OF GOD WITH BOLDNESS

B. RECALL PR 28:1b .. THE RIGHTEOUS ARE AS BOLD AS A LION.

C. YOU NEED GREAT BOLDNESS TO WORK SOME MATH PROBLEMS.
RIGHTEOUSNESS BOLDNESS + HOLY SPIRIT POWER BOLDNESS = SOLVED PROBLEMS IN THIS COURSE.

D. JESUS TOLD HIS DISCIPLES TO WAIT UNTIL THE RECEIVED POWER FROM ON HIGH UNTIL THEY WENT OUT TO WITNESS

E. WHATEVER WE DO IN HIS WILL THIS BOLDNESS IS AVAILABLE.

F. LK 11:13b ... HOW MUCH MORE WILL YOUR HEAVENLY FATHER GIVE THE HOLY SPIRIT TO THOSE WHO ASK HIM.

TRUTH GEM

FOR EVERY PROBLEM, THERE IS A PROBLEM OBLITERATING REVELATION THAT OBLITERATES THE PROBLEM

A. ACTS 9:3 AS HE JOURNEYED HE CAME NEAR DAMASCUS, AND SUDDENLY A LIGHT SHONE AROUND HIM FROM HEAVEN.... GAL 1:23b HE WHO FORMERLY PERSECUTED US NOW PREACHES THE FAITH WHICH HE ONCE TRIED TO DESTROY.

B. PAUL HAD A GREAT PROBLEM. HE WANTED CHRISTIANS JAILED, EVEN HAD A PART IN A STONING. HE GOT A REVELATION. THAT PROBLEM WAS OBLITERATED.

C. FOR EVERY PROBLEM A PERSON HAS, THERE IS A PROMISE OF GOD THAT OBLITERATES THE PROBLEM.

D. II PET 1:4 ... BY WHICH HAVE BEEN GIVEN TO US EXCEEDINGLY GREAT AND PRECIOUS PROMISES, THAT THROUGH THESE YOU MAY BE PARTAKERS OF THE DIVINE NATURE, HAVING ESCAPED THE CORRUPTION THAT IS IN THE WORLD THROUGH LUST.

TG-44

TRUTH GEM

WHEN THERE IS A PROBLEM, LOOK FOR A ROOT FEAR CAUSING THE PROBLEM

A. PS 34:4 I SOUGHT THE LORD AND HE HEARD ME AND DELIVERED ME FROM ALL MY FEARS.

B. SEEK THE LORD ON ANY ROOT FEAR BEHIND A PROBLEM.

1. FEAR OF A BAD GRADE
2. FEAR OF WHAT PEOPLE WILL THINK

C. SEEK THE LORD FOR THE PROBLEM OBLITERATING ILLUMINATION THAT DESTROYS THE FEAR.

1. STAY IN FAITH THAT THE ILLUMINATION WILL COME.
2. STAY IN FAITH WORKING THE ILLUMINATION AFTER IT COMES, THAT VICTORY IS SURE.

D. I JN 5:4b ... AND THIS IS THE VICTORY THAT HAS OVERCOME THE WORLD ... OUR FAITH

E. FAITH IS THE OPPOSITE OF FEAR. OUR JOB IS TO STAY IN FAITH.

F. MK 5:36b DO NOT BE AFRAID; ONLY BELIVE (i.e. HAVE ONLY FAITH - NO MIXTURE)

TG-45

TRUTH GEM

STRONGHOLD OF RIGHTEOUSNESS IN YOUR MIND

A. 2 COR 10:4-5 FOR THE WEAPONS OF OUR WARFARE ARE NOT CARNAL, BUT MIGHTY IN GOD FOR PULLING DOWN STRONGHOLDS, CASTING DOWN ARGUMENTS AND EVERY HIGH THING THAT EXALTS ITSELF AGAINST THE KNOWLEDGE OF GOD, BRINGING EVERY THOUGHT INTO CAPTIVITY TO THE OBEDIENCE OF CHRIST,

B. 2 COR 5:21 FOR HE MADE HIM WHO KNEW NO SIN TO BE SIN FOR US, THAT WE MIGHT BECOME THE RIGHTEOUSNESS OF GOD IN HIM.

C. SOME HAVE STRONGHOLDS IN THEIR MIND THAT THEY ARE MATH FAILURES, RESPONSIBILITY FAILURES, A MISTAKE, A SOCIAL REJECT... ETC. WHEN A CHALLENGE COMES UP THEIR MIND REPLAYS THAT TAPE STRONGLY, THEY YIELD TO THAT AND ARE BOUND — YET THOSE IMAGES IN THE MIND EXALT THEMSELVES AGAINTS THE KNOWLEDGE OF GOD THAT CHRISTIANS ARE THE RIGHTEOUSNESS OF GOD IN CHRIST JESUS, NEW CREATIONS, OLD THINGS HAVE PASSED AWAY, ALL THINGS HAVE BECOME NEW 2 COR 5:17

D. CAST DOWN THE OLD IMAGES!! IN A CHALLENGE HAVE YOUR MIND STRONGLY REPLAYING THE BOLD, RIGHTEOUS, OVERCOMING NATURE YOU HAVE BEEN MADE.

TG-46

TRUTH GEM

LEARN FROM THE CLEAR; CLEARLY DO

A. JOSH. 11:15 a AS THE LORD COMMANDED MOSES HIS SERVANT, SO MOSES COMMANDED JOSHUA, AND SO JOSHUA DID.

B. WHEN HIRED, THE FIRST THING DONE USUALLY IS TRAINING.

C. THERE IS A MAJOR NEED FOR PEOPLE TO BE TAUGHT CLEARLY FROM THOSE WHO SEE CLEARLY AND FOR THE TAUGHT ONES TO FAITHFULLY DO IT IN TUNE AND IN FOCUS.

D. ROM 13:10 a LOVE DOES NO HARM...
(HARM GENERALLY COMES FROM NOT DOING A JOB WELL, THE WAY YOU HAVE BEEN TRAINED)

E. LIFE IS NOT ALL SUBJECTIVE WORD PROBLEMS. MUCH OF IT IS LEARNING FROM THOSE WHO SEE CLEARLY AND DOING IT.

F. SEEK THE CLEAR ONES AND LEARN.

TRUTH GEM

BE STRONG AND OF GOOD COURAGE X ≈ 4

A. "...be strong and of good courage" (Josh 1:6,9,18, ≈7)

B. There are things that some view as very hard to do, that with God being with you, you can do, but you have to be <u>STRONG</u> <u>AND</u> <u>OF</u> <u>GOOD</u> <u>COURAGE</u>!

C. You do not look at the opponent, but <u>begin</u>, doing the best you can do, not relenting on what you are in the will of God for.

D. What is <u>a</u> way to get strength & Courage? You can be commanded to have it!!!

So I command you: BE STRONG AND OF GOOD COURAGE!

NOT DIS-<u>COURAGED</u>
BUT EN-<u>COURAGED</u>

TG-48

TRUTH GEM

ENJOY BEING A STUD(Y)ENT

A. PLP. 4:11 ... FOR I HAVE LEARNED IN WHATEVER STATE I AM, TO BE CONTENT

B. BE CONTENT IN YOUR UPWARD GROWING FLOW (LIKE A 5TH GRADER CONTENT IN THE 5TH GRADE, BUT HEADING FOR 6TH GRADE). THIS IS NOT SETTLING.

C. BE WHAT YOU ARE WHEN YOU ARE IT

D. NOW IS THE SEASON OF YOUR LIFE TO SPEND A BIG PART (NOT ALL) OF YOUR TIME STUDYING.

E. GUARD YOUR TIME. GUARD YOUR JOY

F. THIS IS THE REAL WORLD NOW,

TG-49

TRUTH GEM

HIGHER NATURE EDUCATION

A. JOHN 8:23 AND HE (JESUS) SAID TO THEM, "YOU ARE FROM BENEATH; I AM FROM ABOVE. YOU ARE OF THIS WORLD; I AM NOT OF THIS WORLD.

B. JESUS WAS NOT SPEAKING ABOUT DIRECTIONALLY UP OR DOWN, BUT ABOUT NATURES.

C. MANY EDUCATIONAL IDEAS COME FROM THE REALM OF MAN'S WISDOM AND APPEAL TO THE LOWER NATURE (EXCUSES, WHY YOU CAN'T LEARN, BLAMING OTHERS, LOWERED EXPECTATIONS)

D. TRUE HIGHER EDUCATION APPEALS TO THE HIGHER NATURE WAYS OF THE MOST HIGH. (FAITHFULNESS, DILIGENCE, WISDOM, NO EXCUSES, SUCCESS, VICTORY, OVERCOMING, COURAGE, CHALLENGE, DOMINION OF MATERIAL TO BE LEARNED, WISDOM IN STUDY HABITS)

DISCOURAGEMENT IS <u>NOT</u> AN OPTION

A. BE STRONG AND OF GOOD COURAGE Josh 1:6
" " "
" " "
BE STRONG AND VERY COURAGEOUS Josh 1:7

DIS-COURAGE: TO REMOVE, TAKE AWAY COURAGE
EN-COURAGE: TO INCREASE COURAGE Belinda French

B. IF YOU <u>KNOW</u> AND <u>BELIEVE</u> THE RIGHT THINGS AND <u>DO NOT WAVER</u>, THERE ARE <u>NO GROUNDS FOR DISCOURAGEMENT</u>!

C. WHEN YOU KNOW YOU HAVE
1. IRREVOCABLE ACCESS TO WISDOM
2. IRREVOCABLE ACCESS TO GRACE
3. IRREVOCABLE RIGHT STANDING WITH GOD
4. IRREVOCABLE FORGIVENESS ACCESS

THERE ARE NO GROUNDS FOR DISCOURAGEMENT!

D. DISCOURAGEMENT SLOWS DOWN LEARNING

E. NOW THANKS BE TO GOD
WHO ALWAYS LEADS US IN TRIUMPH
IN CHRIST 2 COR 2:14

F. IS 42:4A HE WILL NOT FAIL NOR BE DISCOURAGED TILL HE HAS ESTABLISHED JUSTICE IN THE EARTH

GOD-PACED LEARNING
NOT SELF-PACED LEARNING

A. (Mt 16:24) IF ANYONE DESIRES TO COME AFTER ME, LET HIM DENY HIMSELF, AND TAKE UP HIS CROSS AND FOLLOW ME.

B. SELF WILL WANT TO STOP AND HAVE PIZZA

C. SELF WILL DECEIVE ITSELF DUE TO HUNGER FOR SELF-ESTEEM

IN ONE STUDY THE U.S. CAME IN LAST IN MATH SCORES, BUT FIRST IN HOW THEY FELT ABOUT MATH

D. HUNGER TO BE GOD-ESTEEMED, GOD-PACED EMPOWERED BY GRACE.

GRACE-BOOSTED CARRYING YOUR OWN LOAD
VS.
PARALYZED DEPENDENCE

A. JN 5:7-8 Parts SIR, I HAVE NO ONE TO PUT ME IN THE POOL... JESUS SAID TO HIM, RISE TAKE UP YOUR BED AND WALK.

B. (K. HAGIN) SOME CONSIDER THEMSELVES HELPLESS DEPENDENT ON OTHER PEOPLE AND DO NOT REALIZE THERE IS TRANSFORMATIONAL WORD THAT CAN TRANSFORM THEM TO WHERE THEY CAN DO IT. THEY THINK THEY CANNOT LEARN WITHOUT AN INDIVIDUAL TUTOR OR GROUP WORK. WHAT IS NEEDED IS TO HEAR, BELIEVE AND FOLLOW THE RIGHT WISDOM WORD AND EMPOWERED BY GRACE THEY DO IT THEMSELF.

C. GAL. 6:2,5 BEAR ONE ANOTHER'S BURDENS... EACH ONE SHALL BEAR HIS OWN LOAD.

GIVE THE RIGHT WISDOM WORD THAT FREES OF HARMING DEPENDENCE, WHEN YOU ARE TO DO IT GRACE BOOSTED, NOT DEPENDENT ON OTHERS,

THEN THERE IS WORK YOU ARE MADE TO DO AS A SOURCE, NOT A DRAIN; DO IT HELPING OTHERS.